

T H E  
PHILOSOPHICAL and MATHEMATICAL  
**COMMENTARIES OF PROCLUS,**  
ON  
THE FIRST BOOK OF EUCLID'S ELEMENTS.

TO WHICH ARE ADDED,  
A History of the Restoration of PLATONIC THEOLOGY,  
BY THE LATTER PLATONISTS:  
And a Translation from the Greek of  
PROCLUS'S THEOLOGICAL ELEMENTS.

IN TWO VOLUMES.



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VOL I.

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LONDON, PRINTED FOR THE AUTHOR:

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LEIGH and CO. G. NICOL; R. FAULDER; and T. and J. EGERTON. 1792.

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*Extracts from Curiosities of Literature.* Second Edition.

Printed for Murray. Page 385.

MR. T. TAYLOR, the Platonic Philosopher  
and *the modern Plethon*, consonant to that philosophy,  
professes Polytheism.\*

\* The Grecian literature of this writer will now prove of real utility ;  
and the graces and the sublimities of PLATO will soon be familiarised  
to the English reader, by a hand that I am persuaded will not appear  
inferior to his great original. Let me also be permitted to recommend  
his version of PLOTINUS ON THE BEAUTIFUL.

The Reader is requested to correct the following Errors.

Page 4. of the Dissertation, Vol. I. line 8. for *admitted*, read *omitted*. Page 16. line 8. for *from*, read *form*. Page 51. Vol. I. of the Commentaries, line 16. instead of *They are surely not the, &c.* read *For surely it cannot be said that there are, &c.* Line 17. for *but we*, read *but that we*. And line 19. for *is by much prior to*, read *is by a much greater priority*.

Vol. II. page 18. line 26. for *and one is*, read *and one part is*. And line 27. for *another*, read *the other*. Page 114. line 13. for *The angle*, read *Let the angle*; and instead of *is bisected*, in the same line, read *be bisected*. Page 411. line 2. for *is filled with intellect*, read *fills intellect*. And line 3. for *it also participates*, read *also it participates*.

**T O**

**T H E      S A C R E D**

**M A J E S T Y**

**O F**

**T R U T H.**



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## P R E F A C E.

THE design of the present work is to bring us acquainted with the nature and end of Mathematics in general, and of Geometry in particular: and in the execution of this design our Author has displayed an uncommon elegance of composition, and a most valuable store of recondite learning. He is not content with every where unfolding the full, and most accurate meaning of Euclid; but he continually rises in his discourse, and leads us into the depths of the Pythagoric and Platonic philosophy. We are surprised to find an use in Geometry, which at present it is by no means suspected to afford. For who would conceive that it is the genuine passage to true theology, and the vestibule of divinity? This, indeed, is by no means the case when it is studied for lucre, and applied to mechanical purposes; for then the soul is neither elevated nor enlightened; but degraded and filled with material darkness. Hence these Commentaries are alone valuable to the liberal part of mankind, who look beyond *sense* for certainty; and who prefer things desirable for their own sakes, before such as minister to the necessities of life.

\* B

THE

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THE translation of this work is attended with great difficulty and labour; not only from the sense of the philosopher, which is always profound, and frequently obscure, but from the great incorrectness of the Greek edition, in which, exclusive of numberless typographical errors, entire sentences, essential to the connection, are frequently omitted; and in one place two pages of the Latin translation are wanting in the original, as will be shewn in our following notes. Indeed, the Latin translation of Francis Barocius the Venetian, (Patav. 1560.) which was made from a variety of manuscripts, is inconceivably valuable; for the diagrams, so necessary to a work of this kind, but which are omitted in the Greek, are here inserted; and the version is every where faithful, and sufficiently perspicuous to those who are conversant in the ancient philosophy. Barocius justly cautions the reader not to compare his version with the printed Greek, which he observes is rather lacerated than printed; as indeed, without his translation, it is impossible for any one to read the half of this invaluable work, even though he should be as perfect in Greek as in his native tongue. If I had not, therefore, fortunately acquired this translation, which is at present very rare, I would have by no means engaged in this arduous undertaking. Barocius, indeed, gives evident proofs of his possessing the philosophical genius, by the excellence of his translation, and his preface to the reader; and it is  
greatly

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greatly to be lamented that he did not adorn his version with explanatory notes, which this profound work frequently demands, and which he was doubtless well qualified to accomplish. This defect I have endeavoured, as far as I was able, to supply ; and at the same time have been cautious neither to weary the reader by prolixity, nor by too much brevity to leave him destitute of proper information. In the distribution of the first book of this work into chapters, I have followed the order of Barocius, because it is natural and obvious ; and must beg leave to solicit the reader's indulgence for using the words *partible* and *impartible*, differently from their common signification. These words I have generally employed to express the meaning of *μεριστος* \*, and *αμεριστος* † in the Greek, as I do not conceive that the words *divisible* and *indivisible* always convey their full signification. I have likewise used *quadrangle* instead of square, and *quinquangle* for the word pentagon. For if *τριγωνος* be rendered *triangle*, why should not *τετραγωνος* be rendered *quadrangle*? And, as Barocius observes, why, for a similar reason, should not *πενταγωνος* and *εξαγωνος* be rendered *quinquangle* and *sexangle*; and so of the rest? Uniformity is always desirable when it can be obtained; and is no where so necessary as in scientifical disquisitions.

\* i. e. Capable of parts.

† i. e. Not capable of parts.

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IT is likewise necessary to inform the reader, that though I have always endeavoured to give the faithful meaning of my Author, yet I have occasionally paraphrased his sense, when most obscure, and added such elucidations of my own, as I either thought necessary to the full comprehension of his matter; or which were naturally excited by the fire and spirit of the Original. If it shall appear that I have succeeded in the execution of this work, and rendered it intelligible to the lovers of truth, I shall rejoice in my success, and consider my labours sufficiently rewarded. The applause of the multitude I am neither likely nor desirous to gain; but I am anxious to procure the approbation of the discerning few, who know that the age of philosophy is past; and who esteem the works of her ancient heroes as the most precious treasures which have escaped the ravages of time.

TIME, indeed, is like a deep and rapid river; whatever is trifling and light, is precipitately borne on its surface, and what is valuable and weighty, sinks to its bottom. Hence; the superficial observer collects nothing more than the rubbish, which it is forever devolving into the abyss of oblivion; while the profound and contemplative genius explores the depths of the stream, and accounts himself happy if he can gather any of the pearl which its bottom contains. Thus the discoveries of experimental philosophy, float

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float like straws on the surface, while the wisdom of Pythagoras and Plato lies concealed in the depths of the river. I am well aware it will be said, that the reverse of this similitude is true; that the modern philosophy is the pearl, and the ancient the stubble; and that the former will be celebrated by posterity, and increase in reputation when the latter shall scarcely be known. But let us attentively examine the truth of this assertion, and shut our ears to the unsubstantial echoes of popular applause. Is it reasonable to suppose that men of such exalted abilities, as the Pythagoric and Platonic philosophers possessed, even in the estimation of their opponents, accompanied with the greatest advantages of birth and fortune, and the most unwearyed attention, have discovered nothing valuable, and have left nothing behind them, but jargon and reveries? Is it to be supposed, that in an age when philosophy was almost adored; when it was esteemed by kings, cultivated by noblemen, and even reverenced by the vulgar; when empire was relinquished for its pursuit, and every danger encountered for its possession: is it to be supposed, that nothing but delusion was the offspring of so glorious a period, and nothing but folly the reward of such generous endurance? Or shall we say, that the discovery of truth was reserved for the age of experiment; and that she is alone to be apprehended in the infinite labyrinth of particulars? That she is to be investigated with the corporeal senses

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senses, and not with the powers of intellect; and that the crucible, the alembic, and the air-pump, are the only mediums of detection? If this be the case, truth is material, and may be calcined, distilled, and rarefied, like any other corporeal substance. It is no longer eternal and immutable, but perishable and fluctuating; the phantastic subject of sensible inspection, and not the steady and real object of the permanent energies of science. Shall we call this the age of philosophy, in which talents are prostituted for sustenance, and learning submits to the impudence of wealth? Shall we say that we have strengthened the cause of philosophy, by demolishing her schools; and increased her independence, by enlarging the empire of commerce? Where shall we find the man, who is at present reverenced for the profession of teaching speculative truth, or indeed who teaches it at all? Or should we chance to meet with such an obsolete character, shall we find him supported by the profession? It is a well known fact, that men formerly lived in the highest esteem by its propagation: it is equally as notorious, that a man at present would starve by such an attempt. Dare we assert, that the reason of this difference must be ascribed to the greater liberality, and more philosophical spirit of the present age? Shall we not rather say, that the period, in which these ancient heroes lived, was the golden age of philosophy;—a period so different from the present, as to appear fabulous on the comparison? For mark the dif

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distinguishing characteristics of our inferiority. The great object of ancient philosophy, was an accurate speculation of principles and causes: but that of the modern, is a confused investigation of effects. And if pursuits participate of the nature of their subjects, and causes are more noble than effects, the ancient philosophy must undoubtedly be more elevated than the modern. Again, the object of the Pythagorean and Platonic philosophy was to make its possessors wise and virtuous; and to elevate them above the common frailties and imperfections of degraded humanity; and this end was happily accomplished in its votaries, as their lives abundantly evince: but the object of modern philosophy, is a promotion of the conveniences and refinements of life, by enlarging the boundaries of traffic; and the Mathematical Sciences are studied solely with a view to this enlargement. The design of the ancient philosophy was to remove the causes of wonder, by contemplating effects in their causes: the grand object of the modern, is to increase admiration, by attempting to investigate causes through the infinity of particular effects. So that philosophy, as Mr. Harris justly observes, now ends where it formerly began. For either there is no such thing as science, or if its existence be admitted, it can never be obtained by experimental enquiries; as these must be liable to all the inaccuracy and imperfection of their material subjects.

IN

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IN short, the philosophy of Pythagoras and Plato will be found, when impartially considered, to contain every thing which can enlighten the mind, improve the morals, and exalt the character of man. It is built on the steady basis of truth, and will survive the wreck of ages. Its foundation is deep, and its summit reaches the heavens. It is a mighty rock, which modern systems *may* assail, like a raging sea ; but, like stormy waves, they will only be broken about its impenetrable sides. To war against wisdom is folly ; for opposition in this case is the destruction of its author. The moderns may, indeed, expect, because their merit is raised by the present age, above that of the ancients, to appear as giants in the eyes of posterity ; but they will only verify the elegant observation of the poet \*, that

Pygmies are Pygmies still, though perch'd on Alps,  
And Pyramids are Pyramids in vales.

\* Dr. Young, in his Night Thoughts.

A D I S-

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A

**D I S S E R T A T I O N**

ON THE

**PLATONIC DOCTRINE OF IDEAS, &c.**

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S E C T I O N I.

THE Platonic doctrine of Ideas has been, in all ages, the derision of the vulgar, and the admiration of the wise. Indeed, if we consider that ideas are the most sublime objects of speculation, and that their nature is no less bright in itself, than difficult to investigate, this opposition in the conduct of mankind will be natural and necessary ; for, from our connection with a material nature, our intellectual eye, previous to the irradiations of science, is as ill adapted to objects the most splendid of all, "as the eyes of bats to the light of day \*." And yet (as I presume, it will appear from the following discourse), unless the existence of these lucid beings is admitted, there can be no such thing as science ; nor, indeed, any genuine knowledge at all. Hence, an enquiry concerning their nature and reality, is highly proper, as an introduction to the ensuing Commentaries, in which they are considered as the stable pillars of all truth, and the prolific principles of the universe.

\* See book the second, of Aristotle's Metaphysics.

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But previous to this enquiry, it is proper to observe, that Plato was not the inventor, though he was a strenuous asserter, of ideas; for, in the *Sophista* he affirms, that ideas were the discovery of men who excelled in wisdom and piety, and who contended for an invisible essence. Diogenes Laertius, indeed, asserts, that Plato received the doctrine of ideas from Epicharmus. But Epicharmus was not their inventor, because Pythagoras, and others of still higher antiquity, were well acquainted with ideas; so that it may be affirmed, with much greater truth, that Plato was instructed in their nature by Philolaus his preceptor, and the disciple of Pythagoras. For Pythagoras, after his mysterious manner, signified ideas by numbers. But, prior to Pythagoras, Orpheus was an asserter of ideas, and called Jupiter, or the *demiurgus* of the world, "the idea of all things." And, according to Syrianus, the mundane sphere, celebrated by Empedocles, is no other than the ideal world; so that the doctrine of ideas is as ancient as that of wisdom itself.

But to begin with our enquiry: in the first place, without universals there can be no science; for the flowing and perishing nature of particulars is perfectly foreign from that stability and duration which is requisite to objects of invariable truth. Neither is it possible, that infinite individuals can exist without the subsistence of one cause endued with infinite power; for all multitude must necessarily originate from one, and must resemble its cause in as great a degree of perfection as its nature can admit; by a diffused infinity, shadowing forth that infinite power which subsists in indivisible union. Hence, if this be the case, and if infinite men, horses, and a multitude of other univocals, are produced in an infinite time, an unity of infinite power must be the source of each, according to which they are generated:

generated in a terminated manner to infinity in the universe. Again, all animals are transmuted from that which is in capacity (i. e. seed), into energy. But if this be true, it is requisite there should be some animal in the universe, subsisting in ever-vital energy, which may call forth that which is concealed in dormant capacity, into perfect actuality. Thirdly, the celestial orbs would not perpetually revolve in the same spaces, and after the same manner, unless one and the same universal number, or idea, ruled in each. So, likewise, there is a natural number in every animal; or those of the same species, would not always (when perfect) be distinguished with the same invariable organs; nor would they be subject to puberty and old age, at the same time, unless they were detained by the same measure of nature. Besides, the participation of universals, is evident in every sensible object. Thus, the rational nature is united with every individual man. Thus, animal subsists in a lion and a horse, in a man and a dog. And thus the pentad, or number five, is participated in the five fingers, and the duad in the nostrils, eyes, hands, and feet. But since these do not subsist without a cause, but are perfected by certain determinate natures, it is necessary there should be an universal animal, in the whole of nature, separate from sensibles, by means of which this sensible animal is generated. And that there should subsist in nature a pentad, through which the hands are always adorned with that number of extremities; and a duad, from which the two eyes and nostrils are derived. But if nature does not possess these numbers from herself, as she is not the first cause of all, but derives them from another cause, in the same manner as matter from nature, it is necessary there should be universals and numbers prior to nature, subsisting in far greater purity and perfection.

2. Again, we may demonstrate the existence of ideas as follows: if the Deity, in fabricating the universe, operated essentially (and there is no other way in which we can conceive him to operate), he must fabricate the universe, an image of himself. But, if this be the case, he contains in himself, in the manner of an exemplar, the causes of the universe; and these causes are no other than ideas. Besides, this consideration is not to be admitted, that the perfect must necessarily antecede and preside over the imperfect; unity over multitude; the impenetrable over the penetrable; and that which is perpetually the same, over that which admits of variation and change. From whence it may be inferred, that things do not originate from baser natures, but that their gradual processions end in these; and that they begin from the most perfect, best, and most beautiful natures. But let us pursue this reasoning more minutely, as it affords the strongest arguments for the existence of ideas.

When the Deity fabricated the various species of animals, and bestowed on them the different senses, it was doubtless with a view to the benefit of their possessors, as he foresaw, that without these, the animal could neither provide for its own support, nor defend itself from surrounding dangers. But may we not enquire from whence this previous perception originated? For it is not to be supposed, that he first made animals destitute of senses, and so, being admonished by their sudden destruction, afterwards assigned them to their nature. Shall we say, this fore-knowledge was the result of a reasoning process? But then, we again ask, What were the principles of this ratiocination? For if they originated from other reasonings, it is necessary, at length, to arrive at something prior to these discursive operations, on which they ultimately depend; since all reasoning must be.

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be founded on indemonstrable principles. Was sense, then, or intellect the principle of this previous perception? But sense, in the present instance, had not then a being, for it could not exist prior to the animal nature: it was, therefore, intellect. But if intellect be the repository of certain propositions, and the conclusion be science, it must follow, that there could not then be a consultation of any thing sensible. For the principle and the conclusion must both depend on something intelligible. Besides, may we not ask, how such a habit of thought arose before the existence of a sensible nature! It is absurd in the extreme, to say from chance, and to resolve it into a sudden volition of the Deity, is an assertion that may, indeed, satisfy vulgar minds, but can by no means quiet the restless spirit of philosophical investigation. Since, to suppose the cause of the universe, actuated by sudden volitions, is to place him on a level with the vilest natures, and subject him to the irrational impulses of the brute. Hence we infer that the formation of animals, and by the same arguments of the world, was not the result of any reasoning process. For, indeed, argument and foreknowledge cannot with propriety be attributed to the Deity; but when they are ascribed to him, we must consider it as nothing more than an indication of his constituting particulars, in a manner somewhat similar to the providence of a wise man, in inferior concerns. For, in subordinate natures, whose operations cannot take effect prior to enquiry, reason is necessary, on account of the inferiority of that power which precedes the reasoning energy. In like manner, foreknowledge is necessary, because a power is wanting to its possessor, which might render him superior to its use. For foreknowledge is directed to this end, that one particular circumstance may take place in preference to another. But if it be requisite

that

that every energy in the Deity should be void of defect, and if it is not lawful that any thing should be present with him, which is not total and universal, it is necessary that all things should be contained in every thing essential to the nature of the Deity. Hence, since even futurity is with him present, there is nothing in him posterior; but what is present in him becomes posterior, by its participation in another. If then futurity be present with the Deity, it is necessary it should be so present, as if foreknown in a posterior nature ; that is, in such a manner that nothing may be wanting to any being ; and that is, lastly, so that every thing may be complete.

Besides, reasoning cannot, by any means, belong to an eternal essence like the deity ; for if this be admitted, he must be forgetful of his former operations. And if, in consequence of reasoning, he produces more perfect natures afterwards, his works could not be perfectly beautiful before : but if they were beautiful before, they must be co-existent with their cause, i. e. they must be eternally beautiful, antecedent to the reasoning energy. Again, if we suppose the supreme intellect, the demiurgus of the world, to operate by enquiry, his energy could not be spontaneous, and truly his own ; but his essence would be similar to that of the artificer, who does not derive his productions from himself, but procures them as something adventitious by learning and enquiry. But if the universe was not formed by deliberation, it must be co-existent with its cause, and reside in his essence ; for if it be not co-existent there must have been some particular time, in which its artificer determined on its production ; and this determination must have been the result of a reasoning process, concluding that it would not be good to produce it before that particular time, (from whence, by the way, we infer the eternity

eternity of the world.) And if the universe be co-existent with its author, it must perpetually emanate from his nature, and be dependent on it, like the shadow on its forming substance. But in this case, its archetype must be contained in the essence of its author; for every cause is that primarily, which its effect is secondarily. And hence we infer, that if the sensible universe be replete with forms of every kind, the exemplars of those forms, must subsist in immaterial perfection, in the artificer of the world.

If this sensible world, then, be formed according to the exemplar of that which is intelligible; may we not say, with the great Plotinus, that it is requisite universal animal should there primarily subsist in perfect vital energy, containing all things in its omniform essence. "Hence (says he \*) the heavens are there a divine animal, replete with ideal stars. Earth too does not there subsist solitary, but is much more vital than this corporeal earth, for it is full of intellectual life. The sea too is there, and all water subsisting in life, and an ever-abiding stream. For how is it possible that any thing not vital, can be the progeny of ~~life~~ itself? He, therefore, who enquires from whence animals originate in the intelligible world, might as well enquire from whence all life, and soul, and universal intellect, arose. For here there is nothing indigent nor defective, but every thing is perfect and exuberant. Here they all flow from one fountain, not as from a certain spirit, or heat, but as if from an universal quality, possessing and preserving in itself, all qualities; such as sweetness, accompanied with fragrance of smell, the vigour of wine, and the strength of all juices, bright colours, and whatever is perceived by the taste."

\* Ennead vi. lib. vii.

3.. Such.

3. Such then are the arguments which the Platonic philosophy affords in defence of ideas; the existence of which was so evident to Plato, that, in the *Sophista*, he compares those who oppose the friends of ideas to the giants of old, warring, as it were, on celestial souls, and such as are engaged in sublime investigations. Let us now consider to what universals these lucid beings are confined; since, according to the Pythagoreans and Platonists, there are not ideas of all universal conceptions. "For, in the first place (says Syrianus \*), there are no ideas of things evil and base, because these subsist in nature rather by a privation and absence of ideas. And, on this account, they are said to exist contrary to nature. Nor, secondly, of negations, for these are destructive of the bound and limitation which is attributed to every thing from the unifying and comprehending nature of ideas; and hence, separation is rather the result of material infinity than of that which is formal or ideal. Nor again, are there any ideas of things which at different times receive a variety of conditions. For these participate of transmutation from a moveable cause, but not from the immovable and stable illustration of ideas. Nor again of parts, such as the hand, head, fingers, and the like. For the causes of things existing entire, produce whole species and forms; not divided about the parts of these, like the reasons of nature. But neither did these wise men place in intellect the determinate causes of accidents in bodies, such as sweetness and whiteness. For they considered that natural reasons were sufficient for the production of accidents. Nor again, of composites, as of a wise man. For

\* In his commentary on the 2d, 12th, and 13th books of Aristotle's Metaphysics, page 60. A Latin translation only of this invaluable work is extant; but I have fortunately a copy in my possession, with the version every where corrected by the learned Thomas Gale, and with large extracts from the Greek.

since

Since ideas are simple, they preside over the simple essence of every thing. But the composition and division of things is the business of our intellect; ideas, at the same time, and that intellection which is co-ordinate to ideas, being exempt from all these, on account of superlative simplicity. Neither, therefore, must we establish ideas of things generated from dissimilars, such as mules; nor of fruit produced by engrafting from different trees. For all these have a posterior and adventitious generation, and are not the work of nature alone, nor of nature proceeding according to her own reasons, but, as it were, compelled to labour contrary to her own determinations. Hence it is manifest, that all art, which imitates nature, and alone ministers to the use of mortal life, is separated from the cause of ideas. But neither are the works which, depending on the purpose of the soul, are perfected by a concourse of many causes, and which we are accustomed to call the operations of fortune, to be conjoined to the cause of ideas. For things which are there perfected, are eternal, and subsist perpetually the same, free from the nature of contingent events. It remains, therefore, that ideas must be confined to universal and perfect essences, and to whatever confers to their natural disposition; as for instance, to man, and every thing perfective of man, such as wisdom and virtue. For ideas existing as the generative and energetic causes of the perfection of every thing, distribute being to essences, and convert them to the inexhaustible plenitude of their own omniform natures."

4. But let us now consider the nature of numbers; for as every form is a number, according to the Pythagoreans\*, a speculation of this kind must afford no small light to the

\* See Proclus on Plato's Theology, p. 226.

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arduous investigation of ideas. Will it not, therefore, be proper, in the first place, to enquire, with the great Plotinus \*, whether multitude is not a departure and distance from *one*, so that infinity itself is a separation from unity in the extreme, because it is no other than innumerable multitude; that on this account it becomes evil; and that we contract a similar nature when departing from intellectual unity, we are divided by sensible multitude? For a being then properly becomes many, when no longer able to remain collected in itself, the same, it is diffused abroad, and thus, being dispersed, is variously extended; so that when, by diffusion, it is absolutely deprived of unity, it becomes perfect multitude, destitute of that universal cement, which unites one part with another. But whenever the conciliating one is present, then that which was scattered and diffused, becoming permanent by its bounding power, passes into magnitude. But if any one should deny the subsistence of unity, asserting that one is nowhere to be found, which is not some particular one; and should hence affirm, that what is called one abstractedly, is only a certain affection of the soul towards any being; we ask, what prohibits the appellation of essence, from being nothing more than an affection of the soul, and consequently the existence of *being*, a delusion? For we predicate unity of particulars with as great propriety as *being*. I am well aware, that philosophers of the present day will answer, that we have an evident proof of the reality of *being*, from its agitating the soul, and becoming apparent in the phantasy: to which we reply, that in like manner, the soul is agitated, and the imagination influenced about the *one*. For every individual as much excites the perception of *one*, as of *being*.

\* Ennead vi. lib. 6.

Besides,

X

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Besides, it is necessary to enquire whether we behold this passion and conception of the soul, as one or multitude. And again, when we say not one, we do not then possess one from the thing itself; for we say that one is not contained in that individual. And hence we must possess one in our own nature, and this must reside in the soul, separate from that which is denominated some particular one. But here it may be objected, that the one we possess is received from externals, and is nothing more than a conception of the mind, produced by the thing itself. For it will be said, that as multitude is nothing besides a number of individuals, which are called many, so one is nothing besides one thing; and is formed by thought separating that one particular from others. To this we reply as follows:

How can it be consonant to reason to suppose that the conception of *one* arises from the sensation of some one particular subject? For one particular man, who is discerned by sense, is by no means the same with one itself, since, if this were the case, thought could never predicate one of that which is not a man. Besides, as cogitation, on beholding the different positions of things, affirms that this is here or there, so when it perceives an individual, it pronounces one; for that passion is not vain, nor does it assert one of a non-entity. Nor must we think it predicates one, because this individual is different from another; for when cogitation affirms such a thing is this, and not another, it declares, in the mean time, that the other is *one*. Likewise when it affirms that any thing is this *alone*, it then declares, that what is alone is one: on which account, it predicates one, prior to alone. Besides, if there be multitude, it is necessary that one should antecede; since when it predicates *many*, it pronounces more than one. And

when it affirms that an army contains a multitude of men, it conceives the soldiers reduced to one order.

For thought, indeed, does not permit multitude to remain perfect multitude, destitute of the conciliating power of unity; in which very circumstance, the subsistence of one is evinced; for acutely and swiftly perceiving the one which results from order, it reduces the nature of the many into one. Besides, we affirm that a house and an army are each one, but that a house is more one than an army, on account of the continuity of its parts. If therefore, one is contained more in that which is continued than in that which is discrete, and still more in what is perfectly indivisible, it is evident that *the one* is a certain nature, and has a real being. For it is impossible that the more and the less should take place among things which have no subsistence. If then it be not possible to understand any thing without one or two, or some other number, it is by no means proper to deny existence to that, without which we cannot comprehend the existence or properties of any being: but it is requisite *that* nature should antecede all discourse, and intelligence, which is every where necessary to their existence.

Again, if unity has no real subsistence, and is nothing more than a name or conception of the mind, it may be destroyed without the destruction of its subject. The unity, therefore, of a house may be taken away, without the ruin of a house. But if a house is nothing more than certain materials, reduced into one form, this is impossible: And, on the contrary, the alteration of that subject, of which unity is predicated, can make no real alteration in unity (on this hypothesis) any more than the death of a man can affect his name. When, therefore, a body, of which *one* was predicated, is divided into a multitude of parts,



parts, there is no real alteration made in the unity of the body, because unity is nothing more than a name.

It was in consequence of this reasoning, and perceiving that unity was participated by every being, that the Pythagoreans placed a super-essential one at the top of the universe, intelligibly abstracted from all beings in simplicity and excellence of nature. For they considered, that unless there was a self-subsisting one in all things, there could neither be universals nor particulars. Not the first, because they are by nature *one* and *many*. But it is requisite that the one itself, should preside over that which is not one alone. Nor again, the second, because they are many and one, (that is, they participate more of multitude than unity, and their nature is determined more by the *many* than the *one*.) And because of things in participation, unless an unparticipated one is added, there can be no cause of union to beings; in the same manner as the cause of essence to beings, is taken away by those who deny that Being itself, is the principle of all essence. For as the good itself, is the one principle of good to the universe, and is nothing besides good; and as a self-motive nature, which is nothing besides self-motion, is the cause of motion to all things; so all things proceed from being itself, and all united natures receive their union from *the one*, abstracted from all things.

Hence (such is the absolute dominion of unity), continued quantities would have no existence without its participation; for when they are divided, so far as they lose unity, they change their being into some other form. Hence, the bodies of plants or animals, which are each of them one, when they fly from unity, and are dissipated into multitude, immediately lose the essence they formerly possessed, and become something else; which new state of being.

being they likewise possess so far as they are one. Add too, that health then flourishes in the corporeal frame, when the body is conciliated into one; then beauty flourishes, when the power of one connects the members into proportion and consent; and then virtue reigns in the soul, when the soul is reduced into one similitude with that which is divine.

5. But let us now investigate the nature of numbers. All number, according to the Pythagoreans, originates from unity and the indefinite duad; the first having the relation of form, and the second, that of matter to all the orders of numbers. But they likewise divided number into two kinds, essential and monadic. The essential number they considered as first subsisting in the intelligible world, together with being, and from thence distributed into all the various gradations of forms. But the monadic, or that which is composed from certain units, they justly considered as nothing more than the image of essential number. And with respect to the numbers which the human soul participates, these from its imperfect condition have a middle subsistence; i. e. they exist in a vital, gnostic, and speculative, but not in an operative manner. Hence, when receiving one thing with another, we affirm, that they are two, as a dog and a man, or two men; or when we compute more than two, as ten, and say that there is a decad of men, this number is not essential to the two or ten individuals, nor is it to be conceived as subsisting in sensible natures; but it is purely quantity. But when we distribute this ten, into units, we produce the principle of quantity, and generate a subject in opinion \*, capable of parti-

\* In giving monadic number a subsistence in opinion, I have followed the distribution of Proclus, in the conclusion of his comment on a point; and, I think, not without sufficient reason. For since monadic numbers are more immaterial than geometrical lines and figures, they

participating the essential decad of our soul. But when, considering man in himself, we affirm that he is a certain number, as the duad, composed of animal and rational, we do not observe one mode in this predication; but so far as by a discursive operation of the soul, we numerate, we effect a particular quantum; but so far as the subjects are two, and at the same time both one (since one fills the essence of both, and in both unity is contained), we pronounce another, and an essential number: and this duad is not of a posterior origin, nor alone signifies a certain quantity, external to the subject, but a duad subsisting in the essence of man, and containing his nature. For here we do not produce a number by a discursive operation, while we pursue essential natures. But when we number any ten things, which are not connected by any conciliating unity, like a choir, or an army, then this decad, which we predicate of the ten particulars, subsists alone in our numerating soul, which renders the ten individuals in opinion, a definite quantum. But in a choir, or an army, essential number is participated exclusive of that which subsists in our soul. And if it be enquired how number subsists in the human soul, we must say, that the soul, by her self-moving energies, procreates number, while she numerates, and by this energy, causes the existence of quantity; in the same manner as in walking, we give rise

they must have a more immaterial subsistence. But as they are correspondent to matter, they cannot reside in the essential reasons of the soul; nor can they subsist in the phantasy, because they are superior to geometrical figures. It remains, therefore, that we must place them between *διάνοια*, or cogitation, and the phantasy; and this middle situation is that of opinion. For cogitation, which Plato defines, in his *Sophista*, to be an inward discourse, without voice, is an energy of the rational soul, extending itself from propositions to conclusions. And, according to Plato, in the same place, opinion is the silent affirmation, or negation of *διάνοια*, or thought. Hence, says he, "opinion is the conclusion of cogitation; but imagination, the mutual mixture of sense and opinion." So that opinion may, with great propriety, be said to contain monadic number, to which it bears the proportion of matter. And hence the reason is obvious, why the Pythagoreans called the duad opinion.

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, to a certain motion. Thus, monadic number, or a collection of units of various kinds, subsists in opinion, in a manner correspondent to that of geometrical figures; and by this means participates the essential number of the soul. For as a triangular figure in the phantasy, is the recipient of a triangular nature, or of triangle itself; so every three units in opinion, receive the essential triad of the soul, and, by this means, ~~from~~<sup>in</sup> a definite quantum.

In short, as in every being we may discern the resemblances of matter and form, so in the pentad, or any other number, the five units, which are the subject of participation, and the quantity of the number, originate from the duad; but the form, that is the pentad itself, from unity. For every form is an unity, which unites its subject quantity, and connects it with its ideal species. It is, therefore, requisite to understand, that the two principles of mathematical numbers are resident in our souls, with which every mathematical number is co-existent; I mean unity, comprehending in itself all the forms of numbers, and which corresponds to unity in intellectual natures; and the duad, endued with a generative power, of a formless nature, and of infinite virtue; and which is called boundless, on account of its being the image of never-failing and intelligible duality. Hence, the unity of the soul, with a never-ceasing energy, continually distinguishes and forms all the orderly processions of her numbers, suffers no vacuum to intervene, and leaves no quantity formless and innumerable. Hence too, no essential number of the soul, as for instance, the pentad, is composed from substance and accident, as a white man; nor from genus and difference, as man from animal and biped; nor again, from five unities mutually touching each other, like a bundle of wood; nor from things mixt, like water and wine,

wine, nor from things subsisting by position, in the manner that stones compose a house; nor lastly, does it subsist like things numerable; for it is not because they are composed from indivisible units, that they possess any thing besides units. For many points are indivisible, yet quantity is not produced on this account; but because they participate of two natures, the one corresponding to matter, and the other to form. Lastly, it is not proper to say, that the number seven (and so of any other number), is composed from the triad and the tetrad; for units, indeed, composed with units, form a subject adapted to the reception of the heptad, or the ideal and essential number seven; but the definite numerical quantity seven, is formed from so many units, and the ideal heptad. Hence, as the soul of the shipwright gives form to the timber, from her inherent art; so the numerative soul, from the unity endued with the relation of a principle which she possesses, gives form and subsistence to all her inherent numbers. But there is this difference between the two, that the shipwright's art is not essential to our nature, and requires manual operation, because it is conversant with sensible matter; but the numerative art is essentially inherent in the soul, and is therefore present with all men, and possesses an intellectual matter, which it easily forms without the assistance of time. And this, perhaps, is what deceives many, who think that the heptad is nothing more than seven units. For the imagination of the vulgar, unless it first perceives a thing destitute of ornament, and afterwards the operations of the adorning artificer supervening its nature; and lastly, beholds the thing perfect, and invested with form, cannot be persuaded that it possesses two natures, the one formless, but the other endued with an energetic and forming power.

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And here it is necessary to observe, that though unity is the form of all arithmetical forms, yet it is chiefly the form of the decad. For what unity is simply to all the series of numbers, that the decad is to the following hundreds, thousands, and millions; from whence, according to a secondary progression, it is denominated unity. As intellect, therefore, is the form of all things, but especially of the soul, so unity, though it is the idea of all numbers, yet especially of the decad. But the reason why the Pythagoreans extended ideal numbers no farther than ten, is because this number is the ultimate perfection of beings, containing all things in its omniform nature. For all proportion subsists within the number ten; the arithmetical in a natural progression of numbers from unity; the geometrical in the numbers 1, 2, 4, and 1, 3, 9, and the harmonical in the numbers 2, 3, 6, and 3, 4, 6. And since the causes of all things are contained in numbers, as far as to the decad \*, it is superfluous to suppose exemplars of the following numbers.

If it should be asked in what manner we must conceive number as subsisting in the intelligible world, we answer, with the great Plotinus, that we must conceive it as subsisting in being itself, with a power of impelling it to the production of multitude. "Hence (says he, Ennead vi. lib. vi.) number is either the essence or the energy of being, and animal itself, and intellect is number. But, perhaps, we must call being, number united (*ἀριθμὸς ηνωμένος*), but beings, number evolved, or unfolded; (*εξεληλεγμένος ἀριθμὸς*); intellect, number moving in itself; (*ἀριθμὸς ἐν ἑαυτῷ κινόμενος*).

\* *Ἄλεων, ἀκαμάτοι, Δικάδας κλίεσσιν μη ἄγνη,  
Αθέτατοι τε θεοὶ καὶ γηγενεῖς ἀνθεώποι.*

Syrian. in Meta. Aristot. p. 113. Gr. ]

i. e. (According to the Pythagoreans) "the immortal gods and earth-born men, call the venerable decad, immutable and unweared."

and

and lastly, animal, number comprehending (*ἀριθμὸς περιέχων.*) It was in consequence of this reasoning, that the Pythagoreans called ideas numbers; because the gradual evolution of these from ineffable unity, produced all the beautiful variety of forms. Their exalted conceptions of numbers, likewise, originated from the same sublime theory. Hence, \* Pythagoras, in the sacred discourse, calls number “*the ruler of forms and ideas.*” But † Philolaus, “*the commanding and self-begotten container of the eternal duration of mundane concerns.*” And ‡ Hippasus, and all those who were called *ἀκερατικοὶ* (or such as were yet under the probation of the quinquennial silence), “*the first exemplar of the mundane fabric, and the judiciary instrument of its artificer.*”

6. And here I cannot but take notice, with regret, of the very unphilosophical mistake committed by that great mathematician Dr. Barrow §: I say, with regret, on account of the extraordinary obligations I am under to his writings, for my proficiency (whatever it may be) in mathematical learning. But respect must yield to the truth. “Unity, says he, is not indivisible. (For how ex. gr. can  $\frac{1}{2}$  added to  $\frac{1}{2}$  be equal to unity, if unity be indivisible and incomposed, and represent a point) but rather only unity is properly divisible, and numbers arise from the division of unity.” Here the Doctor evidently confounds sensible units, which are the subjects of vulgar practical arithmetic, with those units which are the objects of science. Every individual sensible object, is indeed an unit, so far as it

\* Αὐτὸς μὲν Πυθαγόρας ἐν τῷ ιέρῳ λόγῳ διαξήηδην μορφῶν καὶ ιδεῶν κάρτορα τὸν ἀριθμόν θεοὺς εἶναι.

Vid. Syrian. in Arist. Meta. p. 85. Gr.

† Φιλόλαος δέ, τῆς τῶν κοσμικῶν αἰώνιας διαμειᾶς τὴν κρατισίεσσαν καὶ ἀίσθετὴν συνοχὴν εἶναι ἀπειρόνα τον ἀριθμὸν.

Syrian. in eodem loco.

‡ Οἱ δὲ περὶ Ἰππακοῦ ὀκεανοτικοῖ, ἀριθμοῖς εἴποι παράδιγμα πρῶτον κοσμοποιαζον. Καὶ πάλιν κριτικὸν κοσμεργὸν θεῖ βούγανον.

Jamb. in Nicomach. Arith. p. ii.

§ In his Mathematical Lectures, page 48.

participates the connecting and conciliating power of an immaterial *one*: but the unity which stands at the top of speculative arithmetic, is perfectly indivisible, or arithmetic would cease to be a science. The truth of this is evident from Euclid's definition: "Unity (says he) is that according to which each of the things which are, is called one." But if unity be a composite, the definition is false; since a composite, or a certain multitude, can never be the cause of unity, but the contrary. And that this immaterial *one* subsists in sensible natures, has, I hope, been sufficiently proved in the preceding part of this discourse. But the Platonic Theo \* of Smyrna, fully establishes the indivisibility of unity, as follows: "Unity is terminating quantity, the principle and element of numbers, which remains undiminished by the most immense multitude of subtractions, and being deprived of all number, continues firm and fixt, because it is impossible for division to proceed beyond the bound of unity. Thus, if we separate any one corporeal substance into parts, the *one* again becomes *many*; and by subtracting the several parts, we end in one part; and from this remaining part, again divided, arises multitude; and by taking away every part, we again arrive at *one*. So that *one*, considered as *one*, is incapable of diminution, and perfectly indivisible. On the contrary, every number is diminished by division, and is separated into parts less than itself; as the number 6 into 3 and 3, or into 4 and 2, or into 5 and 1. But unity in sensible particulars, if divided, is diminished after the manner of body, and by section is distributed into parts less than itself: but it receives increase after the manner of number; for instead of the one, multitude is produced. In this sense, therefore, is unity indivisible; for nothing is divided into parts greater than

\* In Arithmet. p. 23.

itself.

itself. But that which is cut into parts greater than the whole, and into parts equal to the whole, is divided as number. Thus, for instance, if any one sensible body is divided into six parts, 1, 1, 1, 1, 1, 1, these shall be equal to the whole; but by a section into 4 and 2, it is divided into parts greater than the whole, considered as *one*; for 4 and 2 considered as numbers, exceed unity, and the body was supposed to be one. Unity, therefore, as number is perfectly indivisible. But unity is called by the Greek word *μονας*, only, or alone, either because it remains immoveable, and does not desert itself, nor surpass the bounds of its nature (for it remains the same, however multiplied into itself, through an infinite progression) or because it is placed separate and apart from the multitude of other numbers, it is denominated the *monad*, or *one*."

In consequence of this very mistaken hypothesis, which opposes not only all the wisdom of antiquity, but the sublimest truths, the Doctor asserts, that an arithmetical cypher is the principle of numbers; and that it is analogous to a point in geometry. Just as if a cypher, which is nothing more than a mark expressive by its position with numbers, of a certain quantity, had a real existence, and was productive of number: when, at the same time, any other arbitrary character would serve the same purposes, if applied in a similar manner. It must surely afflict every thinking mind, to see how dreadfully the mechanical system of philosophy, which has been so long in fashion, enslaves and perverts the minds of its votaries; for there cannot, I think, be a more egregious instance of its fatal tendency, than the present, in which *nothing* is considered as the foundation of that noble science, arithmetic; which was deservedly placed by the ancients, in the first rank of the mathematical disciplines. Such a foundation, indeed, *may be*

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be proper to the *mechanical philosophy*; but is very ill adapted to support the solid fabric of the arithmetical science. But let us attend to the arguments of this most learned man, in defence of so strange an assertion, "A cypher, or arithmetical nothing (says he) is really the bound of every number coming between it and the numbers next following, but not as a part. A cypher being added to, or taken from a number, does neither increase nor diminish it; from it is taken the beginning of computation, while itself is not computed; and it bears a manifest relation to the principal properties of a geometrical point." But in what manner are we to conceive the *nothing* which intervenes between any two numbers, to be their term or boundary? For Euclid defines a term to be the extremity of any thing; implying by the extremity, something belonging to that of which it is the bound. But how can a cypher, or *nothing*, in any respect belong to number, or *something*? For if *nothing* be a boundary, merely from its intervention, a point existing between any two disjoined lines, though at the greatest distance from each, must be their common boundary, which is evidently absurd. Besides, what relation does it bear to a point, which is endued with a generative power, by its flux forming the simple extension of a line, and, at the same time, every where limiting its progression, and subsisting in infinite capacity in its every part? Where are the real and divine properties to be found in an arithmetical nothing, which Proclus, in the following Commentaries, exhibits in a point? And how can computation originate from a mere non-entity?

But a little consideration will convince us, that this Saracen, or Indian cypher, is nothing more than an arbitrary character, invented for the purpose of facilitating computation. For, suppose the letter (*a*) to be placed in

its

its stead, and to signify, when connected with the mark for unity, ten, or ten multiplied by one; when connected with the mark for two, ten multiplied by two, and so on. And again, when placed twice after unity, let it express the second power of ten, or one hundred, in this manner,  $\alpha\alpha$ ; when thrice connected, one thousand, or the third power of ten, and so on: shall we say, in consequence of this, that ( $\alpha$ ) is the bound of numbers, and the principle of arithmetic? Or, shall we not rather say, that it is an arbitrary symbol, like any other algebraic character, having no real connection with numbers, and depending, for its existence and application, entirely on the will of its inventor. But this opinion is too absurd to need any farther refutation.

7. It may here, perhaps, be expected, that I should explain how, in the language of Syrianus \*, "divine number proceeds from the immortal retreats of unity, until it arrives at the divine tetrad †;" and that I should unfold the properties of the tetractys, according to the Pythagoreans; but an undertaking of this kind, would not only far exceed the limits of this dissertation, but, perhaps, in the present age, might be justly deemed, by the lovers of wisdom, a prostitution and profanation of the most exalted truths. Enough, I hope, has been said to excite the curiosity, and rouse the attention of the thinking and liberal part of mankind; and those who understand what is here briefly delivered, may apply themselves, with advantage, to Proclus on Plato's Theology, where they will find all the mysteries of numbers unravelled; and to the works of the great Plotinus, who will lead them into the pene-

\* In Aristot. Meta. p. 113. Gr. vel 59. b. Lat.

† For the tetrad contains all numbers within its nature, in the manner of an exemplar; and hence it is, that in monadic numbers, 1, 2, 3, 4, are equal to ten.

tralia of the most recondite wisdom. But, in perusing the works of these great men, the reader must not expect to find the sublimest truths explained in a familiar manner, and adapted, like many modern publications, *to the meanest capacities.* For this, indeed, is impossible to be effected. "Mankind (says Petvin \*), are not to be made any more truly knowing than happy by another's understanding.—There is no man can at once convey light in the higher subjects, to another man's understanding. It must come into the mind from its own motions, within itself: and the grand art of philosophy, *is to set the mind a-going;* and, even when we think nothing of it, to assist it in its labour." After which he observes, that "the ancients never attempt to lead us into knowledge, by a continued chain of reasoning; on the contrary, they write in such a manner, as to force us to think for ourselves." And, previous to this, he remarks, "that there are certain truths acquired by a long exercise of reason, both in *particular,* and likewise in those subjects that are most *general,* as much, perhaps, out of the reach of the greatest mathematician, as Sir Isaac Newton's speculations are above the capacity of some that are now called mathematicians." The truth of this observation is sufficiently evinced, in Plato's definition of a philosopher (in his *Sophista*), "The philosopher (says he) is the man who sufficiently sees one idea every way extended through many, every one of them lying apart; and many ideas different from one another, externally comprehended under one.—And farther, one idea, throughout all many, wrapt up in one; and many ideas, every way separate or discreet. This is to have the knowledge to discern how ideas, as they are general, agree and disagree." Now, he who thinks that a perception of this kind may be acquired

\* Notes to Letters on Mind, page 83.

by

by barely reading an accurate discourse on the nature of ideas, composed in intelligible terms, without, at the same time, employing a long course of profound meditation, and patient thought, knows but little the difficulty of the task, and until he changes his opinion will never be the wiser. But the folly and presumption of men, with respect to this sublime philosophy, is really unpardonable; for there are very few who conceive that much previous instruction is requisite to its acquisition; but almost every man decides peremptorily on the most abstract speculations; and reckons himself sufficient for the most profound investigations. In the sciences and arts they are willing to proceed to perfection by gradual advances; but they consider philosophy as easy, of instant access, and hastily approach to her embraces with an assured confidence of success. Though, like unhappy Ixion, through their presumption, instead of a goddess, they grasp nothing but an empty cloud. Plato was so sensible of this truth, that, in his seventh epistle to Dion, he expressly affirms, that he neither has written, nor ever will write explicitly concerning these sublime speculations; "For a thing of this kind (says he) cannot be expressed by words, like other disciplines, but by a lasting familiarity, and conjunction of life, with this *divine object*, a bright light \*

on

\* This bright light is no other than that of ideas themselves; which, when it is once enkindled, or rather re-kindled in the soul, becomes the general standard, and criterion of truth. He who possesses this, is no longer the slave of opinion; puzzled with doubts, and lost in the uncertainties of conjecture. Here the fountain of evidence is alone to be found.—This is the true light, whose splendors can alone dispel the darkness of ignorance, and procure for the soul undecaying good, and substantial felicity. Of this I am certain, from my own experience; and happy is he who acquires this invaluable treasure. But let the reader beware of mixing the extravagancies of modern enthusiasm with this exalted illumination. For this light is alone brought into the mind by science, patient reflection, and unwearyed meditation: it is not produced by any violent agitation of spirits, or ecstasy of imagination; for it is far superior to the energies of these: but it is tranquil and steady, intellectual and divine. Avicenna, the Arabian, was well acquainted with this light, as is evident from the beautiful description he gives of it, in the elegant introduction of Ebn Tophail, to the Life of Hai Ebn Yokdhan. "When a man's

on a sudden, as it were leaping from a fire, will illuminate the soul, and there preserve and nourish its splendor. He adds, that a publication of such concerns, is alone useful to a few of mankind, who from some small vestiges previously demonstrated, are sufficiently sagacious to their invention. But it will fill others partly with a base contempt, and partly with a rash and vain confidence, as if they had now learned some very excellent things." He then subjoins the following instance of the difficulty attending such an undertaking : " There are three things (says he), from which science must necessarily be produced; but the fourth is science itself. And it is requisite to establish the fifth as that which is the object of knowledge, and has a true existence. One of these is the name of a thing ; the second its definition ; the third the resemblance ; the fourth science. Now take each of these, desiring to learn what we have lately asserted, and think concerning them all, in a similar manner. A circle is called something, whose *name* we have just expressed. After this follows its definition, composed from nouns and verbs. For that which every where is equally distant from the extremes to the middle, is the definition of that which we signify by the name of a round, and a circumference, and

desires (says he) are considerably elevated, and he is competently well exercised in these speculations, there will appear to him some small glimmerings of the truth, as it were flashes of lightning, very delightful, which just shine upon him, and then become extinct. Then the more he exercises himself, the ostener will he perceive them, till at last he will become so well acquainted with them, that they will occur to him spontaneously, without any exercise at all ; and then as soon as he perceives any thing, he applies himself to the divine essence, so as to retain some impression of it ; then something occurs to him on a sudden, whereby he begins to discern the *truth* in every thing ; till through frequent exercise he at last attains to a perfect tranquillity ; and that which used to appear to him only by fits and starts, becomes habitual, and that which was only a glimmering before, a constant light ; and he obtains a constant and ready knowledge." He who desires to know more concerning this, and a still brighter light, that arising from an union with the supreme, must consult the eighth book of Plotinus' fifth Ennead, and the 7th and 9th of the sixth, and his book on the Beautiful, of which I have published a translation.

a circle.

a circle. But the third is the circle which may be painted, or blotted out, which may be made by a wheel, or destroyed. None of which affections, the circle itself, which each of these respects, suffers, as being of a different nature. But the fourth is science, and intellect, and true opinion about these. And this again must be established as one whole, which subsists in voice, nor in corporeal figures, but in intellect and intelligence. It is therefore manifest, that this fourth is different from the nature itself of the circle, and again different from the three we have previously mentioned. But among the number of these, intellect, by its relation and similitude, proximately adheres to the fifth, while the rest are more remote from its nature. The same may likewise be affirmed of a straight and crooked figure, of colour, and of the good, the beautiful, and the just. And again, of every body, whether fashioned by the hand, or the work of nature, whether fire or water, and the rest of this kind; likewise of every animal, and the manners of animals; and of all actions and passions. For unless, among these, some one, after a manner, receives that fourth, he will never perfectly participate the science about the fifth." He then proceeds to shew in what respect each of the preceding four are different from the fifth. " Every circle (says he) which by the hands of men is either painted, or fashioned by a wheel, is plainly contrary to our fifth. For it every where participates of the right-line. But we must affirm, that the circle itself has neither more nor less of any thing whatever; that is, it possesses in itself, nothing of a contrary nature. Besides, none of these are endued with any stability of name. For nothing hinders our applying the appellation of straight to that which we now denominate round, and calling the straight by the denomination of the round; nor will there

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be any less stability in these, when their names are changed into the contrary. The same reasoning is likewise true of definition, since it is composed from nouns and verbs, which possess no stability. And in a variety of ways, it may be proved, that no one of these four is certain and firm." Now, this fifth division of Plato's entirely respects ideas, considered as flourishing in intellect; by a conjunction with which, we acquire true intelligence, and the perfection of human knowledge. The first three of the preceding are obnoxious to various mutations; the fourth less; but the last is perfectly stable and invariable. The three first are rather conversant about the qualities of things, about the image and shadow; the fourth raises us to the participation of truth; but the fifth to truth itself, and permanent essence. In the first degrees almost all are conversant; in the fourth a few; in the fifth, all the gods, but a very small part of mankind, as it is asserted in the Timæus. The four first may be known, indeed, without the fifth, confusedly; but from the knowledge of the fifth they become perfectly manifest, as effects from the knowledge of their cause. But we cannot, by any means, attain to the apprehension of the fifth, unless we have been first accurately conversant with the rest; for from our imperfect condition we are compelled to rise from difference to identity, from multitude to unity, and from shadow to substance. While we investigate the knowledge of things, if we are alone desirous to apprehend their resemblance (which is the case with the multitude) we shall be placed in the third degree, and may easily acquire the object of our pursuit. But if we should fortunately possess the true philosophical genius, which is rare in the extreme, and aspiring to the fifth degree, should, by a happy event, attain to its conjunction, though such a contact is clearer and

more

more certain than all knowledge ; yet it is difficult to express it in words, and to manifest it to others. And the reason of this is obvious : first, because words are wanting, which exactly correspond to the essence of a thing, since these are only the symbols of shadows. Secondly, because we speak with those, who are alone conversant with shadows, and are on this account derided by them, when they find that our fifth does not, by any means, accord with material resemblances, which they consider as the only realities.

8. And here a question very naturally presents itself for our solution, whether the soul, while united with the body, is able to perceive ideas, without the assistance of the phantasy. For it seems difficult to apprehend how the soul, thus depressed and weighed down with the terrene mass, should be able to raise herself to the supernal light of ideas, and become united with their fulgence. The opinion of the Peripatetics is well known, that some phantasm must always accompany intelligence ; but this is denied by the Platonists, and I think with great reason. For the operations of intellect are not dependent on the phantasy, though the perceptions of the latter proceed from the energies of the former. Besides, as Plotinus beautifully observes, our most vigorous energies are accompanied with the least animadversion ; and there is no absurdity in supposing that by increasing the force of intellectual energy, we may speculate free from all imagination ; since the phantasms attending our conceptions, become weak in proportion as the intellectual fight increases in vigour. On this account, the Platonists affirm, that the moral virtues free us from the vehemence of perturbations ; but the contemplative from imagination, and the senses. Hence too, the sciences may be called living waters ; in which the

wings

wings of the soul being dipt, her feathers, which were either separated or broken by her lapse into body, are repaired, and restored to a resemblance of their former perfection. For the wings are the powers of the soul, leading to intelligibles: but the feathers are as well the natural instincts to good and truth, as reasons inserted in the soul; which either fall off, or are broken by her descent into body, and conjunction with its ruinous bonds. But these are repaired and invigorated by the sciences, which, like living streams, flowing from the fountains of ideas, restore life and perfection to the soul. Hence Plato, in the Phædrus, asserts that these wings of the soul are increased by every thing which confers to supernal elevation; as beauty, wisdom, and the like; and by a convenient metaphor, in the same dialogue, he considers the chariot of the souls lives, her charioteer, and the horses by which her car is drawn; and lastly, every thing which contributes to the elevation of the soul, and her conjunction with intellect and ideas. We may therefore conclude, that this conjunction is possible to be effected, though it is rarely obtained; and that it is a flight too arduous and sacred for the groveling and sordid; a splendor too bright for the sensible eye; and a contact too ineffable to be described by the unstable composition of words.

But I cannot conclude this section, without soliciting the reader's attention to a comparison of the difference between the ancient philosophy, and that invented by Mr. Locke, and the moderns. According to Mr. Locke's system ideas are formed from sensible particulars, by a kind of mechanical operation; so that truth is something by its nature, posterior to sensation, and entirely dependent on it for existence. According to Plato, ideas are eternal and immaterial beings, the originals of all sensible forms, and the

The fountains of all evidence and truth; so that on this system truth ranks among the first, and not in the last of things; and would still retain its nature, though the corporeal senses were no more. According to Mr. Locke, the soul is a mere *rāsa tabula*, an empty recipient, a mechanical blank. According to Plato, she is an ever-written tablet, a plenitude of forms, a vital and intellectual energy. On the former system, she is on a level with the most degraded natures, the receptacle of material species, and the spectator of delusion and non-entity \*. Hence, her energies are nothing but somnolent perceptions, and encumbered cogitations; for all her knowledge terminates in sense, and her science in passion. Like a man between sleeping and waking, her visions are turbid and confused, and the phantoms of a material night, continually glide before her drowsy eye. But on the latter system, the soul is the connecting medium of an intelligible and sensible nature, the bright repository of all middle forms, and the vigilant eye of all cogitative reasons. Hence she is capable of rousing herself from the sleep of a corporeal life, and emerging from this dark Cimmerian land, into the regions of light and reality. At first, indeed, before she is excited by science, she is oppressed with lethargy, and clouded with oblivion; but in proportion as learning and enquiry stimulate her dormant powers, she wakens from the dreams of ignorance, and opens her eye to the irradiations of wis-

\* Lest the superficial reader should think this is nothing more than declamation, let him attend to the following argument. If the soul possesses another eye different from that of sense (and that she does so, the sciences sufficiently evince), there must be, in the nature of things, species accommodated to her perception, different from sensible forms. For if our intellect speculates things which have no real substance, such as Mr. Locke's ideas, its condition must be much more unhappy than that of the sensitive eye, since this is co-ordinated to beings; but intellect would speculate nothing but illusions. Now, if this be absurd, and if we possess an intellectual eye, which is endued with a visive power, there must be forms correspondent and conjoined with its vision; forms immovable; indeed, by a corporal motion; but moved by an intellectual energy.

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dom. On Mr. Locke's system, the principles of science and sense are the same, for the energies of both originate from material forms, on which they are continually employed. Hence, science is subject to the flowing and perishable nature of particulars ; and if body and its attributes were destroyed, would be nothing but a name. But on the system of Plato, they differ as much as delusion and reality ; for here the vital, permanent, and lucid nature of ideas is the fountain of science ; and the inert, unstable, and obscure nature of sensible objects, the source of sensation. On Mr. Locke's system, body may be modified into thought, and become an intelligent creature ; it may be subtilized into life, and shrink, by its exility, into intellect. On that of Plato, body can never alter its nature by modification, however, it may be rarefied and refined, varied by the transposition of its parts, or tortured by the hand of experiment. In short, the two systems may be aptly represented by the two sections of a line, in Plato's Republic. In the ancient, you have truth itself, and whatever participates of the brightest evidence and reality : in the modern, ignorance, and whatever belongs to obscurity and shadow. The former fills the soul with intelligible light, breaks her lethargic fetters, and elevates her to the principle of things ; the latter clouds the intellectual eye of the soul, by increasing her oblivion, strengthens her corporeal bands, and hurries her downwards into the dark labyrinths of matter.

Nor is it wonderful there should be so great a difference between the two systems, and so much in favour of the ancients, if we consider the great advantages these ancients possessed over the moderns in every thing which contributes to the advancement of philosophy. For, in the first place, they lived in an age when abstract investigations

were in the greatest request, and the professors of such pursuits in the highest estimation. Besides this, they united the most exalted abilities with the most unwearied attention and obstinate perseverance; they devoted their whole lives to the search of truth; and relinquished every thing which might be an obstacle to its acquisition. We may add, likewise, the advantages of a language extremely philosophical; and a freedom from the toil of learning any tongue but their own. Now the reverse of all this is the portion of the moderns: for in the present age, abstract Speculations are ridiculed; and its professors despised. The pursuit of truth is considered as perfectly consistent with ordinary avocations, and is rather prosecuted as a relief from the toils of business than as a thing desirable for its own sake, and of the greatest dignity and worth. Hence, a few years desultory application at a college, where language is one of the first objects of attention, qualifies a modern for philosophy, raises him above Pythagoras and Plato, and persuades him, with presumptuous confidence, to enter the lists against these venerable heroes. And lastly, all modern languages are barbarous with respect to the Greek; falling far short of its harmony and energy, its copiousness and propriety. If such then be the true state of the case, what judgment must we form of men who, with all these disadvantages, philosophized without the assistance of the ancients, despising their works, and being ignorant of their contents? Shall we call it prudence or presumption, wisdom or folly? Truth will certainly pronounce the latter; and the general voice of posterity will confirm her decision. There are two egregious instances in our own country of this daring presumption; I mean Bacon and Locke. The former of these is celebrated for having destroyed the jargon of the schoolmen, and brought

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experimental enquiries into repute; and for attempting to investigate causes through the immensity of particular effects. Hence, he fondly expected, by experiment piled on experiment, to reach the principle of the universe; not considering that his undertaking was as ridiculous as that of the giants of old, who attempted to invade the heavens, by placing Offa upon Pelion, and Olympus upon Offa; and ignorant that

Heaven still, with laughter, the vain toil surveys,  
And buries madmen in the heaps they raise.

The latter of these, Mr. Locke, is applauded for having, without assistance from the ancients, explained the nature, and exhibited the genuine theory of human understanding. But that this applause is false, the preceding comparison between his and the ancient philosophy, may evince; and the variety of other self-taught systems which, like nocturnal meteors, blaze for a while, and then vanish in obscurity, abundantly confirms. Had these men, indeed, when they justly derided the barbarous writings of the schoolmen, explored the works of antiquity, penetrated the wisdom they contain, and enriched their native language with its illustration, they had doubtless been celebrated by the latest posterity: but, desirous of becoming masters in philosophy by intuition, they disdained the instruction of the ancients, and vainly attempted to soar on artificial wings to the very summit of science. They are, however, destined, like Icarus, to a precipitate fall; for the influence of time, which is continually dissolving the cement of their plumes, is likewise continually weakening their force, and will at last effect their final separation. And thus much concerning the doctrine of ideas, and numbers, according to Pythagoras and Plato.

S. E. C.

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## DEMONSTRATIVE SYLLOGISM. **xxxv**

### S E C T I O N II.\*

**B**UT let us now consider the properties of the demonstrative syllogism, and endeavour to unravel its intricate web; appointing Aristotle for our guide in this arduous investigation. For an enquiry of this kind is naturally connected with the doctrine of ideas, as it enables us to gain a glimpse of the universals participated in mathematical forms, and to rise to the principles of science. It brings us acquainted with the laws which bind demonstration; and teaches us that objects of intellect are alone the objects of science, and the sources of truth.

Previous to the acquisition of all learning and ratiocinative discipline, it is necessary we should possess certain natural principles of knowledge, as subservient to our future progress and attainments. Thus, in every science there are some things which require an immediate assent as soon as proposed; whose certainty is too evident and illustrious to stand in need of any demonstrative proof deduced from that particular science which, like stately pillars, they equally support and adorn. Hence we are informed by the geometerian, that a point is that which is destitute of all parts whatever; but we must previously understand the meaning of the word *part*. Thus the arithmetician defines an odd number, that which is divided according to unequal parts; but it is necessary we should antecedently know the meaning of the word *unequal*. Thus, too, art as well as science operates by antecedent knowledge; and hence the architect,

\* The present section contains an illustration of almost all the first book of Aristotle's last Analytics. I have for the most part followed the accurate and elegant paraphrase of Themistius, in the execution of this design, as the learned reader will perceive: but I have likewise everywhere added elucidations of my own, and endeavoured to render this valuable work intelligible to the thinking mathematical reader.

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the statuary, and the shipwright, learn the names and the use of their respective implements, previous to the exercise of the materials themselves. This is particularly evident in the discursive arts of rhetoric and logic ; thus the logician reasons by syllogism, the rhetorician by induction, and the sophist by digressions and examples ; while each proceeds in an orderly progression from principles simple and evident, to the most remote and complicated conclusions.

2. The antecedent knowledge of things may be divided into two parts : the one a knowledge of their existence, or that they exist ; the other a knowledge of the terms expressive of their existence. Thus, previous to the enquiry why iron is attracted by the magnet, it is necessary we should learn the reality of this attraction, and the general mode of its operation : thus too, in an enquiry concerning the nature of motion and time, we must be previously convinced of their existence in the nature of things. The second division of antecedent knowledge takes place in subjects whose very existence admits of a dispute : thus previous to a solution of the questions, Whether there are any gods or not ? Whether there is a providence or not ? and the like, it is necessary we should first understand the meaning of the terms ; since we in vain investigate the nature of any thing while we are ignorant of the meaning of its name ; although, on the contrary, we may have a perfect conception of the meaning of some words, and yet be totally ignorant whether the things they express have a real, or only an imaginary existence. Thus, the meaning of the word *centaur* is well understood by every one ; but its existence is questioned by most.

3. From hence it will easily appear, that no small difference subsists between *learning* and *knowledge*. He who is about to understand the truth of any proposition, may

be said to possess a previous conception of its truth ; while, on the contrary, it may happen that he who is in the capacity of a learner, has no antecedent knowledge of the science he is about to learn. Thus we attain to the distinct knowledge of a thing which we formerly knew in a general way ; and frequently, things of which we were ignorant are learned and known in the same instant.

Of this kind are the things contained under some general idea, of which we possess a previous knowledge : thus, he who already knows that the three interior angles of every triangle are equal to two right, and is as yet ignorant that some particular figure delineated on paper is a triangle, is no sooner convinced from inspection of its being a triangle, than he immediately learns and knows : he learns it is a triangle ; he knows the equality of its angles to two right ones. That it is now a triangle he both sees and learns ; but the equality of its angles he previously knew in that general and comprehensive idea, which embraces every particular triangle.

Indeed, a definite knowledge of this triangle requires two conditions : the one, that it is a triangle ; and the other, that it has angles equal to two right. The first we receive from inspection ; the second is the result of a syllogistic process ; an operation too refined for the energies of sense, and alone the province of *intellect* and *demonstration*. But demonstration without the knowledge of that which is universal, cannot subsist ; and since the proposition is universal, that in every triangle the angles are equal to two right, as soon as any figure is acknowledged to be a triangle, it must necessarily possess this general property.

Hence we infer, that of the triangle delineated on paper, and concealed, we are partly ignorant of this general property, the equality of its angles (because we are ignorant of

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of its existence); and we partly understand it as included in that universal idea we previously possessed. Hence too, it is evident that actual science arises from a medium between absolute ignorance and perfect knowledge; and that he who possesses the principles of demonstration, possesses in capacity the conclusions also, however complicated and remote; and that by an evocation of these principles from dormant power into energy, we advance from general and abstracted knowledge to that which is sensible and particular.

4. Two acceptations of knowledge may be admitted; the one common and without any restriction; the other limited and peculiar. Since all knowledge, whether arising from accidents, or supported by necessary principles, is called science. Knowledge, properly so called, arises from a possession of that cause from which a thing derives its existence, and by which we infer the necessity of its existence; and this constitutes simple and absolute science. Thus too, the definitions of those general conceptions and suppositions, which from their primary nature are incapable of demonstration, are called science. But the science which treats of the method of arriving at knowledge, is called demonstration; for every demonstration is a syllogism producing science. Hence, if in every syllogism it is necessary that the propositions should be the cause of the conclusion; and to know any thing properly, a knowledge of its cause is requisite; in the propositions of demonstration, both these conditions are required: that they should be effective of the conclusion; and the causes of the thing demonstrated.

Thus, from the ruins of a stately edifice, we may justly infer, that the building was beautiful when entire; and from the smoke we may collect the existence of the fire, though

though concealed : but the ruins of the edifice are not the cause of its beauty; nor does fire originate from smoke, but, on the contrary, smoke is the natural result of fire : the inference, therefore, is in neither case a demonstrative one. Again, since every cause is both prior to, and more excellent than its effect, it is necessary that the propositions should be more peculiar, primary, and excellent than the conclusions. And because we then know a thing properly when we believe it to have a necessary existence, hence it is requisite that the propositions should be true ; for if false, a false conclusion may ensue, such as, that the diameter of a square is commensurable with its side. But if every science arises from antecedent knowledge, demonstration must be founded on something previous ; and on this account it is requisite that the propositions should be more known than the conclusions. The necessary properties, then, of all demonstrative propositions, are these ; that they exist as causes, are primary, more excellent, peculiar, true, and known, than the conclusions. Indeed, every demonstration not only consists of principles prior to others, but of such as are eminently first ; for if the assumed propositions may be demonstrated by other assumptions, such propositions may, indeed, appear prior to the conclusions, but are by no means entitled to the appellation of first. But others, on the contrary, which require no demonstration, but are of themselves probable or manifest, are deservedly esteemed the first, the truest, and the best. Such indemonstrable truths were called by the ancients, axioms, from their majesty and authority ; as the assumptions which constitute the best syllogisms derive all their force and efficacy from these.

And on this account, above all others, they merit the title of the principles of demonstration. But here it is worth

worth observing, that these primary propositions are not the first in the order of our conceptions; but first to nature, or in the nature of things. To us, that which is first is particular, and subject to sensible inspection; to nature, that which is universal, and far remote from the apprehension of sense. Demonstration does not submit itself to the measure of our ingenuity, but, with invariable rectitude, tends to truth as its ultimate aim; and without stopping to consider what our limited powers can attain, it alone explores and traces out the nature of a thing, though to us unperceived and unknown.

This demonstrative syllogism differs not a little from others, by the above property; the rest can as well educe a true conclusion from false premises, which is frequent among the rhetoricians, as that which is prior from that which is posterior; such as, Is every syllogism derived from conjecture?

With respect to the rest, as we have already confessed, they may be formed from principles that are true, but not from such as are proper and peculiar; as if a physician should endeavour to prove an orbicular wound the most difficult to coalesce and heal, because its figure is of all others, the most capacious; since the demonstration of this is not the province of the physician, but of the geometricalian *alone*.

5. That proposition is called immediate, which has none superior to itself, and which no demonstration whatever can confirm: such as these are held together by the embraces of universals. There are some, indeed, united from that which is sensible and particular: thus, that the garment is white, is an immediate proposition, but not of that kind whose principles require to be demonstrative ones; the cause of which we shall hereafter investigate. Of immediate

mediate propositions subservient to the purposes of demonstration, some are of such a superior nature, that all men possess a knowledge of them without any previous instruction; and these are called axioms, or general notions; for without these all knowledge and enquiry is vain. Another species of immediate propositions is position; incapable of being strengthened by demonstration, yet not necessarily foreknown by the learner, but received from the teacher. With respect to the genus of position, one of its species is definition, and another hypothesis. Definition is an oration, in which we neither speak of the existence, nor non-existence of a thing; but alone determine its nature and essence. It is common to every hypothesis, not to be derived from nature, but to be the entire result of the art of the preceptor.

It likewise always affirms the existence or non-existence of its subject; such as, that motion is, and that from nothing nothing is produced. Those which are not so perspicuous are called postulates, or petitions; as that a circle may be described from any centre, and with any radius; and such as these are properly hypotheses and postulates.

6. We have now seen the privilege assigned to the principles of demonstration:—whether or no our decision has been just, the ensuing considerations will evince. We said that the assumptions in demonstration were more known than the conclusions,—not indeed without reason, since through these our knowledge and belief of the conclusion arises. For universally, that quality which is attributed to many different things so as to be assigned to one through the medium of another, abounds most in that medium by which it is transmitted to the rest.

Thus the sun, through the medium of the moon, illuminates the earth by night; thus the father loves the

preceptor through the medium of his child. And in the first instance the moon is more lucid than any object it enlightens: in the second, the child possesses more of the father's regard than his preceptor. If then we assent to the conclusions through our belief of the principles alone, it is necessary that the principles should be more known, and inherit a greater degree of our assent. Hence, if it be true that the principles are more known than the conclusions, it follows, that either our knowledge of them is derived from demonstration, or that it is superior to any demonstrative proof; and after this manner we must conceive of those general self-evident notions which, on account of their indemonstrable certainty, are deservedly placed at the top of all human science.

These propositions not only possess greater credibility than their conclusions, they likewise inherit this property as an accession to their dignity and importance; that no contrary propositions deserve greater belief; for if you give no more assent to any principle than to its contrary, neither can you give more credit to the conclusion deduced from that principle than to its opposite. Were this the case, the doctrine of these propositions would immediately lose its invariable certainty.

7. There are, indeed, some who, from erroneously applying what we have rightly determined, endeavour to take away the possibility of demonstration. From the preceding doctrine it appears that the principles are more aptly known than the conclusions. This is not evident to some, who think nothing can be known by us without a demonstrative process; and consequently believe that the most simple principles must derive all their credit from the light of demonstration.

But

But if it be necessary that all assumptions should be demonstrated by others, and these again by others ; either the enquiry must be continued to infinity, (but infinity can never be absolved), or if, wearied by the immense process, you at length stop, you must doubtless leave those propositions unknown, whose demonstration was declined through the fatigue of investigation. But how can science be derived from unknown principles ? For he who is ignorant of the principles, cannot understand the conclusions which flow from these as their proper source, unless from an hypothesis or supposition of their reality.

This argument of the sophists is, indeed, so far true, that he who does not understand that which is first in the order of demonstration, must remain ignorant of that which is last :—But in this it fails, that all knowledge is demonstrative ; since this is an assertion no less ridiculous than to maintain that nothing can be known. For as it is manifest that some things derive their credit and support from others, it is equally obvious that many, by their intrinsic excellence, possess indubitable certainty and truth ; and command our immediate assent as soon as proposed. They inherit, indeed, a higher degree of evidence than those we assent to by the confirmation of others ; and these are the first principles of demonstration : propositions indisputable, immediate, and perspicuous by that native lustre they always possess. By means of these, we advance from proposition to proposition, and from syllogism to syllogism, till we arrive at the most complicated and important conclusions. Others, willing to decline this infinite progression, defend the necessity of a circular or reciprocal demonstration. But this is nothing more than to build error upon error, in order to attain the truth ; an attempt no less ridiculous than that of the giants of old. For since, as we shall hereafter

accurately prove, demonstration ought to consist from that which is first, and most known; and since it is impossible that the same thing should be to itself both prior and posterior: hence we infer the absurdity of circular demonstration; or those syllogisms in which the conclusions are alternately substituted as principles, and the principles as conclusions. It may, indeed, happen, that the same thing may be both prior and posterior to the same; but not at one and the same time, nor according to the same mode of existence. Thus, what is prior in the order of our conceptions, is posterior in the order of nature; and what is first in the arrangement of things, is last in the progressions of human understanding. But demonstration always desires that *first* which is prior in the order and constitution of nature. But the folly of such a method will more plainly appear from considering its result: let us suppose every *a* is *b*, and every *b* is *c*; hence we justly infer, that every *a* is *c*. In like manner, if we prove that every *a* is *b*, and by a circular demonstration, that every *b* is *a*, the consequence from the preceding is no other than that every *a* is *a*; and thus the conclusion terminates in that from which it first began; a deduction equally useless and ridiculous. However, admitting that, in the first figure, circular demonstration may be in some cases adopted, yet this can but seldom happen from the paucity of reciprocal terms.

But that reciprocal terms are very few, is plain from hence: let any species be assumed, as *man*; whatever is the predicate of man, is either constitutive of his essence, or expressive of some accident belonging to his nature. The superior genera and differences compose his essence, among which no equal predicate can be assigned reciprocal with man, except the ultimate differences which cannot be otherwise than one, i. e. risibility, which mutually reciprocates.

ciprocates with its subject; since every man is risible, and whatever is risible is man. Of accidents some are common, others peculiar; and the common are far more in number than the peculiar; consequently the predicates which reciprocate with man, are much fewer than those which do not reciprocate.

8. It is now necessary to enumerate the questions pertaining to demonstration; and for this purpose, we shall begin with propositions, since from these, syllogisms are formed; and since every proposition consists of a subject and predicate, the modes of predication must be considered, and these are three which I call *total*, *essential*, and *universal*; a *total* predication takes place when that which is affirmed or denied of one individual is affirmed or denied of every individual comprehended under the same common species.

Thus, animal is predicated of every man, and it has this farther property besides, that of whatever subject it is true to affirm man, it is at the same time true to affirm animal.

Those things are said to be *essentially* predicated; first, when the predicate is not only total, but constitutes the essence of the subject; instances of this kind are, animal of man; a tree of the plantain; a line of a triangle; for a triangle is that which is contained under three right-lines. But here we must observe, that not every total predicate is an essential one; thus, whiteness is predicated of every swan, because it is inherent in every swan, and at every instant of time; but because whiteness does not constitute the essence of a swan, it is not essentially predicated; and this, first, is one of the modes of essential predication of the greatest importance in demonstration. The second mode is of accidents, in the definition of which their common

mon subject is applied : thus, a line is essentially inherent in rectitude, because in its geometrical definition, a line is adopted ; for rectitude is no other than a measure, equally extended between the points of a line. In the same manner, imparity is contained in number ; for what is that which is odd, but a number divided into unequal parts ? Thus, virtues are resident in the soul, because, in their definition, either some part of the soul, or some one of its powers, is always applied. The third mode of essential predicates pertains to accidents which are inseparably contained in some particular subject, so as to exclude a prior existence in any other subject ; such as colour in superficies. The fourth mode is of things neither contained in another, nor predicated of others ; and such are all individuals, as Callias, Socrates, Plato. Causes are likewise said to exist substantially, which operate neither from accident nor fortune.

Thus, digging up the ground for the purposes of agriculture, may be the cause of discovering a treasure, but it is only an accidental one. But the death of Socrates, in despite of vigilance, is not the result of a fortuitous cause, but of an essential one, viz. the operation of poison.

9. These posterior significations of essential predicates are added more for the sake of ornament than use ; but the two former have a necessary existence, since they cannot but exist in the definition of names which predicate the essence of a thing, and in subjects which are so entirely the support of accidents, that they are always applied in their definition. But it is a doubt with some, whether those accidents are necessary, which cannot be defined independent of their common subject ? To this we answer, that no such accident can, from its nature, be contained in every individual of any species ; for curvature is not contained in

every line; nor imparity in every number; from whence we infer, that neither is curvature necessarily existent in a line, nor parity in number. The truth of this is evident from considering these accidents abstracted from their subjects; for then we shall perceive that a line may exist without curvature, and number without imparity.

Again, I call that an *universal* predicate, which is predicated of a subject totally and essentially, and considered as primarily and inseparably inherent in that subject: for it does not follow that a predicate, which is total, should be immediately universal; for whiteness is affirmed of every swan, and blackness of every crow, yet neither universally. In like manner, a substantial predicate is not consequently an universal one; for the third mode of essential predicates, and the two following (instanced before) cannot be universal. Thus, colour, although inherent in superficies essentially, is not inherent in every superficies, and consequently not universally. Thus again, Socrates, Callias, and Plato, though they exist essentially, are not universals, but particulars; and thus, lastly, the drinking of poison was an essential cause of the death of Socrates, but not an universal one, because Socrates might have died by other means than poison. If then, we wish to render an accurate definition of an universal predicate, we must not only say it is total and essential, but that it is primarily present to its subject and no other. Thus, the possession of angles equal to two right, primarily belongs to a triangle; for this assertion is essentially predicated of triangle, and is inherent in every triangle. This property, therefore, is not universally in figure, because it is not the property of every figure, not of a square, for instance; nor as universal in a scalene triangle: for although it is contained in every scalene, and in every equilateral, and isosceles,

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celes triangle, yet it is not primarily contained in them, but in triangle itself; because these several figures inherit this property, not from the particular species to which they belong, but from the common genus *triangle*. And thus much concerning *total*, *essential*, and *universal* predicates.

10. Concerning that which is universal, we are frequently liable to err; often from a belief that our demonstration is universal, when it is only particular; and frequently from supposing it particular when it is, on the contrary, universal. There are three causes of this mistake; the first, when we demonstrate any particular property of that which is singular and individual, as the sun, the earth, or the world. For since there is but one sun, one earth, and one world, when we demonstrate that the orb of the earth possesses the middle place, or that the heavens revolve, we do not then appear to demonstrate that which is universal.

To this we answer: when we demonstrate an eclipse of the sun to arise from the opposition of the moon, we do not consider the sun as one particular luminary, but we deduce this consequence as if many other suns existed besides the present.

Just as if there were but one species of triangles existed; for instance, the *isosceles*; the equality of its angles at the base would not be considered in the demonstration of the equality of all its angles to two right ones: but its triangularity would be essential, supposing every species of triangles but the *isosceles* extinct, and no other the subject of this affection. So when we prove that the sun is greater than the earth, our proof does not arise from considering it as this particular sun alone, but as sun in general; and by applying our reasoning to every sun, if thousands besides the present should enlighten the world. This will appear

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still more evident, if we consider that such conclusions must be universal, as they are the result of an induction of particulars : thus, he who demonstrates that an eclipse of the sun arises from the opposition of the moon between the sun and earth, must previously collect, by induction, that when any luminous body is placed in a right-line with any two others opaque, the lucid body shall be prevented, in a greater or less degree, from enlightening the last of these bodies, by the intervention of the second; and by extending this reasoning to the sun and earth, the syllogism will run thus :

Every lucid body placed in a right-line with two others opaque, will be eclipsed in respect of the last by the intervention of the second ;

The sun, or every sun, is a luminous body with these conditions ;

And consequently the sun, and so every sun, will be eclipsed to the earth by the opposition of the moon.

Hence, in cases of this kind, we must ever remember, that we demonstrate no property of them as singulars, but as that universal conceived by the abstraction of the mind.

Another cause of deception arises, when many different species agree in one ratio or analogy, yet that in which they agree is nameless. Thus number, magnitude, and time, differ by the diversities of species; but agree in this, that as any four comparable numbers correspond in their proportions to each other, so that as the first is to the second, so is the third to the fourth; or alternately, as the first to the third, so is the second to the fourth: in a similar manner, four magnitudes, or four times, accord in their mutual analogies and proportions. Hence, alternate proportion may be attributed to lines as they are lines, to numbers as they are numbers, and afterwards to *times* and

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# I D I S S E R T A T I O N O N T H E

to bodies, as the demonstration of these is usually separate and singular; when the same property might be proved of all these by one comprehensive demonstration, if the common name of their genus could be obtained: but since this is wanting, and the species are different, we are obliged to consider them separately and apart; and as we are now speaking of that universal demonstration which is properly one, as arising from one first subject; hence none of these obtain an universal demonstration, because this affection of alternate proportion is not restricted to numbers or lines, considered in themselves, but to that common something which is supposed to embrace all these, and is destitute of a proper name. Thus too we may happen to be deceived, should we attempt to prove the equality of three angles to two right, separately, of a scalene, an isosceles, and an equilateral triangle, only with this difference, that in the latter case the deception is not so easy as in the former; since here the name triangle, expressive of their common genus, is assigned. A third cause of error arises from believing that to demonstrate any property inherent after some particular manner in the whole of a thing, is to demonstrate that property universally inherent. Thus, geometry proves \* that if a right-line falling upon two right-lines makes the outward angle with the one line a right-angle, and the inward and opposite angle with the other a right one, those two right-lines shall be parallel, or never meet, though infinitely extended. This property agrees to all lines which make right-angles: but they are not primarily equidistant on this account, since, if they do not each make a right-angle, but the two conjointly are equal to two right, they may still be proved equidistant. This latter demonstration, then, is primarily and universally conceived; the other,

\* See the twenty-eighth proposition of the first book of Euclid's Elements.

which

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which always supposes the opposite angles right ones, does not conclude universally; though it concludes totally of all lines with such conditions: the one may be said to conclude of a greater *all*; the other of a lesser. It is this greater *all* which the mind embraces when it assents to any self-evident truth; or to any of the propositions of Euclid. But by what method may we discover whether our demonstration is of this greater or lesser *all*? We answer, that general affection which constitutes universal demonstration is always present to that subject, which when taken away, the predicate is immediately destroyed, because the first of all its inherent properties.

Thus, for instance, some particular sensible triangle possesses these properties:—it consists of brass; it is scalene; it is a triangle. The query is, by which of these we have just now enumerated, this affection of possessing angles equal to two right is predicated of the triangle? Take away the brass, do you by this means destroy the equality of its angles to two right ones? Certainly not:—take away its scaleness, yet this general affection remains: lastly, take away its triangularity, and then you necessarily destroy the predicate; for no longer can this property remain, if it ceases to be a triangle.

But perhaps some may object from this reasoning, such a general affection extends to figure, superficies, and extremities, since, if any of these are taken away, the equality of its angles to two right can no longer remain. It is true, indeed, that by a separation of figure, superficies, and terms, from a body, you destroy all the modes and circumstances of its being; yet not because these are taken away, but because the triangle, by the separation of these, is necessarily destroyed; for if the triangle could still be preserved without figure, superficies, and terms, though these

were taken away it would still retain angles equal to two right ; but this is impossible. And if all these remain, and triangle is taken away, this affection no longer remains. Hence the possession of this equality of three angles to two right, is primarily and universally inherent in triangle, since it is not abolished by the abolition of the rest:—such as to consist of brafs ; to be scalene, or the like. Neither does it derive its being from the existence of the rest alone ; as figure, superficies, terms ; since it is not every figure which possesses this property, as is evident in such as are quadrangular, or multangular. And thus it is preserved by the preservation of triangle, it is destroyed by its destruction.

ii. From the principles already established, it is plain that demonstration must consist of such propositions as are universal and necessary. That they must be universal, is evident from the preceding ; and that they must be necessary, we gather probably from hence ; that in the subversion of any demonstration we use no other arguments than the want of necessary existence in the principles.

We collect their necessity demonstratively, thus ; he who does not know a thing by the proper cause of its existence, cannot possess science of that thing ; but he who collects a necessary conclusion from a medium not necessary, does not know it by the proper cause of its existence, and therefore he has no proper science concerning it. Thus, if the necessary conclusion  $c$  is  $a$ , be demonstrated by the medium B, not necessary ; such a medium is not the cause of the conclusion ; for since the medium does not exist necessarily, it may be supposed not to exist ; and at the time when it no longer exists, the conclusion remains in full force ; because, since necessary, it is eternal. But an effect cannot exist without a cause of its existence ; and hence

hence such a medium can never be the cause of such a conclusion. Again, since in all science there are three things, with whose preservation the duration of knowledge is connected; and these are, first, he who possesses science; secondly, the thing known; and thirdly, the reason by which it is known; while these endure, science can never be blotted from the mind, but on the contrary, if science be ever lost, it is necessary some of these three must be destroyed.

If then you infer that the science of a necessary conclusion may be obtained from a medium not necessary, suppose this medium, since capable of extinction, to be destroyed; then the conclusion, since necessary, shall remain; but will be no longer the object of knowledge, since it is supposed to be known by that medium which is now extinct. Hence, science is lost, though none of the preceding three are taken away; but this is absurd, and contrary to the principles we have just established. The thing known remains; for the conclusion, since necessary, cannot be destroyed:---he who knows still remains, since neither dead, nor forgetful of the conclusion:---lastly, the demonstration by which it was known, still survives in the mind; and hence we collect, that if science be no more after the corruption of the medium, neither was it science by that medium before its corruption; for if science was ever obtained through such a medium, it could not be lost while these three are preserved. The science, therefore, of a necessary conclusion can never be obtained by a medium which is not necessary.

I 2. From hence it is manifest, that demonstrations cannot emigrate from one genus to another; or by such a translation be compared with one another. Such as, for instance, the demonstrations of geometry with those of arithmetic. To be convinced of this, we must rise a little

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higher in our speculations, and attentively consider the properties of demonstration: one of which is, that predicate which is always found in the conclusion, and which affirms or denies the existence of its subject: another is, those axioms or first principles by whose universal embrace demonstration is fortified; and from whose original light it derives all its lustre. The third is the subject genus, and that nature of which the affections and essential properties are predicated; such as magnitude and number. In these subjects we must examine when, and in what manner a transition in demonstrations from genus to genus may be allowed. First, it is evident, that when the genera are altogether separate and discordant, as in arithmetic and geometry, then the demonstrations of the one cannot be referred to the other. Thus, it is impossible that arithmetical proofs can ever be accommodated with propriety to the accidents of magnitudes: but when the genera, as it were, communicate, and the one is contained under the other, then the one may transfer the principles of the other to its own convenience. Thus, optics unites in amicable compact with geometry, which defines all its suppositions; such as lines that are *right*, angles acute, lines equilateral, and the like. The same order may be perceived between arithmetic and music: thus, the double, sesquialter, and the like, are transferred from arithmetic, from which they take their rise, and are applied to the measures of harmony.

Thus, medicine frequently derives its proofs from nature, because the human body, with which it is conversant, is comprehended under natural body. From hence it follows, that the geometrician cannot, by any geometrical reasons demonstrate any truth, abstracted from lines, superficies, and solids; such as, that of contraries there is the same science;

## DEMONSTRATIVE SYLLOGISM. 14

science; or that contraries follow each other; nor yet such as have an existence in lines and superficies, but not an essential one, in the sense previously explained.

Of this kind is the question, whether a right-line is the most beautiful of lines? or whether it is more opposed to a line perfectly orbicular, or to an arch only. For the consideration of beauty, and the opposition of contraries, does not belong to geometry, but is alone the province of metaphysics, or the first philosophy.

But a question here occurs, If it be requisite that the propositions which constitute demonstration should be peculiar to the science they establish, after what manner are we to admit in demonstration those axioms which are conceived in the most common and general terms; such as, if from equal things you take away equals, the remainders shall be equal:—as likewise, of every thing that exists, either affirmation or negation is true? The solution is this: such principles, though common, yet when applied to any particular science for the purposes of demonstration, must be used with a certain limitation. Thus the geometrician applies that general principle, if from equal things, &c. not simply, but with a restriction to magnitudes; and the arithmetician universally to numbers.

Thus too, that other general proposition:—of every thing, affirmation or negation is true; is subservient to every art, but not without accommodation to the particular science it is used by. Thus number *is* or *is not*, and so of others. It is not then alone sufficient in demonstration that its propositions are true, nor that they are immediate, or such as inherit an evidence more illustrious than the certainty of proof; but, besides all these, it is necessary they should be made peculiar by a limitation of their comprehensive nature to some particular subject. It is on this account

account that no one esteems the quadrature of Bryso \*, a geometrical demonstration, since he uses a principle which, although

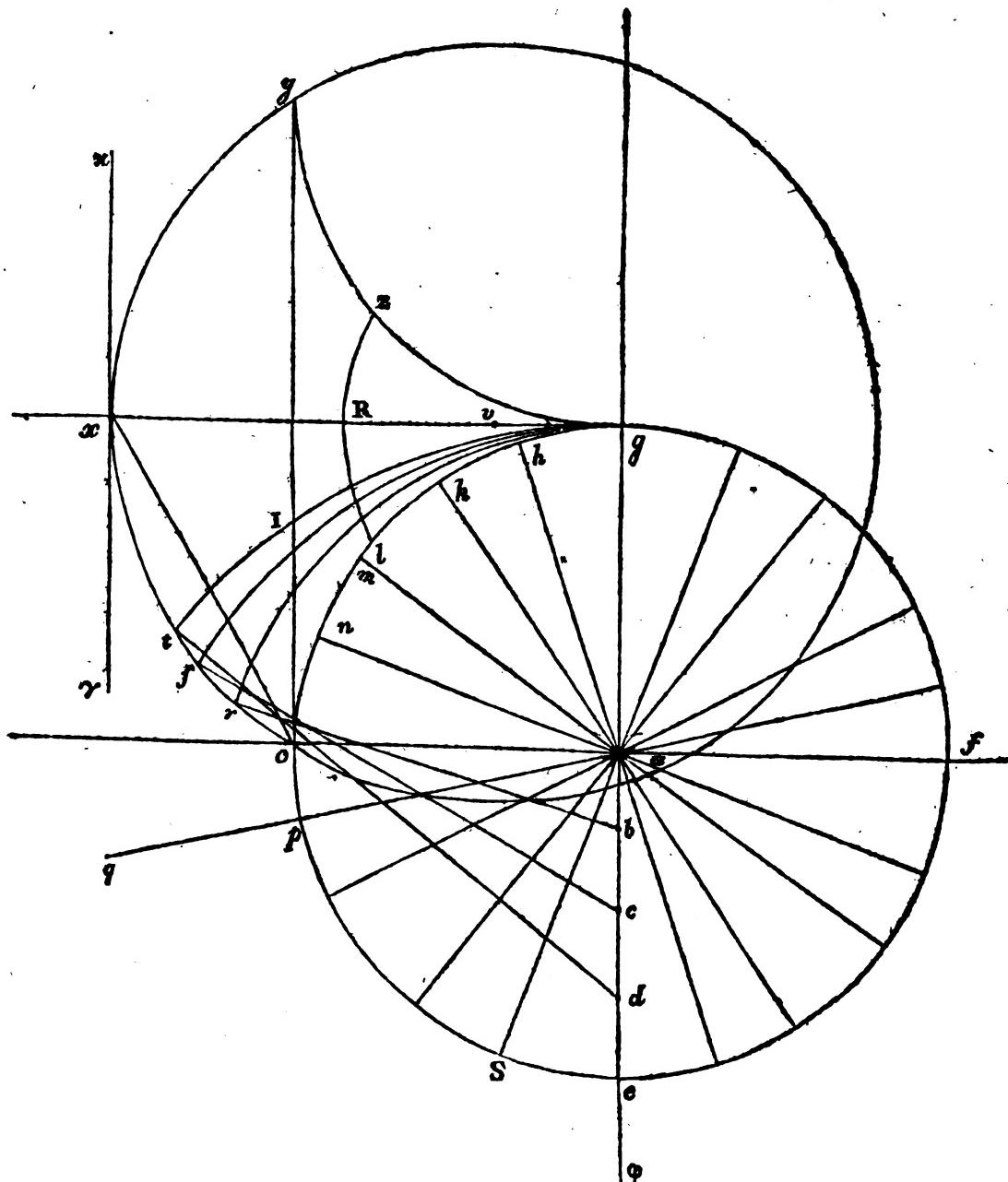
\* We are informed by Simplicius, in his Commentary on Aristotle's third Category of Relation, "that though the quadrature of the circle seems to have been unknown to Aristotle, yet, according to Jamblichus, it was known to the Pythagoreans, as appears from the sayings and demonstrations of Sextus Pythagoricus, who received (says he) by succession, the art of demonstration; and after him Archimedes succeeded, who invented the quadrature by a line, which is called the line of Nicomedes. Likewise, Nicomedes attempted to square the circle by a line, which is properly called *tetragrāphos*, or *the quadrature*. And Apollonius, by a certain line, which he calls the sister of the curve line, similar to a cockle, or tortoise, and which is the same with the *quadratix* of Nicomedes. Also Carpus wished to square the circle, by a certain line, which he calls simply formed from a twofold motion. And many others, according to Jamblichus, have accomplished this undertaking in various ways." Thus far Simplicius. In like manner, Boethius, in his Commentary on the same part of Aristotle's Categories (p. 166.) observes, that the quadrature of the circle was not discovered in Aristotle's time, but was found out afterwards; the demonstration of which (says he) because it is long, must be omitted in this place. From hence it seems very probable, that the ancient mathematicians applied themselves solely to squaring the circle geometrically, without attempting to accomplish this by an arithmetical calculation. Indeed, nothing can be more ungeometrical than to expect, that if ever the circle be squared, the square to which it is equal must be commensurable with other known rectilineal spaces; for those who are skilled in geometry know that many lines and spaces may be exhibited with the greatest accuracy, geometrically, though they are incapable of being expressed arithmetically, without an infinite series. Agreeable to this, Tacquet well observes (in lib. ii. Geom. Pract. p. 87.) "Denique admoneendi hic sunt, qui geometriæ non satis periti, sibi persuadent ad quadraturam necessarium esse, ut ratio lineæ circularis ad rectam, aut circuli ad quadratum in numeris exhibeat. Is sane error valde crassus est, et indignus geometræ, quamvis enim irrationalis esset ea proportio, modo in rectis lineis exhibeat, reperta erat quadratura." And that this quadrature is possible geometrically, was not only the opinion of the above mentioned learned and acute geometerian, but likewise of Wallis and Barrow; as may be seen in the Mechanics of the former, p. 517, and in the Mathematical Lectures of the latter, p. 194. But the following discovery will, I hope, convince the liberal geometrical reader, that the quadrature of the circle may be obtained by means of a circle and right-line only, which we have no method of accomplishing by any invention of the ancients or moderns. At least this method, if known to the ancients, is now lost, and though it has been attempted by many of the moderns, it has not been attended with success.

In the circle  $g o e f$ , let  $g o$  be the quadrant arch, and the right-line  $g x$  its tangent. Then conceive that the central point  $a$  flows uniformly along the radius  $a e$ , infinitely produced; and that it is endowed with an uniform impulsive power. Let it likewise be supposed, that during its flux, radii emanate from it on all sides, which enlarge themselves in proportion to the distance of the point  $a$  from its first situation. This being admitted, conceive that the point  $a$  by its impulsive power, through the radii  $a n$ ,  $a m$ , &c. acting every where equally on the arch  $g o$ , impells it into its equal tangent arch  $g r$ . And when, by its uniform motion along the infinite line  $a \varphi$ , it has at the same time arrived at  $b$ , the centre of the arch  $g r$ , let it impel in a similar manner the arch  $g r$ , into its equal tangent arch  $g s$ , by acting every where equally through radii equal to  $br$ . Now, if this be conceived to take place infinitely (since a circular line is capable of infinite remission) the arch  $g o$  will at length be unbent into its equal, the tangent line  $g x$ ; and the extreme point  $o$ , will describe by such a motion

of

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although true, is entirely common. Previous to his demonstration he supposes two squares described, the one



of unbending a circular line &c. For since the same cause, acting every where similarly and equally, produces every where similar and equal effects; and the arch  $g$   $o$ , is every where

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circumscribing the circle, which will be consequently greater; the other inscribed, which will be consequently less than the given circle. Hence, because the circle is a medium between the two given squares, let a mean square be found between them, which is easily done from the principles of geometry; this mean square, Bryso affirms, shall be equal to the given circle. In order to prove this, he reasons after the following manner: those things which compared with others without any respect, are either at the same time greater, or at the same time less, are equal among themselves: the circle and the mean square are, at the same time, greater than the internal, and at the same time less than the external square; therefore they are

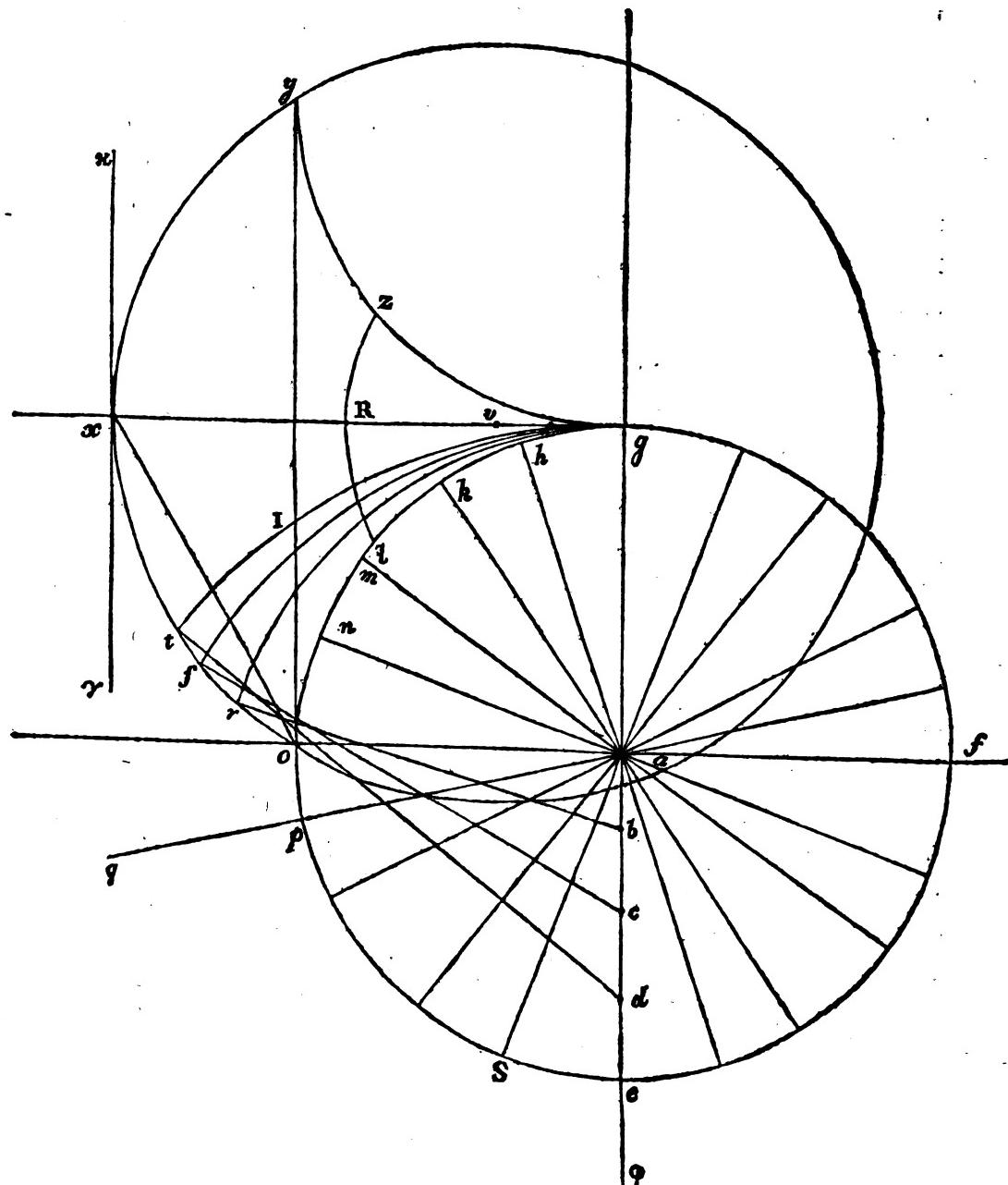
equally remitted or unbent; it will describe a line similar in every part. Now, on account of the simplicity of the impulsive motion, such a line must either be straight or circular; for there are only three lines everywhere similar, i. e. the right and circular line, and the cylindric helix; but this last, as Proclus well observes in his following Commentary on the fourth definition, is not a simple line, because it is generated by two simple motions, the rectilineal and circular. But the line which bounds more than two equal tangent arches cannot be a right line, as is well known to all geometers; it is therefore a circular line. It is likewise evident, that this arch  $ox$  is concave towards the point  $g$ : for if not, it would pass beyond the chord  $ox$ , which is absurd. And again, no arch greater than the quadrant can be unbent by this motion: for any one of the radii, as  $ap$  beyond  $g o$ , has a tendency from, and not to the tangent  $gx$ , which last is necessary to our hypothesis. Now if we conceive another quadrantal arch of the circle  $goef$ , that is  $gy$ , touching the former in  $g$  to be unbent in the same manner, the arch  $xy$  shall be a continuation of the arch  $xo$ ; for if  $yxz$  be drawn perpendicular to  $xg$ , as in the figure, it shall be a tangent in  $x$  to the equal arches  $yx$ ,  $xo$ ; because it cannot fall within either, without making the sine of some one of the equal arches, equal to the right-line  $xg$ , which would be absurd. And hence we may easily infer, that the centre of the arch  $yxo$ , is in the tangent line  $xg$ . Hence too, we have an easy method of finding a tangent right-line equal to a quadrantal arch: for having the points  $y$ ,  $o$  given, it is easy to find a third point, as  $s$ ; and then the circle passing through the three points  $o$ ,  $s$ ,  $y$ , shall cut off the tangent  $xg$ , equal to the quadrantal arch  $go$ . And the point  $s$  may be speedily obtained, by describing the arch  $gs$  with a radius, having to the radius  $ag$  the proportion of 6 to 4; for then  $gs$  is the sixth part of its whole circle, and is equal to the arch  $go$ . And thus, from this hypothesis, which, I presume, may be as readily admitted as the increments and decrements of lines in fluxions, the quadrature of the circle may be geometrically obtained; for this is easily found, when a right-line is discovered equal to the periphery of a circle. I am well aware the algebraists will consider it as useless, because it cannot be accommodated to the farrago of an arithmetical calculation; but I hope the lovers of the ancient geometry will deem it deserving an accurate investigation; and if they can find no paralogism in the reasoning, will consider it as a legitimate demonstration.

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equal among themselves. This demonstration can never produce science, because it is built only on one common principle, which may with equal propriety be applied to numbers in arithmetic, and to times in natural science. It



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is defective, therefore, because it assumes no principle peculiar to the nature of the circle alone, but such a one as is common to quantity in general.

13. It is likewise evident, that if the propositions be universal, from which the demonstrative syllogism consists, the conclusion must necessarily be eternal. For necessary propositions are eternal; but from things necessary and eternal, necessary and eternal truth must arise. There is no demonstration, therefore, of corruptible natures, nor any science absolutely, but only by accident; because it is not founded on that which is universal. For what confirmation can there be of a conclusion, whose subject is dissoluble, and whose predicate is neither always, nor simply, but only partially inherent? But as there can be no demonstration, so likewise there can be no definition of corruptible natures; because definition is either the principle of demonstration, or demonstration differing in the position of terms, or it is a certain conclusion of demonstration. It is the beginning of demonstration, when it is either assumed for an immediate proposition, or for a term in the proposition; as if any one should prove that man is risible, because he is a rational animal. And it alone differs in position from demonstration, as often as the definition is such as contains the cause of its subjects existence. As the following: an eclipse of the sun is a concealment of its light, through the interposition of the moon between that luminary and the earth. For the order of this definition being a little changed, passes into a demonstration: thus,

The moon is subjected and opposed to the sun:

That which is subjected and opposed, conceals:

The moon, therefore, being subjected and opposed,  
conceals the sun.

But *that* definition is the conclusion of demonstration,  
which

which extends to the material cause; as in the preceding instance, the conclusion affirming that the subjection and opposition of the moon conceals the sun, is a definition of an eclipse including the material cause.

Again, we have already proved that all demonstration consists of such principles as are prior in the nature of things; and from hence we infer, that it is the business of no science to prove its own principles, since they can no longer be called principles if they require confirmation from anything prior to themselves; for, admitting this as necessary, an infinite series of proofs must ensue: On the contrary, if this be not necessary, but things most known and evident are admitted, these must be constituted the principles of science. He who possesses a knowledge of these, and applies them as mediums of demonstration, is better skilled in science, than he who knows only posterior or mediate propositions, and demonstrates from posterior principles. But here a doubt arises whether the first principles of geometry, arithmetic, music, and of other arts, can ever be demonstrated? Or shall we allow they are capable of proof, not by that particular science which applies them as principles or causes of its conclusions? If so, this will be the office of some superior science,—which can be no other than the first philosophy, to whose charge the task is committed; and whose universal embrace circumscribes the whole circle of science, in the same manner as arithmetic comprehends music, or geometry optics.—This is no other than that celebrated *wisdom* which merits the appellation of science in a more simple, as well as in a more eminent degree than others: not, indeed, that all causes are within its reach, but such only as are the principal and the best, because no cause superior to them can ever be found.. Hence the difficulty of knowing whether we possess science or not,

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not, from the difficulty of understanding whether it is founded on peculiar or common principles; since it is necessary that both these should be applied in the constitution of all real knowledge and science.

\* Again, axioms differ from postulates in this:---they demand our assent without any previous solicitation, from the illustrious certainty they possess. Their truth may, indeed, be denied by external speech, but never from internal connection. He who denies that equal things shall remain from the subtraction of equal, dissent, as Euripides says, with his tongue, and not with his heart. But demonstration depends not on external speech, but on intellectual and internal conviction; and hence, axioms derive all their authority from intrinsic approbation, and not from public proclaim. For the prompt decisions of the tongue are frequently dissonant from the sentiments concealed in the secret recesses of the heart. Thus the † geometrician does not speculate those lines which are the objects of cor-

\* Axioms have a subsistence prior to that of magnitudes and mathematical numbers, but subordinate to that of ideas; or, in other words, they have a middle situation between essential and mathematical magnitude. For of the reasons subsisting in soul, some are more simple and universal, and have a greater ambit than others, and on this account approach nearer to intellect, and are more manifest and known than such as are more particular. But others are destitute of all these, and receive their completion from more ancient reasons. Hence it is necessary (since conceptions are then true, when they are consonant with things themselves) that there should be some reason, in which the axiom asserting, *if from equals you take away equals, &c.* is primarily inherent; and which is neither the reason of magnitude, nor number, nor time, but contains all these, and every thing in which this axiom is naturally inherent. Vide Syrian. in Arist. Meta. p. 48.

† Geometry, indeed, wishes to speculate the imparible reasons of the soul, but since she cannot use intellections destitute of imagination, she extends her discourses to imaginative forms, and to figures endued with dimension, and by this means speculates immaterial reasons in these; and when imagination is not sufficient for this purpose, she proceeds even to external matter, in which she describes the fair variety of her propositions. But, indeed, even then the principal design of geometry is not to apprehend sensible and external form, but that interior vital one, resident in the mirror of imagination, which the exterior inanimate form imitates, as far as its imperfect nature will admit. Nor yet is it her principal design to be conversant with the imaginative form; but when, on account of the imbecility of her intellection, she cannot receive a form destitute of imagination, she speculates the immaterial reason in the purer form of the phantasy; so that her principal employment is about universal and immaterial forms. Syrian. in Arist. Meta. p. 49.

poreal

poreal sight, but such as are exhibited by mental conception, and of which the delineations on paper, or in the dust, are no more than imperfect copies, notes, and resemblances. Thus, when he draws a pedal line which is not pedal, or an equilateral triangle which is not equilateral, we must pay no regard to the designations of the pen, but solely attend to the intellection of the mind; for the property demonstrated of some particular line, is in the conclusion applied to one that is universal, and this true line could be no otherwise signified to the learner than by a material description.

The certainty of axioms is, indeed, in a measure obvious to every one. For what more evident than that nothing exists of which it is possible, at the same time, to affirm and deny any circumstance of being? Indeed, so illustrious and indubitable is the light of this axiom, that in any demonstration we are ashamed to assign it the place of an assumption. It would almost seem prolix and superfluous, since there is nothing more manifest and certain; and yet there are cases in which it is necessary to rank it among assumptions. And these take place whenever the intention is to conclude the existence of something as true, and of its opposite as false. Thus, for instance, in the demonstration that the world is finite, we assume this principle, and then reason as follows:

Bound and infinite cannot be at the same time affirmed and denied of any *body*:

The world is a body:

Therefore the world is not at the same time finite and infinite.

And in this genus of demonstration, the major proposition ought always to assimilate with the conclusion. But the above axiom is not the only one obvious, for the following.

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owing possesses equal certainty; that of every thing which exists, either affirmation or negation is true. This axiom is of great use in demonstrations leading to an absurdity; for he who demonstrates the impossibility of any opposite assertion, necessarily establishes his own. Hence it is we affirm that the diameter of a square is either commensurable or incommensurable with its side; and this general principle is accommodated, and, as it were, descends into its proper matter as often as that which it possesses of universal is contracted to a certain genus; for, as we have previously observed, common principles are not admitted in demonstration without any restriction; but then only when their general nature is limited to some particular subject, by which they become peculiar and apposite.

I.4. \* Wisdom, or the first philosophy and logic, agree in not using axioms after the same manner as other arts; but on the contrary, they confirm and establish their certainty, though with this difference, that the logician reasons only from probabilities, but the metaphysician from the highest certainty and evidence. Besides, we do not

\* Syrianus, in his excellent Commentary on Aristotle's Metaphysics, (which does not so much explain Aristotle, as defend the doctrine of ideas, according to Plato, from the apparent if not real opposition of Aristotle to their existence), informs us that it is the business of wisdom, properly so called, to consider immaterial forms or essences, and their essential accidents. By the method of resolution receiving the principles of being; by a divisive and definitive method, considering the essences of all things; but by a demonstrative process, concluding concerning the essential properties which substances contain. Hence (says he, because intelligible essences are of the most simple nature, they are neither capable of definition nor demonstration, but are perceived by a simple vision and energy of intellect alone. But middle essences, which are demonstrable, exist according to their inherent properties: since, in the most simple beings, nothing is inherent besides their being. On which account we cannot say that *this* is their essence, and *that* something else; and hence they are better than definition and demonstration. But in universal reasons, considered by themselves, and adorning a sensible nature, essential accidents supervene; and hence demonstration is conversant with these. But in material species, individuals, and sensibles, such things as are properly accidents are perceived by the imagination, and are present and absent without the corruption of their subjects. And these again being worse than demonstrable accidents, are apprehended by signs, not indeed by a wise man, considered as wise, but perhaps by physicians, natural philosophers, and all of this kind.

rank

## DEMONSTRATIVE SYLLOGISM. Ixv

rank logic in the order of the sciences, because it is destitute of some determinate genus or subject, as it is neither conversant about lines, nor numbers, nor proportions. And its chief concern is about apparent properties, and not such as are essential to a subject.

Hence, in logical disquisitions, we confidently employ interrogations, as equally subservient to the affirmation or negation of an opinion:---a method utterly impracticable, if we only employed those principles which are universally acknowledged; since it is impossible of the same thing to prove contrary properties,---as of the soul, that it is mortal and immortal; but he who demonstrates, assumes one definite part of a question, because his purpose is not to interrogate, but to trace out the latent paths of truth. And hence, if any one affirms that the soul is moved, and immediately after denies it, he is no longer a subject worthy the exercise of our discursive and reasoning powers.

Again, it may so happen, that the same science at one time considers *why* a thing is, at another only explains its existence, or that it exists, without considering the cause. Thus, the syllogism which concludes by mediate propositions, demonstrates without assigning the proper cause: but that which determines by immediate ones, in a great measure explains the cause or reason of existence. Thus, he who infers that trees do not breathe because they are not animals, reasons from a mediate and secondary cause, because there are many animals, such as insects, which exist without breathing: but he who infers this from their want of lungs, demonstrates from the immediate and primary cause.

Thus, the following syllogism is a mediate one, or such as requires one or more mediums to establish its certainty:

Every thing that is not an animal does not breathe;

A tree is not an animal;

Therefore a tree does not breathe.

Here the major proposition is evidently mediate, because we are still to seek why that which is not an animal does not breathe, which the following immediate syllogism solves.

Every thing that is not endued with lungs does not breathe;

Every thing that is not an animal is not endued with lungs; ergo,

Every thing that is not an animal does not breathe.

Again, the same science may demonstrate the existence of a thing, or that it exists, and the cause of such existence, as often as it assigns two immediate reasons; but the one from the proper cause, the other only from a sign. Thus, he who demonstrates the increase of the moon, from the plenitude of her orb, infers the cause of such increase; but on the contrary, he who collects the plenitude of her orb from her increase, reasons only from a sign, and can alone declare its existence. And, indeed, it often happens that the *cause* and *sign* reciprocate, so that as from the sign we advance to the cause, demonstration from the cause frequently recurs to the *sign*. Thus, from the breadth and firmness of the basis, we collect the permanent duration of the pyramid; and from its extended existence we infer the strength of its support. Whenever, then, the argument originates from a sign, it gives evidence to the conclusion, as from something more known than its cause. When it begins from the cause, it proceeds from that which is first in the order of nature, to that which is last, and reasons as from the proper principle of the thing.

Sometimes the *cause* and *sign* do not reciprocate. Thus, although wherever there is smoke, we infer the existence of fire; yet we cannot infer, that wherever there is fire smoke exists. Thus, from the palace and the picture we collect the existence of the architect and painter; but the last may exist without the first;—the living architect without

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out the actual palace; and the living painter without the energies of his art. And thus it is that the cause is illustrated by its sign; but not always the sign by its cause.

Hence then, as all causes do not reciprocate with their effects; so neither is it always causes and effects which do reciprocate: because a multitude of signs, mutually inferring each other, may accompany a certain cause. Thus, the signs which attend the causes of a fever, are a quick pulsation of the artery, and an intense heat: and these signs mutually assert each other; but no syllogism can be composed from either expressing the *why*, but only simply *that* the other exists.

15. We now propose to consider the mode in which the two preceding demonstrations are distributed in different sciences. When sciences then are so related, that the one is dependent on the other, as optics on geometry, navigation on astronomy, and music composed by the arbitration of the ear, on that which consists in the knowledge of mathematical proportions: in this case, the demonstration of simple existence, or *that* they *exist*, pertains to the science of sensibles; but the demonstration *why* they *exist* to the science which is speculative and mathematical.

Thus the mathematician speculates the causes of a certain sensible effect, without considering its actual existence; for the contemplation of universals excludes the knowledge of particulars; and he whose intellectual eye is fixed on that which is general and comprehensive, will think but little of that which is sensible and singular. Thus, by mathematics we may learn the responsive harmony of the *last chord*, and its consonance with the *mean*; but we cannot perceive this concord, if unaccustomed to the practice of the musical art. In fine, those sciences which are more of a mathematical nature, I mean such as are more amply

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conversant with the inspection of things, considering their forms abstracted from every material subject, always demonstrate the *wby*; and such is geometry in respect of optics. Thus geometry considers only such things as are peculiar to right-lines, independent of every sensible connection. For the geometrician does not investigate a right-line as contained in stone or brass; but considers it as entirely detached and unconnected with any object of sense.

On the contrary, optics receives a right-line just as it is perceived in a rule, or engraved in brass. And, indeed, in treating of some particulars, natural science has the same relation to optics, as optics to geometry. Thus, in considering the reason of the appearance of the rainbow, the natural philosopher defines the bow to be an image refracted from a certain cloud against the sun; but why it is endued with such a form, and seen with such a colour, must be assigned by him who is skilled in optics. There are, again, sciences, one of which is not subordinate to the other, because founded on principles totally different; yet, in some particulars they agree with the preceding. Thus, to know that an orbicular wound is the most difficult of cure, belongs to the physician; but to know *wby*, to the geometrician.

16. Of all syllogistic figures, the first is the best adapted to science, since the arithmetician, geometrician, and lastly all those who demonstrate any effect from its proper cause, fabricate their reasonings according to this figure. For the middle figure is seldom used, because only adapted to a few occasions: and since the knowledge of the *wby* is of all others the most important, which is alone obtained by this figure: hence, in the pursuit of science, it is always preferred before the rest. Besides, it is equally accommodated to the knowledge of final causes; to which it alone tends

tends : for it composes definitions from words universal, and affirmative. In the second figure, a complex negative is conceived ; and in the last, a particular one. Add to this, that mediate propositions are no other ways reducible to immediate ones than by this figure, in which the mediate proposition tends, by a continued series, to that which is immediate. But the second does not conclude affirmatively, nor the last universally ; from whence it appears, that a mediate proposition can never become immediate by these figures : not that all affirmative propositions are immediate ones, since some negatives are of this kind ; for all propositions are equally immediate, which cannot be confirmed by syllogism ; and such are those negatives, of whose terms it is impossible any genus can be affirmed. Thus the proposition, *no substance is quality*, is an immediate negative of this kind, whose terms are two of the most universal genera of things.

Again, as we have frequently affirmed that he who demonstrates, always assumes such things as are essentially predicated ; but that he who argues dialectically or topically, not always, but generally assumes such as are accidentally predicated, and which appear more probable and known than such as are essentially inherent ; it is proper we should define what is meant by accidental predication ; or something predicated by means of another. Indeed, the term has a diffuse signification : for, first, a body is said to be white by something else, because by its superficies ; and in this manner vines are white, because their branches are white. Thus, if accident be predicated of accident, it is by means of another ; as when we say the musician is fair ; for the being a musician is an accident of man, and the being fair of the musician : and man is the subject of each. The predicate of substance is equally accidental, when not included

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included in the number of things substantially inherent; as when we affirm of any particular man that he is red, or black. But the predication is especially accidental, as often as, by perverting the order of nature, substance is predicated of accident; as when we say something white is an animal: for this assertion differs from that other, *animal is white*. In the latter, the subject *animal* is neither inherent in another, nor subsists by another, but has an essential existence. In the former, what is assumed as a subject derives its existence from that of which it is the accident. It is only dialectically, therefore, that we can argue from predicates as probable and known without any distinction: but in demonstration, all that are preposterous and accidental must be carefully avoided, excepting such accidents as being essentially in a subject, admit of an essential predication; and some of these we have enumerated before.

17. We are now entering on a disquisition neither ignoble nor useless: it is this, whether the number of things predicated essentially of a subject is finite, or whether things in a continued series run on to infinity. For instance, let us suppose some ultimate subject, which is not the predicate of any thing besides; and let *c* represent such a subject, of which *b* is the first and immediate predicate; and in the same manner *d* of *b*, and *e* of *d*: the query is, Whether or not this extraction must necessarily stop, or will admit of an immense progression, so that *f* may be predicated of *e*, and *g* of *f*, and so on infinitely; the power of the predicates, which supplies the common identity, still remaining inexhaustible and undiminished? The second query is this, Supposing some general subject, which we call *a*, of such a nature as to be no longer the subject of any farther predication, but to be itself the supreme and primary predicate; and supposing that it is immediately inherent

inherent in  $f$ , and  $f$  in  $e$ , and  $e$  in  $g$ ; whether or not the process must stop, or extend to infinity, and no subject be found which is not directly predicable of another? There is a remarkable difference in the two considerations; for, in the former we enquire whether any ultimate subject can supply an infinite ascent of predicates; in the latter, whether any first predicate can exist in an infinite descending series of subjects. The third question is, supposing two extremes constituted from a first predicate and last subject, whether it is possible an infinite number of mediums can intervene? And this is no other than to enquire whether demonstrations admit of an infinite progression, so that whatever is assumed in proof of another, must be proved itself? Or whether it is not more agreeable to truth, that there should be some immediate propositions and ultimate terms, whose discovery may give respite to enquiry, and stay the elaborate process of demonstration? The same question occurs in negatives. But that some of these are immediate, the instance lately alledged sufficiently evinces. The solution of this enquiry is not so difficult in subjects which mutually reciprocate; for in these, when the ultimate subject is given, no one can doubt the existence of their primary predicate; nor when the primary predicate is admitted, can there be any doubt of the existence of some ultimate subject. For, in things which mutually reciprocate, whatever is enquired of the one, is immediately questioned of the other; and wherever there is a last subject, there must be a first predicate; for by the conversion of the ultimate subject you effect the primary predicate.

Previous to the discussion of the first question, it is necessary to know that infinite intermediates cannot intervene between two finite terms in an ascending and descending series of predication. I call the series ascending which rises.

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rises to universals; but descending, which, by a contrary process, stops at particulars. Thus, if any one admits that  $a$  is some first predicate, and  $g$  some ultimate subject, and should contend, that between these terms there may be infinite mediums, he contradicts himself; since he who begins from  $a$  in a descending progression, will never, by this means, arrive at  $g$ ; and he who departs from  $g$  in an ascending series, can never finally rise to  $a$ . So that the extremes can be no longer finite, as the hypothesis admitted. Indeed, the absurdity of such a supposition is the same as to contend that between one and ten, an infinity of numbers may exist; which is evidently impossible, because the discrete nature of numbers excludes their actual existence in infinitum, between any finite limits; since they can only become infinite from their actual existence and precedence, and not from any dormant power or capacity they possess: for between any two given numbers there is nothing similar to number in capacity, which can ever become number in energy; as in quantity continuous between any two points there are always parts in capacity, which, whenever a proper agent is at hand, become immediately actual. In like manner, he who admits the terms finite, but believes that the mediums are infinite, asserts what is impossible, since these logical predication are of the same discrete nature with numbers themselves. Thus all the predicates which can exist between Socrates and substance, must exist actually, or not at all; for surely between these two terms, or periods, no predicate in capacity can ever be supposed to subsist. If it be urged, that the capacity of receiving these predicates exists between Socrates and substance, still we reply, it is not that kind of capacity in which these predicates can retain the most shadowy existence; out of which they can ever be called forth into energy,

energy, as from some latent retreat; or into which they can finally retire, when energy is no more. And hence we conclude it impossible that infinite mediums can exist between any finite terms.

18. It now remains that we prove, first, by probable arguments, and then by such as are demonstrative, that the extremes in any series of predication are finite; and that an infinite progression is impossible, not only in substantial predicates, but in such as are accidental. For every thing predicated of another is either essentially or accidentally inherent; and is predicated in a natural or preposterous order. It is predicated according to nature, when accident is declared of substance; contrary to nature, when substance of accident. That essential predicates are finite, appears from hence, because a contrary hypothesis excludes the existence of definition, by admitting that all things are contained in some superior genus, and acknowledge some farther definition; since it is impossible that the definitions of genus can ever be circumscribed, while there is a continual supply of other genera, which can never be known without definition; for thus we shall never obtain either a beginning or an end. But to define all things is not possible, because infinity can never be absolved by the most unwearied progression. Predications then, of this kind, are always circumscribed by a certain number of terms, which prevent their infinite process, and cause all the strength of demonstration, and all the certainty of human knowledge. The same may be proved in accidents; for such as are predicated of substance, are either predicated as qualities or quantities, as relatives, or as actions and passions; as expressive of some habit, or significant of some place; or as connected with some time. Thus we say the wood is white, the triangle is scalene; whiteness being accidental

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to the wood, and scalenity to the triangle. It is therefore certain, that every accident is predicated of substance; and it is no less certain that the predicates of substance are finite, since they are all included in the ten universal genera of things.

19. We have hitherto defended the impossibility of an infinite progression of logical predicates and subjects, in a demonstrative process, by such arguments as are dialectical and common: it now remains that we adopt such as are peculiar and certain. Demonstrations, then, are derived from affections essentially inherent in a subject; and these are either such as take place in definitions of a subject, as multitude and quantity, are essentially predicated of number; or, secondly, accidents which are defined from their subjects, as imparity by number. But the predication cannot, in either case, be extended to infinity. For it is not necessary that in the same manner that imparity is predicated of number, something else, suppose *c*, should be predicated of imparity; and so imparity be contained in its definition, similar to number in the definition of imparity. For in predications of this kind, the terms are always assumed more contracted than their subject; and at length, by a continued procession, must terminate in an indivisible. Thus, as imparity is more contracted than number, *c* must be more contracted than imparity. Hence, these predications either finally stop, for the reasons we have assigned; or because whatever is predicated of imparity, is necessarily predicated of number; so that one thing as number would be actually contained in the definition of an infinity of things; and so actual infinity must ensue, which is absurd. Lastly, whatever is said to reside in the terms, must be allowed to reside in the subject; so number must be applied in the definition of every affection;

and

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and an infinite number of properties will be essentially inherent in number; and number will inherit infinite definitions. But affections essentially resident in a subject cannot be infinite, because it is necessary they should exist in energy. Thus, imparity cannot exist potentially in number; nor reason in man; nor rotundity in a circle, because wherever these subjects have an actual being, it is necessary these essential attributes should be actually inherent. Again, in the definitions of a subject, an infinite process is impossible, because from such an hypothesis nothing could ever be defined; and thus it appears that neither can demonstrations be infinitely extended, nor every thing admit of demonstration, an opinion we have already noticed in the beginning of this section: for if neither universally, nor in every proposition a middle term can be assumed, but as soon as we arrive at immediate propositions, the labour of investigation is finished, the possibility of demonstrating every thing can no longer be defended; since it is proved above, that by limiting the extremes, an infinite number of mediums is necessarily excluded.

And thus, by taking away infinity from the reasoning art, we have given a support to science, which the most vigorous efforts of subtle sophistry can never finally subvert. We have set bounds to that restless spirit of enquiry which wanders uncontrouled in the mind unenlightened by science, by every where circumscribing its progress within the limits of that which is most particular, and most universal, a first predicate, and an ultimate subject: and finally, by asserting that all the evidence of human knowledge results from the lustre of primary and immediate principles, we have held up a steady and permanent light, ever sufficient to direct our steps through the dark mazes of ignorance and error, into the bright paths of certainty and truth.

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- 20. Let us next consider whether universal demonstration is preferable to particular, or not: And first, in favour of particulars we may say that their evidence is more exquisite and certain than that of universals. Thus, the knowledge, from inspection, that Callias is a rational animal, is superior to that acquired by a reasoning process which infers his rationality, because every man is a rational animal. By particular demonstration a thing is known as it is, by universal only in common. Besides, particulars possess some solidity, universals none: and the demonstration of things which have a real existence, is more excellent than that of things which have none. And there are no errors more frequent than those about universals; demonstration considering them as things entirely abstracted from singulars. On the contrary, particulars are usurped by the sight, grasped, as it were, by the hand, and the general subject of every sense; so that concerning these, demonstration affirms nothing false or inconstant. But these reasons, however plausible, are easily confuted. And, first, the term essential is more closely connected with universals than particulars. Thus the possession of three angles equal to two right, is an affection more essential to the triangle itself, than to one equilateral or scalene. Add too, that in the demonstration of universals we always infer some property of a subject from its simple existence, or because it is such a subject. Again, many affections are contained in singulars assumed from no particular nature, but from that which is universal; as rationality in Socrates, which is not inferred from his existence as Socrates, but from his existence as man. Farther, that demonstration is the more excellent which is derived from the better cause: but an universal cause is more extended and excellent than a particular one; since the arduous investigation

of

of the *why* in any subject is stopt by the arrival at universals. Thus, if we desire to know why the exterior angles of a triangle are equal to four right ones; and it is answered, because the triangle is isosceles; we again ask, But why because isosceles? And if it be replied, because it is a triangle, we may again enquire, But why because a triangle? To which we finally answer, because a triangle is a right-angled figure; and here our enquiry rests at that universal idea which embraces every preceding particular one, and is contained in no other more general and comprehensive than itself. Add too, that the demonstration of particulars is almost the demonstration of infinites; of universals, the demonstration of finites.—We add farther, *that* demonstration is the best, which furnishes the mind with the most ample knowledge; and this is alone the province of universals. Again, the principles of science become immediate only in proportion as the demonstration becomes universal; and he who knows universals, knows particulars in capacity: but we cannot infer, that he who has the best knowledge of particulars, knows any thing of universals. Lastly, that which is universal, is the province of intellect and reason, particulars are the offspring of sense; and hence we conclude that universal demonstration exceeds particular both in dignity and excellence, and is first in the nature of things, although last in the progressions of the reasoning power.

Again, That affirmative demonstration is superior to negative, appears from hence: the affirmative does not require the assistance of the negative; but the negative cannot exist without the affirmative; on which account, the demonstration composed from negatives alone, is incapable of producing real evidence and conviction. Besides, affirmation exceeds negation both in priority and simplicity of existence.

Again, the demonstration which concludes *directly*, is better than that which confirms a proposition by evincing the absurdity of its contrary. The first proceeding in a regular order, establishes, by a natural deduction, the truth which was first advanced. The second taking a wider circuit, yet with the same intentions produces a conclusion quite opposite to its apparent design. The one may be compared to the open attack of a valiant and skilful soldier, who expects the conquest of his enemy from strength and courage alone: the progress of the other resembles the same soldier, uniting force with stratagem, and advancing, by an irregular march, which his foe mistakes for a retreat, but finds the secret cause of his destruction. The first is simple and impromiscuous, as composed from propositions alone: the second is compound and miscellaneous, calling in hypothesis to its assistance.

21. One science is said to be prior to, and more certain than another in many respects; —when the one reasons from primary causes, but the other from such as are secondary: —when the one may be ranked in the genera of intelligibles and universals; but the other in the genera of sensibles and particulars. And such is the relation of arithmetic to music; of geometry to optics; and lastly, of every superior to every subordinate science. Again, this happens when the one reasons from simple principles, the other from such as are complex and connected; on which account arithmetic seems to possess greater certainty than geometry. For the principle of arithmetic is unity; but of geometry a point; and unity is without position, with which a point is always connected. And in this manner geometry inherits greater evidence than astronomy; for the one considers body simply, the other as connected with a circular motion. The science is called one which contemplates

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templates actions belonging to one genus: the genus is one which possesses the same first principles; and hence geometry and stereometry form one science. On the contrary, the sciences are called different which have different principles, such as geometry and optics; the latter of which does not originate from the principles of the former.

Again, the same thing may admit of many demonstrations, and may be known from many mediums: at one time from the application of such as are congenial: at another, from those of a different order or genus. From congenials, as when we demonstrate that the plantain is a substance, first, by the medium of a tree, and then by the medium of a plant, thus:

Every tree is a substance;

The plantain is a tree:

Therefore the plantain is a substance. And again,

Every plant is a substance:

The plantain is a plant:

Therefore the plantain is a substance.

We demonstrate, from mediums, of a following order or genus, as when we prove man to be a substance, at one time from his being rational, at another from his being a biped; and these mediums, in part, mutually contain each other.

22. Fortuitous events can never, in any science, become the subject of demonstration; since they are neither limited by necessity, nor admit the arrangement of syllogism. Indeed, so far from obtaining a necessary, they do not possess a frequent existence, but every syllogism is composed from one or other of these.

Again, science is not the business of sense, since that which is universal is the object of perception in particulars themselves. For the object of sight is colour in general,  
and

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and not this particular colour: the object of hearing is found in general, and not any particular sound; and on this account we see or hear not only this or that colour or sound, but likewise every other which falls under the cognizance of these senses. Hence, if it were possible for any one to discern by his sight, the equality of the three angles of some particular triangle to two right, he would not by this means possess a demonstration of the conclusion which affirms this to be the property of every triangle; but his knowledge would extend no farther than the triangle he inspects. Thus too, if we could perceive an eclipse of the moon to arise from the interposition of the earth, we could not universally conclude that this is the cause of every eclipse, but only of the particular one we behold. For the explication of causes extends to universals; and comprehends not only the knowledge of one particular defect of the moon, but simply of every eclipse; since the interposition of the earth is not so much the cause of any present eclipse, as of all which can possibly exist in every age. Whenever, then, the cause is universal, the knowledge of any effect deduced from such a cause is, in every respect, superior to the evidence arising from the perceptions of sense. It is likewise more excellent than the apprehension which subsists independent of the proper cause; as if any one should give absolute credit to the proposition, that the three angles of a triangle are equal to two right, without a previous conviction that the external angle of a triangle, is equal to the two interior opposite ones; and without applying this last proposition as the cause of the first. The comprehension, then, which is conjoined with the proper cause, far exceeds the strongest evidence of sense.

But perhaps it may be said that science consists in sense, because the science of any particular, fails from a defect of the

the sense by which it is apprehended. To this we reply, that science, indeed, is not acquired without the assistance of sense, but it does not follow from hence, that *to perceive* is *to know*; because the object of science is that which is universal; but of sense, that which is particular. Thus, if we could see light penetrating the pores of glass (on the atomical hypothesis) the cause why it illuminates would be manifest from sensible inspection as the means, and from the universal apprehension of science, by which we should understand this to be universally true.

Again, the principles of all sciences cannot be the same neither considered as remote or proximate. Not considered as proximate, because the principles always correspond to the demonstrated conclusions; but these are not the same, since they are often generically different; and consequently the propositions from which they result must be derived from discordant genera. But propositions consist of such things as essentially exist; and hence we infer, that the principles of geometry are essentially distinguished from those of arithmetic, that they cannot admit of reciprocal accommodation, so that the one may be predicated, or become the subject of the other, and that the one can never be subservient as a medium to the other. Again, common and first principles are not applied in every science; such as this, that every thing must either be affirmed or denied. Nor can any thing be proved by their assistance alone, but as often as these are required in demonstration, other principles more proximate and peculiar to the given proposition, must always be adopted. Again, axioms universally conceived, cannot be assumed in syllogism, but they must be contracted, as it were, to some subject genus. Of this kind is that common axiom, that as often as any four quantities are proportionable, by permutation, or changing

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the order of the terms, the same ratio will result. For the arts apply this axiom in a restricted sense; geometry, by considering the relatives as four magnitudes, and arithmetic as four numbers; but the natural philosopher, by adapting the comparison to four motions, or four times. Besides, if the principles of all sciences were the same, it is necessary they should be comprehended by some certain number, similar to the limitation of the elements: but every science is capable of immense increase from the many different modes of amplification the conclusions will admit; and consequently it is requisite to establish a correspondent number of proper principles; for such as are common cannot be alone sufficient. Lastly, if the same principles accord with every science, it follows, that any thing may be demonstrated from such principles: but the certainty of geometrical conclusions cannot be established from the principles of music; and from hence it follows, that although the principles of every science are not the same, they do not possess an entire diversity, nor yet an absolute affinity of nature.

23. There is a remarkable difference between science and opinion. Whatever is the subject of science must have a necessary existence; on the contrary, opinion is conversant with things liable to mutation and decay. Again, as science depends on necessary propositions for support, so opinion on such as possess only a possibility of existence; and so there is one mode of approbation in subjects of opinion, and another in those of science. Hence science is distinguished from opinion by two discriminations, the one arising from their subjects, the other from the mode of approbation. That opinion is conversant with things possible or contingent, we may learn from hence; contingencies cannot belong to science, because their existence is not

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not necessary ; nor to intellect, or that principle of science by which its terms are known ; nor to the apprehension or belief of immediate propositions, called indemonstrable science. Hence, if every habit by which truth is known, is either science, or intellect, or opinion, it remains that opinion alone consists of things which are, indeed, true ; but not necessary. It is, therefore, inconstant and unstable, from the mutable nature of its subjects. Besides, no one thinks he possesses an opinion of things which he believes to have a necessary existence, so that they cannot be otherwise than they are ; but to such conviction he properly gives the name of knowledge, and to its contrary the name of opinion.

Again, the same thing from the same propositions may at one time become the subject of knowledge, at another, of opinion ; and this happens according to the different formation of the syllogism which the propositions compose ; whether reasoning from the proper cause it explains the *why*, or only simply declares a thing exists. Hence a doubt arises why opinions of this kind may not be called science, since both the subjects and propositions are the same ? The solution is obvious. If it is believed that the propositions cannot be otherwise than they are, or that they have a necessary existence, such an assent of the mind is not opinion, but science ; because things which inherit an essential existence are the ornaments of science alone. On the contrary, if we are convinced that the propositions are true, but at the same time not necessary, such conviction is not science, but opinion. Hence, it is impossible that science and opinion can be the same, since they vary in their definition and mode of approbation, and in a different manner demand our belief. Similar to this, although it may happen that of the same thing a true and a false opinion may arise, it

will not therefore follow, that true and false opinions are the same. For that which is firm and constant can never be the same with that which is mutable and frail; and that which is always true must be essentially different from that which may be changed into false. By the power of habit indeed in different men, the same thing may be comprehended by opinion and science. Thus it was opinion in Epicurus when he said that the sun was eclipsed by the moon passing under its orb, because he thought it might otherwise happen, and that the moon might be interposed without obscuring the light of the sun. It was science in Hipparchus, because he knew it as a necessary event. But in the same mind, at the same time, and of the same thing, it is impossible that science and opinion can exist. And thus much concerning the difference of the two.

24. Lastly, sagacity is an acute and sudden apprehension of the medium, or proper cause of a certain effect: as if any one, beholding the moon, should in a moment conjecture the cause of the part opposite to the sun being lucid, and the other parts obscure, because she derives her splendor from the sun: Hence he is universally called acute and sagacious, who, from the aspect or hearing of the extremes, can readily perceive the medium which exists between them: as the term imports a certain revolution of the conclusion into its first propositions, and, as it were, a swift comprehension and continuation of the medium.

S E C-

## S E C T I O N III.

**I**N the ensuing Commentaries, the soul is considered as immaterial ; and as possessing a middle nature between intelligibles and sensibles ; but that this important assertion may not remain without proof, the following demonstrative arguments, derived from the Platonic philosophy, are offered to the reader's consideration. And first, that the soul is an unextended, and consequently immaterial essence, may be thus proved. If, after the manner of magnitude, it consisted of continuous parts, it would be impossible that any one part could be sentient from the passion of another; but the soul, for instance, which is situated in the finger, would be sensible of passion, as if detached from soul in the other parts of the body, and existing by itself ; for the soul, from this hypothesis, would be sentient by parts, and not considered as a whole. Besides, there must be many souls governing each part of us, different among themselves, and endued with their own peculiar energies. For whatever may be said of continuity, is to no purpose, unless it conduces to unity of sensation ; so that the hypothesis which supposes that the sensations gradually arrive at the principal part of the soul by a certain continued succession, is not to be admitted, since it may be reasonably asked, How is the principal part to be peculiarly distinguished ? By what rule of quantity can the parts be discerned, by what difference are they to be distinguished, where the quantity is one, and the bulk continuous ? Besides, is the principal part alone, or are the other parts, sentient ? If the principal part alone be perceptive, the soul can then alone be sentient when a sensible passion meets with this principal

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cipal part, situated in its peculiar seat; but if a sensible passion falls upon any other part of the soul destitute of sense, it is impossible that such a part should be able to transmit the same passion to the principal, or be at all sentient. For how can that which is void of sense, receive passion, and convey it entire to a sentient part? Besides, if passion accedes to the principal, it either falls on one of its parts, and so either one part alone will be sentient, and the rest without sensation, and consequently superfluous, or there must be innumerable and dissimilar sensations; for if the sensation of each of the parts singly, is the same with the aggregate of them all, of what use is a multitude of parts? But if the sensations are various, a man may say, as it were, I am primarily sentient in this place, and secondarily in another; and every sentient part besides the first, will be ignorant where sensation is most powerful: or perhaps, (from such an hypothesis) every part of the soul will be equally deceived, each part thinking the passion to arise in the place where it is situated. But if not the principal part alone, but every part of the soul be sentient, a principal part is superfluous for the purpose of sensation; and if the soul be divisible like magnitude, how is it able to recognize, as belonging to one subject, the qualities which flow, as it were, through many senses, as through the eyes and ears? For that part of the soul which is perceptive through the eyes, distinguishes nothing but colours; that which energizes through the ears, nothing but sounds; and that which acts through the medium of the touch, nothing but the surfaces of bodies: what is it then which perceives all these properties of bodies united in one subject, or what is that which perceives any thing as a certain one? For unless the intentions of the senses, and of sensible objects, were collected together in one, the soul could never

never be able to judge of the peculiar and different properties of bodies. Hence it is necessary that the soul should be, as it were, a general centre; that the several senses should be extended on all sides to this, similar to lines verging from the circumference of a circle to the centre; and that a power of this kind, comprehending all things, should be truly one. For if the soul was any thing divisible, and the intentions of the senses reached the soul, and ended in its essence like the extremities of various lines, they must either again concur in one and the same as a medium, or have different situations, in such a manner that each sense may perceive different from one another: as if, for instance, the sense of sight should perceive the form of Socrates, and the sense of hearing recognize his voice; that essence which pronounces the whole to be one person, that of Socrates, must be something different from each of the senses. Hence it is necessary that the soul should be an indivisible essence; for if she possessed magnitude, she must be divided along with every sensible object she perceives; so that one part of the soul would perceive a certain part of a sensible object, and we should possess no sentient power capable of perceiving the whole, or of pronouncing any thing one. Thus, in the perception of a man, considered as one, how is it possible that the soul can be divided, so that the perception of a part shall be the same with the perception of the whole. But if we suppose the soul divisible in all her ~~part~~ions, since it is impossible she should be co-extended with every sentient object, in how many parts is the division to be made? Is the soul to be distributed into the same number of parts as the sensible object she perceives, so that every part of the soul may perceive the same part of the object? Or shall we say that the parts of the soul have no sensation of the parts of the object?

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object? But this would be absurd and impossible. If every part then of the soul perceived every part of a magnitude as a whole, since magnitude is divisible to infinity, and since, on this hypothesis, there must be innumerable sensations of every sensible object, there must be innumerable images, as it were, of the same thing in our principal part. Besides, if that which perceives is corporeal, it will not be possible for it to perceive in any other manner than as if certain images were impressed from a seal in wax, or in brads, or in any other sensible substance. But if the images of sensation exist as in humid bodies (which is most probable), they will certainly be confounded like images in water; nor can there be any memory, the image departing with its forming substance. And if we suppose the figures to remain like impressions in solid bodies, either it will not be possible for others to succeed while the former endure; and thus, sensations of other things cannot take place; or if others succeed, the former images must be immediately destroyed, and memory be no more. So that if we allow it possible to remember, and, besides this, to perceive other things, without any hindrance from former impressions, it is impossible that the soul should be corporeal. Since the soul, therefore, is an unextended, indivisible, and immaterial substance, it is consequently incorruptible and immortal; for every thing capable of dissolution and dispersion is either corporeal and composite, or exists in some subject from which it is inseparable. And indeed, whatever may be dissolved is corruptible, as being compounded from many. But whatever naturally subsists in something different from itself, when it is separated from its subject, immediately vanishes into non-entity. But the soul, as we have proved, is incorporeal; it is likewise removed from every subject, and naturally reverts to itself, and is therefore immortal and incorruptible.

2. Let

2. Let us now consider how, and on what account the foul is said to be of a middle nature, and to be the receptacle of all middle energies, both vital and gnostic. Since, then, there is a long gradation of beings, proceeding from the first being, even to formless matter, which is nothing more than the dark shadow of essence, it is requisite to enquire what the properties are of the first and last beings, and what the condition is of the middle orders; for thus we shall know where the essence of the soul ought to be placed. The properties of intelligible natures, therefore, are as follows: true being, eternal, indivisible, immovable, total, perfect, full of essence, replete with life, free, moving all things, similitude, presiding over all things, and at the same time separated from all; for each of these properties appears in intelligibles, according to the processions of being. But the properties of sensible natures, different from these by the greatest interval, are such as, not-true-being, temporal according to essence, partible, moveable, particular, indigent of another, always replete with subsistence, living by participation, moved by another, dissimilitude, and occupying place by parts. But the middle properties of these are, not-true-being, an essence better than non-being, and inferior to true-being, according to essence eternal, but according to its energies extended with time, indivisible according to its divine part, but divisible according to the various processions of reasons, self-motive, governing things moved by another, but subordinate to such as are immovable, bearing before itself a particular nature, together with its totality; (for, because it contains in itself all reasons, it is after a manner a whole, but because it is diminished and fallen, ends in parts, and suffers a transition of its energy, it must be esteemed a particular nature): and again, perfecting itself, yet, nevertheless,

perfected by natures prior to its own; filling itself with power and strength, and at the same time filled by others; living from itself, and receiving life from others, being more divine, indeed, than things which live only by participation, but inferior to things primarily vital; moving other things, and itself moved by others; at the same time similar and dissimilar; and separated, at the same time, from last natures, and co-ordinated with them. Such then, being the properties of the first, middle, and last orders, let us consider where we ought to place the soul, whether in the first order, or in those which retain the last place: but if we establish it among the first, it must be true being, every way eternal and immoveable, and it must consequently possess every thing which we have attributed to the first beings; add too, that on this hypothesis we can no longer attribute to the soul a power of self-motion, nor the discursive processions of reasoning, nor a variety of other particulars, which manifestly belong to the soul. But can we place it with propriety among the last of beings? The least of all: for on this hypothesis we shall make it alone moved by others, divisible, composite, and alone possessing perfection from others, the opposite of which is evident in all our souls; since they move and perfect themselves, and are led wherever they please. Since then it is not possible to place the soul either in the first, or last order of beings, it is requisite to assign it a middle place, in imitation of its divine cause Rhea (according to the theologists), who is the conciliating band of the two parents Saturn and Jupiter, and is reported, from her prolific bosom to produce the life of the soul. But though the soul is thus the extremity of intelligible, and the principle of sensible natures, we must not conceive it to be such a principle or extremity as a point in a line, for it is not in both the

natures

natures it terminates, like a point in both the sections of a line; but it is to be called the extremity of intelligibles, because it appears after an intelligible essence, and the principle of sensibles, as being abstracted from them, and the source of their motion. And thus it will preserve to us a certain proportion, that as the natures which are moved by others, are to those which are moved by themselves, so are these last to immoveable natures; and hence it will obtain the condition of a bond, on account of its peculiar mediocrity, unfolding, indeed, united causes, but reducing the dissipated powers of sensibles into *one*, and being contained by an immoveable and perpetually abiding cause; but containing itself the generation, which, moved by another, is subject to continual mutation. It is likewise intelligible, if we regard generated natures; but generated, if we compare it with intelligibles; and thus it exhibits in its middle nature both extremes, imitating also, by this means, (according to the Greek theologists) its divine cause, for it is said to be on both sides resplendent, *αμφιφαντ*, and to be endowed with two faces *αμφιπρόσωπος*, and to receive in its bosom the processions of intelligible natures. It is likewise said to be replenished with intellectual life, and to be the fountain of the ever-running streams of corporeal life, and to contain in itself the centre of the processions of all beings. On this account it is, with great propriety, affirmed to be generated, and at the same time without generation. For true being, according to the Platonists, is without generation, because it has an infinite power of being totally present at the same time: and body is said to be generated, because it always possesses in itself an infinite flowing power, which it cannot at once totally receive. The soul, therefore, because it is incorporeal, abiding in itself, has an infinite power of being, and this total with respect to its

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essence, and immortal without generation; but according to parts it may be considered in infinite production. For it has not the same total infinity ever present, or there would be the same infinity of the whole and part, of the perfect and imperfect, of the contained and containing, which is impossible. But neither is it possible that the whole of its essence should be in the act of perpetual production, any more than that a part of it should be eternal being, lest the part should be more worthy and better than the whole. Hence the *ὑποχώρη*, or subsistence of the soul, is at the same time of infinite power, and is generated in infinitum; for by this means it participates of being, and obtains the first place among generated natures; while body alone, both with respect to its whole and parts, is obnoxious to a perpetual generation.

3. But let us now enquire from what genera Plato composes this nature of the soul, which contains in itself the bond of all beings; previous to which it will be requisite to explain what these genera are, and from whence they originate. Of the species, then, existing in the intelligible world, or the divine intellect, which contains in itself the causes of all posterior natures, some are most general, extending themselves to the universality of things; but others are more particular, like the most special species, and others subsisting between these, expand themselves, indeed, to a multitude of things, but not to all, according to the division of the Elean guest in the Sophista. For man is produced from the ideal man, and horse from the ideal horse, in the intelligible world; but the similitude which is found in man and horse, and other animals, is produced from likeness itself, or the ideal similitude, as dissimilitude from unlikeness itself; but the sameness and difference which are found in all beings, proceed from the sameness and difference

difference which subsist in ever-vital energy and perfection, in the supreme intellect, or the ideal world. Now, as among the sciences some are especially universal, so in intelligible causes some are perfectly particular, presiding alone over the proper and peculiar number of one species ; but others extend themselves to a multitude, such as equality, likeness, totality, (for the whole considered as a whole is not common to all things, since the part is not a whole) ; but others, again, expand themselves to all things, as all beings participate of these, considered as beings, and not considered as vital or animated, or possessing any other property exclusive of the denomination of being. Because, therefore, *being* is the first, the causes of *being* obtain the most universal order among genera ; and these are five in number, as follows, *essence*, *sameness*, *difference*, *motion*, and *station*. For every being is endued with *essence* ; is united to itself ; is by itself, or its own sameness, separated from others ; proceeds from itself, and its own state and principle, and no less appears to participate of a certain *abiding*, in preserving its own proper species. All things, therefore, whether intelligibles or sensibles, or subsisting between both, depend on these genera for their existence. For without the being of *essence*, nothing could subsist ; in like manner, without *sameness* every whole would be dissipated, and divided from itself ; and *difference* being taken away, all things would be one alone, and multitude be destroyed. But without *motion* and *station*, all things would either be inefficacious and dead, or, losing their proper state and stability, would end in non-entity.

4. Such then being the middle nature of the soul, Plato, with great propriety, in the Phædrus, and in his tenth book of laws, defines it to be number moving itself; which definition he received from Philolaus, and Philolaus from Pythagoras.

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Pythagoras. For since mathematical species have a middle subsistence, as is proved in the following Commentaries, they are of all things most accommodated to the nature of the soul. Hence Plato, in imitation of Pythagoras, sometimes explains the soul by number, as in the present instance; and sometimes by figure and magnitude, as in the *Timæus*; while he considers in the soul the intersection of lines, and a twofold circle. For since mathematical forms are separated from the flux and inconstancy of matter, they participate of a certain, exact, sure, and exquisite condition, by means of which they eminently confer to the elevation of our ingenuity, and the explication of latent concerns; and, on this account, as they pertain to numbers, we may say, preserving the analogy, that there are five orders of numbers, *the divine*, *the essential*, *the animative*, *the natural*, and *the mathematic*. The first of these is uniform, the second immovable, the third self-motive, the fourth moved by another, and the last the image of the others, and their external measure. The divine number is considered eminently in the deity, as in the principle of all things; the essential belongs to intellect through ideas, and is called *essence*, *unity*, and *the first being*; the animative number belongs to the soul, through the medium of her inherent reasons; the natural to physical concerns, through the seeds of nature; and lastly, the mathematical belongs to opinion, as it is nothing more than the image of essential number, formed by the energies of the rational soul. The soul, therefore, is number, not limited by quantity, and mathematical, but animative; it is number, not indeed numbering, but numbered, generating and converted into itself. Hence too, because harmony arises from number, the soul is called harmony; not, indeed, a harmony of the parts of the body, nor the harmonic quantity which

subsists in sound or in voice ; but a harmony arising from its essential numbers, placed in its inherent reasons, and in the genera which constitute its nature. It is this harmony which produces, as from its proper cause, the harmony of the corporeal parts, the rhythm of motions, and the melody of voices and sounds. It is this which produces that delight in the soul from sensible harmony, which sufficiently indicates it to be something familiar and domestic to her nature. From hence it may be inferred, that Plato is not inconsistent with himself when, in the Phædo, he denies, and in the Timæus affirms, the soul to be harmony ; for he denies that it is a harmony of a definite quantity, or such as arises from the parts of the body ; but he asserts it to be a harmony in the manner already explained. It may likewise be inferred, that Plato is ignorantly accused by a many, for affirming that the soul is harmony, or number ; for they only regard vulgar mathematical number, and sensible harmony ; while Plato, far more elevated, discourses of intelligible numbers, and ideal harmony, subsisting in immaterial energy and perfection.

5. And here it is necessary to consider what number, in a particular manner belongs to the soul ; for various numbers, differently considered, accord with her self-motive nature. In the first place, union and unity may be considered in the soul, as in her proper degree she participates of divine unity ; and likewise with relation to her totality, for she is one certain whole. And because *a whole* may be considered in a triple respect, one before the parts, another rising from the aggregate of parts, and a third subsisting in the single parts ; the soul is a whole in each of these respects. Thus she is a whole prior to the parts, while she is considered as divisible into them, in an incorporeal manner ; she is a whole rising from parts, while assuming the  
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parts in the first place, we consider how her nature is fabricated from their conjunction; and she is a whole in the single parts, since she is total in the whole, and in every part. Besides, the duad belongs to the soul, because she contains in her nature bound and infinite, sameness and difference; and lastly, a conversion to intelligibles and sensibles. And, indeed, the duad conjoined with unity, very properly accords with the soul; for to intellect above soul, unity particularly belongs; to body beneath soul, the infinite alone; and to soul situated in the middle, duality properly agrees, being, as it were, infinite, connected with unity. Again, the ternary number is attributed to the soul, as well on account of her beginning, middle, and end, as because she abides in herself, proceeds to inferiors, and returns to supernal natures. Besides, she flows from *the one*, recedes from him, and is reflected into the one when she acquires her proper perfection. Lastly, as Proclus observes, the nature of the soul is divided into essence, power, and energy; so that she may be said to rejoice in the ternary number, and to be replete with its perfection. But the quaternary number belongs to the soul so far as she is connected with matter, which is tempered with four qualities, and four elements; and she is endued with four principal faculties, nutrition, sensation, local motion, and intellection. But to omit other numbers, and their conformity with the soul, the quinary, and septenary numbers are especially attributed to the soul. The quinary, because the soul is composed from the five genera of things, we have previously explained; and because five particulars merit a principal consideration in the soul; first, her essence; secondly, the harmony of her reasons; thirdly, the species arising from the concord of her parts or reasons; fourthly, her virtue; and, lastly, her energies: and on this account,

Proclus.

Proclus observes, the consideration of the soul ought to receive a quintuple distribution. Besides, as the soul consists from a divisible and indivisible nature; so the quinary is composed from the first even and the first odd number. Lastly, as the soul is the connecting medium of the universe; so the quinary obtains the middle situation in universal number, that is in the decad. But the septenary number belongs to the soul, because, as Plato shews, in his Timæus, all harmonical reasons are contained in the seven numbers, 1, 2, 3, 4, 9, 8, 27; and from these the soul is composed. Hence Proclus elegantly observes, that the septenary number is dedicated to Apollo, the parent of all harmony; because in one, two, and four, from which the septenary results, the first bisdiapason is found. Besides, the writers on harmony affirm that all the difference of voices proceeds as far as to the seventh degree. So that Plato uses, with great propriety, the septenary number for the composition of the soul. Again, in these numbers of the soul, every medium is found in a convenient proportion; in these the geometric medium is contained, corresponding to the right institution of laws; of which Plato, in his Republic, says, that by this cities are properly governed: there is found too, an harmonical medium, which is the similitude of justice: and lastly, we may discover an arithmetical medium, which is called the illustrious symbol of peace. After this manner, therefore, Plato, with a wonderful fecundity of significations, affirms that the soul is composed from numbers. He likewise considers the figures she contains, I mean the circle and triangle; because as the soul is the first nature which verges to body, so these are the first of all figures, as well rectilinear as curvilinear. Besides, an orbiculation agrees to the soul, through intellect; but progression and rectitude according to her

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own proper nature ; and on this account she contains both a straight and circular figure. I omit other correspondencies of a circle and triangle with the soul, as they are exhibited in the following Commentaries ; and particularly by Proclus, in the third book of his Commentaries on the Timæus : it is sufficient to the design of this Dissertation, just to have mentioned this analogy, that the nature of the numbers and figures may appear, which are considered by Plato in the composition of the soul.

6. Let us now pass from contemplating the nature of the soul, to a survey of its various gradations of knowledge, and the means by which it acquires the illuminations of science ; as this is a speculation perfectly essential to a full comprehension of the ensuing Commentaries. According to Plato, then, in the sixth and seventh books of his Republic \*, there are four degrees of the internal cognitions of our soul ; imagination, or assimilation ; faith, cogitation (*dianoia*) ; and lastly, science or wisdom. The two first degrees conjoined constitute opinion ; but the two last equally joined produce intelligence in its large acceptation. I say in its large acceptation, because the word intelligence is considered by the Platonists in a triple respect. First, as it rises from opinion and science ; as Plato asserts, in the seventh book of his Republic. Secondly, as it passes into the same with science ; for thus, in the end of the sixth book, he considers intelligence and science as the same. Lastly, as it is distinguished from science, and intelligible from that which is the object of cognition : thus, science regards the essential reasons of the soul ; but intelligence elevates us to ideas, and this is intelligence in its proper acceptation. The distinction of these four degrees, especially depends on the distinction of things with which the

\* See Note to Chap. i. Book i. of the ensuing Commentaries.

soul

soul is conversant. For these four degrees of things are, the image of a sensible object, the sensible object itself, the image of an intelligible object, and the intelligible itself. Imagination or assimilation regards the image of the sensible object ; which image is nothing more than the shadow or resemblance appearing in water, or other lucid and polished bodies. Faith is conversant with sensible objects ; and these are animals, plants, and every thing subject to the energies of sense. From the junction of imagination and faith, opinion is produced. Cogitation is conversant with the image of an intelligible object ; which is nothing more than a certain universal, collected from sensibles, related to the reasons of things existing in the soul, and constituted by their assistance ; but not elevated to ideas, and resolved into their lucid nature. Those who are conversant with this image of an intelligible object, use hypotheses ; which do not elevate us to principles, which are reasons and ideas, but bring us down to subordinate objects. Lastly, science, considered as the same with intelligence, is conversant with that which is intelligible, or the essence of things ; and of this kind are reasons pertaining to the soul, and ideas to intellect. Hence, as *intelligible* is to *sensible* in splendor and truth, and form to its image ; such is the relation of the superior to the inferior degrees of cognition. And as these four degrees may be resolved into five, by separating intelligence from science ; so they are reduced by Plato into two principal degrees ; so far as the two first are conversant about generation ; but the two last about essence. And thus much for the first particular proposed.

It now remains that we investigate the mode in which science is produced in the soul, according to the doctrine of Plato. For this purpose, I think it will be necessary to

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consider the rational soul, in whose nature all reasons corresponding to ideas, have been inserted from eternity, received into the human body, as into the plain of oblivion ; bordering on the river of negligence, that is placed near to the flux of humours ; which producing in its nature various kinds of perturbations, are the causes of its self-oblivion and neglect. Hence the soul, thus constituted, as it were, sleeping and intoxicated ; sleeping before it is roused ; intoxicated before it is purified ; begins from external sensible objects to be moved and excited, and to seek with avidity, the knowledge congenial to its nature. The soul now, enquiring after truth and the sciences, is first conversant with the external images of things, in which the glimmering light of similitude to truth, presents itself to the view ; then it distinguishes these images among themselves by its reasoning power ; and if they agree in any particular, collects them into one. Thus, being employed in separating into many, things united, and reducing many into one, it advances from shade to substance, and is elevated from similitude to truth itself ; and thus apprehends the essence of a thing free from every foreign or contrary quality, shining in *reason* and *idea*. Afterwards, from this contact, especially salutary to itself, it experiences an ineffable joy, as from a return into its proper nature, and best disposition ; and so great is its exultation, that it neglects and despises the shadows which it formerly pursued. Then the soul truly knows, that while a man regards corporeal natures, he is employed in ~~resemblances~~ ; and that though he may esteem himself knowing in many things, he knows nothing in reality ; but is then alone elevated to the sublime degree of science, when he arrives at ideas. Hence it appears, that there are four subordinate dispositions of mankind in order to science. For, in the first place, children, as new

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guests of this world, are ignorant of every thing, without being conscious of their ignorance. But as they advance in years, they are employed in the shadows and images of natural concerns; and not being yet converted to the essences of things, though they possess no real knowledge, yet they conceive themselves to abound in knowledge. But in the third place, being elevated to supernal natures, and judging these to be alone true, they affirm themselves to be ignorant of all they formerly imagined themselves to know; in which degree Socrates professed to find himself, when he said, *this one thing I know, that I know nothing*; an ignorance preferable to all the knowledge gained by the most unwearyed experimental enquiries. Indeed, this order is indicated by Socrates himself, in the Phædo, when he says, that on his first acquaintance with natural concerns, he thought he abounded in knowledge, but as he advanced in these pursuits, he perceived that he knew nothing. Lastly, when, through the piercing sight of the soul, men are elevated to ideas, they become illustrated with the splendors of true science, and pass into the regions of perfect reality. And hence it appears how true science, which is the same with intelligence and wisdom, produces piety and religion: for science elevates us to intellect and divinity; and copulates the soul with natures of the greatest purity and perfection; so that an union of this kind cannot take place without piety, sanctity, and religion; as dissimilars can never be blended in amicable conjunction.

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## iii. DISSERTATION ON THE

## SECTION IV.

WE are informed by Proclus, in the ensuing Commentaries, that the end of geometry, and, indeed, of mathematics in general, is to be referred to the energies of intellect; and that it is degraded when made subservient to the common utilities of a mere animal life. But as the very opposite to this is the prevailing opinion of the present age, let us examine the truth of this doctrine, and attend to the arguments which the Platonic philosophy affords in its defence. For if we can prove that this assertion of Proclus is supported by the strongest evidence, we shall vindicate the dignity of true geometry, restore it to its ancient esteem in the minds of the liberal, and shew how much it is perverted by applying it to contrary purposes.

In order to this, I shall endeavour to prove the following position, that things valuable for their own sakes, are preferable to such as refer to something else. Now, this may be demonstrated, by considering that every natural production was made with reference to some end, as is evident from an induction of particulars; and if this be the case, it may be safely inferred, that every thing exists for the sake of the end. But that for the sake of which any being subsists is the best of all; and the end, according to nature, is that which is perfected the last of all, from the birth of any being. Hence the human body receives its end or perfection first, but the soul last. And hence the soul is posterior to the body, in the accomplishment of its nature; and its ultimate perfection is wisdom. It is on this account that old age alone pursues and desires the goods of prudence and wisdom. Hence, wisdom is a certain end to us according to nature; and to be wise, is the extreme or

or final cause for which we were produced. It was, therefore, beautifully said by Pythagoras, that man was constituted by divinity, that he might know, and contemplate: If then wisdom be the end of our nature, to be endued with wisdom must be the best of all. So that other things are to be performed for the sake of the good which this contains. But to enquire in every science something besides this, and to require that it should be useful, is alone the employment of one ignorant of the great difference between the most illustrious goods, and things necessary. For they differ, indeed, widely; since things are to be called necessaries, which are the objects of desire for the sake of others, and without which it is impossible to live. But those concerns alone are properly good, which are loved by themselves, though nothing else should fall to the lot of their possessor; for one thing is not to be desired for the sake of another infinitely, but it is requisite to stop at some limited object of desire, of which it would be ridiculous to require any utility abstracted from itself. But you will ask, What is the emolument of contemplative wisdom; what the good it confers on its possessor? What if we should say (for such is the truth of the case) that it transports us by intellect and cogitation, to regions similar to the fortunate islands; for utility and necessity are strangers to those happy and liberal realms. And if this be admitted, ought we not to blush, that having it in our power to become inhabitants of the fortunate islands, we neglect the pursuit; through a sordid enquiry after what is useful and profitable according to vulgar estimation? The rewards of science; therefore, are not to be reprehended, nor is it a trifling good which results from its acquisition. Besides, as men travel to the mountain Olympus for its spectacle alone; preferring a view of its lofty summit to much wealth; and

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as many other spectacles are desired for their own sakes, and valued beyond gold, in like manner the speculation of the universe is to be prized above every thing which appears useful to the purposes of life: for it is surely shameful that we should eagerly frequent the theatre, and the race, for the sake of the delight afforded to our corporeal sight, and should look for no farther utility in these than the pleasure they produce; and yet should be so sordidly stupid as to think that the nature of things, and truth itself are not to be speculated without some farther reward than the sincere delight their contemplation affords.

It is on this account that the apprehension of truth is compared to corporeal vision; for the sight is the most liberal of all the senses, as is confirmed by the general testimony of mankind. Hence, the sight of the sun and moon, and the glorious spectacle of the stars is desired by the most illiterate as well as the most knowing, for the delight such visions afford; while, on the contrary, the desires of the other senses are for the most part directed to something farther than the mere objects of their energy. Thus, even the sense of hearing, which is the next in dignity to the sight, is not always desirable for its own sake; for light is the general object of sight, and sound that of hearing; but it is evident that light is more universally desired than sound, since all light, when not excessive, is always pleasing, but this is by no means the case with every kind of sound. Hence it is, that all contemplation is so delightful, and this in proportion as it becomes abstracted from sensible objects; for the most beautiful forms do not produce genuine delight, until they are strongly represented in the phantasy, as is evident in the passion of love; since the fairest face then alone causes love when it presents itself clearly to the inward eye of thought, in the mirror of imagi-

imagination, accompanied with living elegance, and a resistless energy of form.

Indeed, so liberal and so exalted an employment is contemplation, that Plotinus, with his usual profundity, proves that the universe subsists for its sake; that all the productions of nature originate from this; and that even actions themselves are undertaken with a view to the enjoyment of after-speculation. May we not, therefore, say that the sportsman follows the chace for the sake of a subsequent review of his favourite pursuit? That the glutton for this rejoices in the meal; and even the miser in his wealth? And that conversation is alone solicited, that it may recal past images to the soul? In short, contemplation is the first spring of action, and its only end; since we are first incited to any external object by speculating its image in the phantasy: and our subsequent conduct tends, without ceasing, to the energy of reflection; for destroy prior and posterior contemplation, and action is no more.

Now if this be the case, and if geometry is a speculative science (I mean the geometry of the ancients), it is both desirable for its own sake, and for still higher contemplations, the visions of intellect, to which it is ultimately subservient. For, when studied with this view, it opens the eye of the soul to spectacles of perfect reality, and purifies it from the darkness of material oblivion. Away then, ye sordid vulgar, who are perpetually demanding the utility of abstract speculations, and who are impatient to bring down and debase the noblest energies, to the most groveling purposes; ignorant of that mighty principle of action, which influences every part of the universe, and through which even division and discord tend as much as possible to union and consent; ignorant that from the depravity of your nature, and the blindness of your inward eye, you

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are incapable of speculating the substance of reality, and are therefore eagerly gazing on its shadow: and lastly, unconscious that this is the point about which you are continually making excentric revolutions, mistaking the circumference for the centre, motion for rest, and a departure from good for a tendency to felicity.

It was for the sake of this most exalted and liberal contemplation that Heraclitus yielded his right of succession to a throne, to his brother; and that Anaxagoras neglected his patrimony, esteeming one drop of genuine wisdom preferable to whole tuns of riches. Led by a desire of this, as by some guiding star, Pythagoras travelled into Egypt, and cheerfully encountered the greatest difficulties, and maintained the most obstinate perseverance, until at length he happily penetrated the depths of Egyptian wisdom, and brought into Greece a treasury of truth for future speculation. But these were happy days; this was the period destined to the reign of *true philosophy*, and to the advancement of the human soul to the greatest perfection its union with this terrene body can admit. For in our times, the voice of wisdom is no longer heard in the silence of sacred solitude; but *folly* usurping her place, has filled every quarter with the barbarous and deafning clamours of despicable sectaries; while the brutal hand of commerce has blinded the liberal eye of divine contemplation. For unfortunately, the circle of time, as it produces continual variations, at length reverses the objects of pursuit; and hence, that which was once deservedly first, becomes at length, by a degraded revolution, the last in the general esteem.

2. If geometry, therefore, be both valuable for its own sake, and for its subserviency to the most exalted contemplations, there can be no doubt but that the great perfection

fection to which this science was brought by the Greeks, was entirely owing to their deep conviction of this important truth. Euclid, we are informed by Proclus, in this work, was of the Platonic sect; and Archimedes is reported, by Plutarch, in his Life of Marcellus, to have possessed such elevated sentiments of the intrinsic dignity of geometry, that he considered it perverted and degraded, when subservient to mechanical operations; though, at the request of king Hiero, he fabricated such admirable engines for the defence of Syracuse. From this source alone, the great accuracy and elegance of their demonstrations was derived, which have been so deservedly applauded by the greatest modern mathematicians, and the warmest advocates for the farrago of algebraic calculation. Algebra, indeed, or as it is called, *specious analysis*, is the modern substitute for the perfect method adopted by the ancients in geometrical demonstrations; and this solely, because it is capable of being applied with greater facility to the common purposes of life. Hence, hypotheses have been eagerly admitted in geometry, which the ancients would have blushed to own: I mean the multiplications and divisions of lines and spaces as if they were numbers, and considering geometry and arithmetic as sciences perfectly the same. But we have fortunately the testimony of the first mathematicians among the moderns against the unlawfulness of this ungeometrical invasion. And to begin with the great sir Isaac Newton, in his Universal Arithmetic \*: "Equations (says he) are expressions of arithmetical computation, and properly have no place in geometry, except so far as quantities truly geometrical (that is, lines, surfaces, solids, and proportions), may be said to be some equal to others. *Multiplications, divisions, and such sort of computations, are newly received*

• Page 227.

*into geometry, and that unwarily, and contrary to the first design of this science.* For whoever considers the construction of problems by a right-line and a circle, found out by the first geometers, will easily perceive that geometry was invented that we might expeditiously avoid, by drawing lines, the tediousness of computation. Therefore, these two sciences ought not to be confounded. The ancients so industriously distinguished them from one another, that they never introduced arithmetical terms into geometry. And the moderns, by confounding both, have lost the simplicity in which all the elegancy of geometry consists." And in another part \* of the same work he observes, that "the modern geometers are too fond of the speculation of equations." To this very high authority we may add that of Dr. Halley, in the preface to his translation of Apollonius de Sectione Rationis, for which work he conceived so great an esteem, that he was at the pains to learn Arabic in order to accomplish its translation into Latin †: "This method, says he, (of Apollonius) contends with specious algebra in facility, but far excels it in evidence and elegance of demonstrations; as will be abundantly manifest if any one compares this doctrine of Apollonius de Sectione Rationis, with the algebraic analysis of the same problem, which the most illustrious Wallis exhibits in the second volume of his mathematical works, cap. liv. p. 220." And in the conclusion of his preface, he observes ‡, "that it is one thing to give the resolution of a problem some how or other, which may be

\* Page 250.

† Methodus hæc cum algebra speciosa facilitate contendit, evidentiâ vero et demonstrationum elegantiâ eam longe superare videtur: ut abunde constabit, si quis conferat hanc Apollonii doctrinam de Sectione Rationis cum ejusdem Problematis Analyti Algebraicâ, quam exhibuit clarissimus Wallisius, tom. ii. Operum Math. cap. liv. p. 220.

‡ Verum pendendum est, aliud esse problema aliquantum resolutum dare, quod modis variis. plerumque fieri potest, aliud methodo elegantissimâ ipsum efficere; Analyti brevissimâ et finaliter perspicuâ, Synthesi concintâ et minime operosâ.

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accomplished by various ways, but another to effect this by the most elegant method; by an analysis the shortest, and at the same time perspicuous; by a synthesis elegant, and by no means operose." And Dr. Barrow, notwithstanding he was so great an advocate for the identity of arithmetic and geometry, expressly asserts \*, that *algebra is no science*. To these authorities we may add Simson and Lawson, who, sensible of the superior skill of the ancients, both in analysis and synthesis, have made laudable attempts to restore the *Greek geometry* to its pristine purity and perfection.

Again, the greatest men of the present times have been of opinion, that algebra was not unknown to the ancients; and if this be true, their silence respecting it is a sufficient proof of their disapprobation. Indeed, if we consider it when applied to geometry, as an art alone subservient to the facility of practice, as conveying no evidence, and possessing no elegance of demonstration, we shall not wonder at its being unnoticed by the ancients, with whom practice was ever considered as subservient to speculation; and in whose writings elegance of theory and accuracy of reasoning are found perpetually united.

3. But the lives of the first cultivators of this science (I mean the Egyptian priests) as well as of the Pythagoreans and Platonists, by whom it afterwards received such improvements, sufficiently evince that this science advanced to perfection from an intellectual theory as its source, and from being referred to contemplation as its end; and this will be evident, by attending to the following history of the Egyptian priests, as preserved to us by Porphyry, in his excellent work on abstinence †; a translation of which will not, I presume, be unacceptable to the philosophical

\* In his Mathematical Lectures, p. 44.

† Lib. iv.

reader.

reader, "Chæremon, the Stoic (says he) explaining the rites of the Egyptian priests, who, he says, are accounted philosophers by the Egyptians, relates, that they choose a place best adapted to the study and performance of sacred rites; so that a desire of contemplation is excited by only frequenting those recesses which are dedicated to their use, and which procure safety to the priests, on account of that reverence of the divinity, whose sacred mysteries they perform; so that all possible honour is paid to these philosophers, in the same manner as to some sacred animals. But he says they live entirely solitary, except at particular times, when they mix with others in such assemblies as are usually held, and in public feasts; and that on all other occasions they are scarcely to be approached. For he who desires to converse with them must first purify himself, and abstain from a multitude of things after the manner of these Egyptian priests. He adds, that these men, renouncing every other occupation, and all human affairs, give themselves entirely, through the whole of life, to the contemplation of divine concerns, and to enquiring into the divine will: by the latter of these employments procuring to themselves honour, security, and the estimation of piety; by contemplation, tracing out the latent paths of science; and by both these occupations united, accustoming themselves to manners truly occult, and *worthy of antiquity*. For to dwell always on divine knowledge, and be disposed for divine inspiration, removes a man beyond all immoderate desires, calms the passions of the soul, and raises her intellectual eye to the perception of that which is real and true. But they studied tenuity of aliment, and frugality in their apparel, and cultivated temperance and patience, together with justice and equity, in all their concerns. Indeed, a solitary life rendered them perfectly venerable; for during that period which

which they call the time of purification, they scarcely mixed with the associates of their own order, or saw any one of them, except him who was conversant with them in that exercise of purity, on account of necessary uses. But they by no means concerned themselves with those who were unemployed in the business of purification. The remaining part of their time they conversed familiarly with those similar to themselves; but they lived separate and apart from those who were estranged from their ceremonies and manner of living. He adds, they are always seen employed among the resemblances of the gods, either carrying their images, or preceding them in their accustomed processions, and disposing them with gravity of deportment, and in a graceful order. In all which operations they did not indicate any pride of disposition; but exhibited some particular natural reason. But their gravity was conspicuous from their habit; for when they walked, their pace was equable, and their aspect so perfectly steady, that they refrained from winking whenever they pleased. Their risibility too, extended no farther than to a smile. But their hands were always contained within their garments; and as there were many orders of priests, every one carried about him some remarkable symbol of the order he was allotted in sacred concerns. Their sustenance too was slender and simple; and with respect to wine, some of them entirely refrained from it; and others drank it very sparingly, affirming that it hurt the nerves, was an impediment to the invention of things, and an incentive to venereal desires. They also abstained from many other things, never using bread in exercises of purity; and if they ate it at other times, it was first cut in pieces, and mingled with hyssop. But they abstained, for the most part, from oil; and when they used it mixt with olives, it was only in

small quantities, and as much as was sufficient to mitigate the taste of the herbs.

In the mean time, it was not lawful for any one to taste of the aliment, whether solid or fluid, which was brought into Egypt from foreign parts. They likewise abstained from the fish which Egypt produced; and from all quadrupeds having solid or many fissured hoofs; from such as were without horns; and from all carnivorous birds: but many of them abstained entirely from animal food. And at those times when they all rendered themselves pure, they did not even eat an egg. But when the time drew near in which they were to celebrate some sacred rites, or festival, they employed many days in previous preparation, some of them setting apart forty-two days, others a greater length of time than this; and others again a shorter; but never less than seven days; abstaining, during this period, from all animals, and from all leguminous and oily nutriment, but especially from venereal congress. Every day, they washed themselves three times in cold water; after rising from bed, before dinner, and when they betook themselves to rest. And if they happened to be polluted in their sleep, they immediately purified their bodies in a bath. They made cold water too subservient to the purposes of purification at other times, but not so often as the bath. Their beds were composed from the branches of palm, which they called *βαις*, *bais*. A piece of wood, of a semi-cylindrical form, and well planed, served them for a pillow. But through the whole of life, they were exercised in the endurance of hunger and thirst, and accustomed to a paucity and simplicity of nutriment.

But as a testimony of their temperance, though they neither used the exercise of walking nor riding, yet they lived free from disease, and were moderately strong. For, indeed,

indeed, they endured great labour in their sacred ceremonies, and performed many services exceeding the common strength of men. They divided the night between observations of the celestial bodies, and offices of purity; but the day was destined by them to the cultivation of the divinities, whom they worshiped with hymns each day three or four times; in the morning and evening, when the sun is at his meridian, and when he is setting. But the rest of their time they were occupied in arithmetical and geometrical speculations, always laborious and inventing, and continually employed in the investigation of things. In winter nights also, they were diligent in the same employments, and were ever vigilant to literary studies; since they were not solicitous about external concerns, and were freed from the base dominion of intemperate desires. Their unwearied and assiduous labour, therefore, is an argument of their great patience; and their continence is sufficiently indicated by their privation of desire. Besides this, it was esteemed very impious to sail from Egypt, as they were particularly careful in abstaining from the manners and luxuries of foreign nations; so that to leave Egypt was alone lawful to those who were compelled to it by state necessities. But they discoursed much concerning a retention of their native manners; and if any priest was judged to have transgressed the laws in the least particular, he was expelled the college. Besides, the true method of philosophizing was preserved in Commentaries and Diaries, by the prophets and ministers of sacred concerns: but the remaining multitude of priests, *Pastophori*, or priests of Isis and Osiris, governors of temples, and servants of the gods, studied purity, yet not so exactly, nor with so great continence as those we have mentioned. And thus much is related of the Egyptians, by a man who is equally a lover of truth, and of accurate

diligence, and who is deeply skilled in the Stoic philosophy."

4. But the lives of the Pythagoreans and Platonists, who carried this divine science to its ultimate perfection, no less eminently evince the truth of our position. For, as Porphyry informs us, in the same invaluable treatise \*, "some of the ancient Pythagoreans, and wise men, inhabited the most desert places; and others retired into temples, from which the multitude and every tumult were expelled. But Plato was willing to fix his academy in a place not only solitary, and remote from the city, but, as they report, insalubrious. Others, again, have not spared their eyes, through a desire of more perfectly enjoying that blissful contemplation, from which they wished never to be separated." After this, he presents us with a description from Plato † of those intellectual men, by whom the world has been enlightened with the sublimest wisdom and truth: "For it was not falsely, or in vain (says he), that a certain philosopher, speaking of contemplative men, affirms, that such as these are ignorant, from their early youth, of the way which leads to the forum, or in what place the court or senate-house is situated, or any public council of the state. They neither see nor hear the laws, whether decreed or promulgated, or written; and with respect to the factions and contentions of their companions for magistracy, for assemblies and splendid entertainments, luxurious eating and minstrels, they do not even think of these as in a dream. Such an one knows no more of the evil which has happened to some one of his ancestors, whether male or female, or any thing belonging to them, than how many pitchers of water are contained in the sea. Nor does he abstain from things of this nature for the sake of acquir-

\* Lib. i. p. 30.

† In Theæteto.

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ing fame ; but in reality, his body alone abides in the city, and wanders about from place to place, but his intellect esteeming all these as of small importance, or rather as non-entities, he despises them, and, according to Pindar, "from these on every side he soars :" by no means applying himself to things which are near him, and to sensible concerns."

If such then were the lives of the men who brought this contemplative science to its present perfection, and who are to this day our masters in geometry ; if such were the exalted sentiments they entertained of its dignity and worth, what greater proof can we require of its being valuable for its own sake, and as subservient to the energies of intellect ? We have ample evidence too, of its being degraded when brought down to the common purposes of life, in the example of those who, with this view, have disguised it with the dark and sordid involutions of algebraic calculation ; for it was solely to facilitate practice, that this barbarous invasion has been admitted by the moderns. Let me then be permitted to persuade the few who study geometry in its ancient purity, and who consider the ruins of Grecian literature on this, as well as on every other science, the models of perfection, to enter with avidity on the study of the ensuing Commentaries, and endeavour to fathom the depth of our profound and elegant philosopher : for by this means they may happily obtain the end of all true science, the purification of the soul ; and be able to draw the light of perfect wisdom, from the undecaying and inexhaustible fountain of good.

But if it should be asked in what these energies of intellect consist, to which all science ultimately refers ? I answer, in the contemplation of true being, or those ideal and divine forms, with which the intelligible world is replete.

Now this great end is not to be accomplished without previous discipline, a long exercise of the reasoning power, and a continued series of philosophic endurance. For this end, when attained, is no other than the enjoyment of that felicity congenial to the soul previous to her immersion in body. But, for the further information of the liberal reader on this important subject, the following paraphrases from Porphyry and Proclus are subjoined; the former instructing us in the various purifications necessary to this end; and the latter exhibiting the gradations by which we may rise to the speculation of reality, and (leaving all *multitude* behind) ascend to the divinely solitary principle of things, the ineffable *One*.

5. "In the first place, then (says Porphyry \*) my reasons are not addressed to those who are occupied in illiberal arts, nor to those engaged in corporeal exercises, neither to soldiers nor sailors, neither to rhetoricians nor to those who have undertaken the duties of an active life. But I write to the man continually employed in thinking what he is, from whence he comes, and whither he ought to tend: and who, with respect to every thing pertaining to food, and other offices of life, is entirely changed from those who propose to themselves a different manner of living; for to a man of this kind alone is my present discourse addressed. Indeed, in this common state of existence, one and the same mode of persuasion cannot be addressed to the sleeper, who, if it was possible, would conciliate to himself perpetual sleep, and who, for this purpose, seeks on every side for soporiferous incentives, as to him who studies continually to drive away sleep, and to dispose every thing about him to vigilance and intellectual activity. But to the former, it is necessary to advise intoxication, surfeiting,

\* In his most excellent work on Abstinence, lib. i. p. 22, &c.

and

and satiety, and to recommend a dark house; and, as the poets say, *a bed luxurious, broad, and soft.* Such a one should chuse whatever tends to produce stupor, and give birth to indolence and oblivion, whether consisting of odours, ointments, or medicaments which are accustomed to be eat or drank. But it is necessary that the intellectual man should use sober drink, unmixed with the lethargic fumes of wine; nutriment slender, and almost approaching to fasting; a lucid house, receiving a subtle air and wind; that he should be continually agitated with cares and griefs; and lastly, that he prepares for himself a small and hard bed, while thus employed in purifying his soul from the stains contracted by corporeal involution. But whether we are born for this exalted purpose, I mean for vigilant intellectual energies, allowing as small a part of our life as possible to sleep; (since we do not exist in a place where souls perpetually vigilant abide), or whether we are destined to a contrary purpose, I mean, to sleep and oblivion, would be foreign from our design to explain; and would require a longer demonstration than the limits of our work will admit.

But whoever once cautiously surmizes the delusions of our life in the present world, and the enchantments of this material house in which we are employed, and who perceives himself naturally adapted to vigilant energies; lastly, who apprehends the soporiferous nature of the place in which he acts, to such a one we would prescribe a diet congruous to his suspicion of this fallacious abode, and to the knowledge he possesses of himself; in the mean time, advising him to bid a long farewell to the sleeper, stretched on his couch, as on the lap of oblivion. Nevertheless, we should be careful lest, as those who behold the bleer-eyed, contract a similar defect, and as we gape when present with those who are gaping, so we should be filled with drowsiness

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drowsiness and sleep, when the place in which we reside is cold, and adapted to fill the eyes with watery humours, from its abounding with marshes and vapours, which incline their inhabitants to heaviness and sleep. If then, legislators had composed the laws with a view to the utility of the state, and had referred these to a contemplative and intellectual life as their end, we ought to submit to their institutions, and acquiesce in the diet they have prescribed for our subsistence. But if they, only regarding that life which is according to nature, and is called of the middle kind, ordain such things as the vulgar admit, who only estimate good and evil as they respect the body, why should any one, adducing these laws, weary himself in endeavouring to subvert a life which is far more excellent than every law written and composed for the sake of the vulgar, and which follows a law not written, but divinely delivered? For such is the truth of the case.

*That* contemplation which procures us felicity, is not a mass of discourses, and a multitude of disciplines; or, as some may think, consisting from hence; nor does it receive any increase from a quantity of words: For if this was the case, nothing could hinder those from being happy, who comprehend all disciplines, and *are accurately skilled in a variety of languages*. But the whole circle of the sciences cannot by any means accomplish this blissful contemplation, nor even those disciplines which are conversant with true and substantial being, unless there is also a conformation of our nature and life to this divine end. For since there are, as they say, three ends of living, if we regard the particular objects to which mankind tend, the end with us is to follow the contemplation of *true being*, promoting, as much as possible, by an acquisition of this kind, an intimate union of the contemplating individual with

with the object of contemplation. For, in nothing else besides *true being*, is it possible for the soul to return to its pristine felicity; nor can this be effected by any other conjunction. But intellect is *true being itself*: so that the proper end is to live according to intellect. And on this account, exoteric discourses and disciplines, regarding the purgation of the soul, are far from filling up the measure of our felicity. If then, felicity was defined by the comprehension of words or sciences, they who do not pay a proper attention to the kind and quantity of their food, nor to any thing else pertaining to their present existence, might obtain this end: but since it is requisite to change our life, and to be pure both in speech and action, let us consider what discourses and what works may render us partakers of this most necessary means of acquiring substantial felicity.

Are, then, those things which separate us from sensible objects, and from the affections which they excite, and which lead to a life intellectual, and void of imagination and passion, are these the means we are in pursuit of? So that every thing contrary is foreign from our purpose, and worthy to be rejected? And in such proportion as it draws us aside from intellect? Indeed, I think it is consonant to truth, that we should eagerly contend where intellect leads; for in this material abode, we are similar to those who enter or depart from a foreign region, not only in casting aside our native manners and customs, but from the long use of a strange country, we are imbued with affections, manners, and laws foreign from our natural and true region, and with a strong propensity to these unnatural habits. Such an one, therefore, should not only think earnestly of the way, however long and laborious, by which he may return to his own, but that he may meet with a

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more favourable reception from his proper kindred, should also meditate by what means he may divest himself of every thing alien from his true country, which he has contracted; and in what manner he may best recal to his memory, those habits and dispositions without which he cannot be admitted by his own, and which, from long diffuse, have departed from his soul. In like manner, it is requisite, if we wish to return to such things as are truly our own, and proper to man considered as a rational soul, to lay aside whatever we have associated to ourselves from a mortal nature, together with all that propensity to material connections, by which the soul is allure<sup>d</sup>, and descends into the obscure regions of sense; but to be mindful of that blessed and eternal essence intellect, our true father, and hastening our return to the contemplation of the uncoloured light of *good*, to take especial care of these two things; one, that we divest ourselves (as of foreign garments) of every thing mortal and material; the other how we may return with safety, since thus, ascending to our native land, we are different from ourselves before we descended into mortality. For we were formerly intellectual natures; and even now we are essences purified from every stain contracted by sense, and from that part which is destitute of reason: but we are complicated with sensible connections, on account of our impotence and infirmity, which is the cause that we cannot always be conversant with intellectual concerns; but with mundane affairs we can be present with frequency and ease: for all our energetic powers are stupified and clouded with oblivion, through body and sense; the soul not remaining in an intellectual state; (as the earth when badly affected, though good fruit is deposited in its bosom, produces nothing but weeds); and this, through the improbity of the soul, which does not,

indeed, destroy its essence, while it acquires brutality ; but by such an accession it becomes complicated with a perishing nature, is bound in the dark folds of matter, and is drawn aside from its proper state, into one that is foreign and base.

So that it is highly requisite to study, if we are solicitous of returning to our pristine state of felicity, how to depart from sense and imagination, and her attendant brutality, and from those passions which are raised by her phantastic eye, as much as the necessity of our nature will permit. For the intellect must be accurately composed ; and it is proper it should obtain a peace and tranquility free from the contentions of that part which is destitute of reason, that we may not only hear with attention concerning intellectual and intelligible objects, but to the utmost of our ability, may enjoy their contemplation ; and thus, being reduced into an incorporeal nature, may truly lead an intellectual life, and not in a false delusive manner, like those who are at the same time entangled with corporeal concerns. We must, therefore, divest ourselves of the various garments of mortality by which our vigour is impeded ; as well this visible and fleshly garment, as that more interior one with which we are invested contiguous to the skin. We must enter the place of contest naked, and without the incumbrance of dress, striving for the most glorious of all prizes, the Olympiad of the soul. But the first requisite, and without which it is not lawful to contend, is, that we strip off our garments. And since our vestments are some of them exterior, and some interior, so with respect to the denudation of the soul, one process is by things more open, another by such as are more occult. For instance, not to eat, or not to accept what is offered, is among things obvious and open ; but not to desire is more obscure ; so that

it is here requisite not only to abstain from things improper in deeds, but likewise in desire. For what does it profit to abstain in actions from what is base, in the mean time adhering to the causes which produce such actions, as if bound in indissoluble chains?

But this receding from material affections is brought about partly by force, and partly by persuasion; and by the assistance of reason the affections languish, and are, as it were, buried in oblivion, or in a certain philosophical death; which is, indeed, the best mode of desertion, without oppressing the terrene bandage from which the soul departs. For in things which are the objects of sense, a violent devulsion cannot take place without either a laceration of some part, or at least a vestige of separation. But vice steals in upon the soul through continual negligence: and carelessness is produced by not sufficiently attending to intelligible objects; the affections in the mean time being excited by the drowsy perceptions of sense, among which must be also reckoned the sensations arising from food. We must therefore abstain, not less than from other things, from such food as usually excites the passions of our soul. Let us then in this particular enquire a little farther.

There are two fountains, whose noxious streams detain the soul in matter, and with which, as if saturated with lethargic potions, she forgets her own proper speculations: I mean pleasure and grief, the artificer of which is sense and its perceptions, together with the operations attendant on the senses, imaginations, opinions, and memory. The passions, roused by the energies of these, and the irrational part, now fattened with noxious nutriment, draw down the soul, and avert her inclinations from her native love of true being. It is requisite, therefore, that we revolt  
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from these to the utmost of our ability. But true defections can alone take place by avoiding the passions and rash motions produced by the senses. But, sensation respects whatever moves the sight, or the hearing, or the taste, or the smell. And sense is, as it were, the metropolis of that foreign colony of passions which reside in the soul, and which must be expelled by him who wishes, while connected with body, to become an inhabitant of the royal regions of intellect. Let us then enquire how much fuel of the passions enters into us through each of the senses; and this either when we behold the spectacles of horses in the race, and the labours of the athletic, or the contests of those who twist and bend their bodies in leaping, or when we survey beautiful women. For all these insnare us, unconscious of the danger, and subject to their dominion the irrational appetite, by proffered enchantments of every kind.

For by all such enchantments the soul, as if driven into fury, compels the compound man to leap rashly, and without reason, and full of the brutal nature to bellow and exclaim. In the mean time, the perturbation appearing from without, being inflamed by the internal, which was first of all roused by sense. But the vehement motions excited by the hearing, arise from certain noises and sounds, from base discourse, and mixed assemblies; so that some, exiled from reason, behave as if struck mad; and others, enervated by effeminate softness, agitate themselves by a multitude of trifling gesticulations. And who is ignorant how much the soul is fattened, and infested with material grossness, by the ointments and perfumes which commend lovers to each other? But why is it necessary to speak of the passions originating from the taste: in this respect especially, binding the soul in a double band; one of which is

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thickened by the passions excited by the taste; the other becomes strong and powerful by the different bodies which we receive in food. For as a certain physician observed, those are not the only poisons which are prepared by the medical art, but such things as we daily receive for food, as well liquid as solid, are to be reckoned among this number; and much greater danger arises to our life from these, than to our bodies from poisons. But the touch does all but transmute the soul into body, and excites in it, as in a dissonant body, certain broken and enervated sounds. The remembrance, imagination, and cogitation of all these raise a collected swarm of passions, i. e. of fear, desire, anger, love, emulation, cares, and griefs, they fill the soul with perturbations of this kind, cloud its intellectual eye with oblivion, and bury its divine light in material darkness.

On which account it is a great undertaking to be purified from all this rout of pollutions; and to bestow much labour in meditating day and night, what measures we shall adopt to be freed from these bonds, and this because we are complicated with sense, from a certain necessity. From whence, as much as our ability will permit, we ought to recede from those places in which we may (perhaps unwillingly), meet with this hostile rout; and it is requisite we should be solicitous not to engage in combat with these dangerous foes, lest, through too great a confidence of victory and success, instead of vigorous contention, we produce only unskilfulness and indolence."

And in the conclusion of the first book, he adds, " For, indeed, if it be lawful to speak freely, and without fear, we can by no other means obtain the true end of a contemplative, intellectual life, but by adhering to the Deity (if I may be allowed the expression), as if fastened by a nail, at the same time being torn away and separated from body

body and corporeal delights; having procured safety from our deeds, and not from the mere attention to words. But if friendship is not to be conciliated with a divinity, who is only the governor of some particular region, with any kind of food, or by the use of animal nutriment, much less can a gross diet effect an union with that God who is exalted above all things, and who is superior to a nature simply incorporeal; but after every mode of purgation, and the greatest chastity of body, and purity of soul, we shall scarcely be thought worthy to obtain the vision of his ineffable beauty; though this is sometimes permitted to him whose soul is well disposed, and who has passed through life with the greatest sanctity and purity of manners. So that, by how much the Father of all exceeds every nature in simplicity, purity, and self-sufficiency, as being infinitely remote from all suspicion of material contagion, by so much the more ought he who approaches to the Deity, to be entirely pure and holy, first in his body, and afterwards in the most secret recesses of his soul; having distributed a purgation adapted to every part, and being completely invested with purity, as with a transparent garment, fit for the intimate reception of divine illumination." Thus far Porphyry, whose excellent sentiments on this subject are a lasting monument of the elevation and purity of soul which the Platonic philosophy affords; and at the same time sufficiently prove the arrogance and ignorance of those who deprecate the wisdom of the ancients, and consider their greatest philosophers as involved in mental darkness and delusion. But presumption of this kind is continually increased by indolence, and strengthened by interest; and it is common to find *scribblers of every kind*, laughing at Plato and his philosophy, who are too mean for criticism; and even too insignificant for contempt. Let us, therefore, leave such

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in their native inanity, and listen to the instructions of the divinely elegant Proclus, by which we may ascend to the contemplation of true being, and the ineffable principle of things.

6. \* "Pythagoras and Plato command us to fly from the multitude, that we may pursue the most simple truth, and apply ourselves wholly to the contemplation of real being. From the multitude of exterior people drawing us aside in various ways, and deceiving us by fallacious appearances. But much more to shun the multitude of interior people; for this much more distracts and deceives. We must, therefore, fly from the various multitude of affections, the obscure informations of sense, the shadowy objects of imagination, and the dusky light of opinion. For every multitude of this kind is so different in itself, that its parts are contrary to one another; from whence it is necessary to betake ourselves to the sciences, in which multitude has no contrariety. For though affections are contrary to affections, one perception of sense to another, imaginations to imaginations, and opinions to opinions, yet no one science is found contrary to another. In this multitude, therefore, of propositions and notions, we may collect into one the number of sciences binding them in one according bond. For they are so remote from contrariety to each other, that notion is subservient to notion, and inferior sciences minister to superior, depending on them for their origin. Above all, it is here necessary, from many sciences which presuppose one, to betake ourselves to one science itself, no longer supposing another, and in an orderly series to refer them all to this original one. But after science, and its study, it will be necessary to lay aside compositions, divi-

\* See the Excerpta of Ficinus from Proclus, on the first Alcibiades of Plato; his Latin version only of which is extant. *Ficini Opera*, tom. ii.

fions,

fions, and multiform discourses, and from thence to ascend to intellectual life, to its simple vision, and intimate perception. For science is not the summit of knowledge, but beyond it is intellect; not that intellect only which is separated from soul, but the illustration infused from thence into the soul, which Aristotle affirms to be the intellect by which we acknowledge the principles of science; and Timæus says, that this exists in no place but the soul. Ascending, therefore, to this intellect, we must contemplate together with it intelligible essence, by indivisible and simple perceptions, speculating the simple genera of beings. But after venerable intellect itself, it will be proper to contemplate that summit of the soul, by which we are one, and under whose influence our multitude is united. For as by our intellect we touch the divine intellect; so by our unity, and as it were the *flower* of our essence, it will be lawful to touch that first one, from whom all subordinate unities proceed. And by this our *one*, we are especially conjoined with divinity. For similitude may be every where comprehended by that which is similar; the objects of knowledge by science; things intelligible by intellect; and the most unifying measures of being, by the unity of the soul. But this unity and its energy is the summit of our actions; for by this we become divine, when, flying from all multitude, we retire into the depths of our unity, and, being collected into one, uniformly energize. Thus far we admonish to shun the multitude, by steps proceeding from the order of knowledge: in the next place, we shall proceed in the same design by the series of knowable objects. Fly then every sensible species, for they are heaped together, are divisible, and perfectly mutable, and incapable of affording sincere and genuine knowledge. From these dark informations, therefore, betake yourself

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to incorporeal essence ; since every sensible object possesses adventitious unity, is by itself scattered and confused, and full of formless infinity. Hence its good is divisible, and adventitious, distant and separated from itself, and residing in a foreign seat. When you have ascended thither, and are placed among incorporeal beings, you will behold above the fluctuating empire of bodies, the sublime animal order, self-moving, spontaneously energizing in itself, and from itself possessing its own essence, yet multiplied, and anticipating in itself a certain apparition or image of the essence divisible about the unstable order of bodies. You will there perceive many habitudes of reasons, various proportions, and according bonds. Likewise the whole and parts, vivid circles, and a multiform variety of powers ; together with a perfection of souls not-eternal, not subsisting together as a whole, but unfolded by time, gradually departing from their integrity, and conversant with continual circulations. For such is the nature of the soul.

But after the multitude belonging to souls, betake yourself to intellect, and the intellectual kingdoms, that you may possess the unity of things. There remain in contemplation of a nature ever abiding in eternity, of life ever flourishing, intelligence ever vigilant, to which no perfection of being is wanting, and which does not desire the chariot of time, for the full energy of its essence. When you have beheld natures of this exalted kind, and have seen by how great an interval they are superior to souls ; in the next place enquire whether any multitude is there, and if intellect, since it is one, is also universal ; and again, since it is uniform, if not also multiform : for you will find it subsists after this manner. When, therefore, you have intimately beheld this intellectual multitude, though profoundly indivisible and united, transport yourself again

## TRUE END OR GEOMETRY. cxix

to another principle, and having considered, as in a more exalted rank, the *unities* of intellectual essences, in the last place proceed to unity perfectly separate and free from all things. And when advanced thus far, lay aside all multitude, and you will at length arrive at the ineffable fountain of *good*. And since it appears, from these various gradations, that the soul then properly obtains perfection, when she flies from all external and internal multitude, and the boundless variety of the universe; we may likewise conclude from hence, that our souls do not alone collect their knowledge from the obscure objects of sense, nor from things particular and divisible discover a perfect whole, and a perfect one, but draw forth science from their ~~most~~ recesses, and produce accuracy and perfection from whatever in appearances is inaccurate and imperfect. For it is not proper to suppose that things false and obscure, should be the principal sources of knowledge to the soul; and that things discordant among themselves, which require the reasonings and arguments of the soul, and which are ambiguous and confused, should precede science which is immutable; nor that things variously changed, should generate reasons abiding in one; nor that indeterminate beings should exist as the causes of determinate intelligence. It is not, therefore, fit to receive the truth of eternal entities from boundless multitude; nor from sensible objects the judgment of universals; nor from things destitute of reason, accurate discrimination of that which is good: but it is proper that the soul, retiring into her immortal essence, should there scrutinize the *good* and the *true*, and the immutable reasons of all things: for the essence of the soul is full of these, though they are clouded by oblivion. The soul, therefore, beholding exteriors, enquires after truth, in the mean time possessing it in the depths of her essence,

and deserting herself, explores the good in the dark regions of matter. Hence, every one in the pursuit of reality ought to begin with the knowledge of himself. For, if we constantly extend our views among the multitude of men, we shall never discern the one species man, obscured by the multitude, and distracted by the division and discord, and the various mutations of those who participate the species. But if we turn our eye inwards, there, remote from perturbation, we shall behold one reason and nature of men ; since multitude is an impediment to the conversion of the soul into herself. For here variety darkens unity, difference obscures identity, and dissimilitude clouds similitude ; since species are confused in the folds of matter ; and every where that which is excellent is mixed with the base." Thus far Proclus ; and thus much for our intended Dissertation.

L I F E

A N D

COMENTARIES

O F

P R O C L U S.



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T H E  
L I F E   O F   P R O C L U S,  
B Y   M A R I N U S\*;  
O R,  
C O N C E R N I N G   F E L I C I T Y.

WHEN I consider the magnitude of mind, and dignity of character belonging to Proclus, a philosopher of our time, and attend to those requisites, and that power of composition which those ought to possess who undertake a description of his life; and

\* Marinus, the author of the ensuing life, was the disciple of Proclus; and his successor in the Athenian school. His philosophical writings were not very numerous, and have not been preserved. A commentary ascribed to him, on Euclid's data, is still extant; but his most celebrated work, appears to have been, the present life of his master. It is indeed in the original elegant and concise; and may be considered as a very happy specimen of philosophical biography. Every liberal mind must be charmed and elevated with the grandeur and sublimity of character, with which Proclus is presented to our view. If compared with modern philosophical heroes, he appears to be a being of a superior order; and we look back with regret on the glorious period, so well calculated for the growth of the philosophical genius, and the encouragement of exalted merit. We find in his life, no traces of the common frailties of depraved humanity; no instances of meanness, or instability of conduct: but he is uniformly magnificent, and constantly good. I am well aware that this account of him will be considered by many as highly exaggerated; as the result of weak enthusiasm, blind superstition, or gross deception: but this will never be the persuasion of those, who know by experience what elevation of mind and purity of life the Platonic philosophy is capable of procuring; and who truly understand the divine truths contained in his works. And the testimony of the multitude, who measure the merit of other men's characters by the baseness of their own, is surely not to be regarded. I only add, that our Philosopher flourished 412 years after Christ, according to the accurate chronology of Fabricius: and I would recommend those who desire a variety of critical information concerning Proclus, to the Prolegomena prefixed by that most learned man to his excellent Greek and Latin edition of this work, printed at London in 1703.

V O L. 1.

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lastly,

lastly, when I regard my own poverty of diction, I am inclined to believe it more proper to refrain from such an undertaking, not to leap over the fosse (according to the proverb), and to decline a discourse involved in so much difficulty and danger. But my scruples are something diminished when I consider, on the other hand, that even in temples, those who approach to the altars do not all sacrifice alike; but that some are solicitously employed in preparing bulls, goats, and other things of a similar kind, as not unworthy the beneficence of the Gods to whom those altars belong: likewise that they compose hymns, some of which are more elegant in verse, but others in prose; while some, who are destitute of all such gifts, and sacrifice with nothing more perhaps than a cake and a small quantity of bread, with frankincense, and who finish their invocations with a short address to the particular divinity they adore, are not less heard than others. While I thus think with myself, I am afraid, according to Ibucus \*, lest I should not offend against the Gods (for these are his words) but against a wise man, and thus obtain the praise of men.

For I do not think it lawful, that I who was one of his familiars, should be silent concerning his life; and should not, according to my utmost ability, relate such particulars concerning him as are true, and which perhaps ought to be published in preference to others. And indeed by such a neglect I shall not perhaps obtain the esteem and honour of mankind, who will not entirely ascribe my conduct to the desire of avoiding ostentation, but will suppose I avoided such a design from indolence, or some, more dreadful disease of the soul. Incited, therefore, by all these considerations, I have taken upon me to relate some illustrious particulars of this philosopher, since they are almost infinite, and may be depended on for their undoubted reality.

I shall begin therefore not according to the usual manner of writers, who are accustomed to distribute their discourse into chapters; but I consider that the felicity of this blessed man ought, with the greatest propriety, to be placed as the foundation of this treatise. For I regard him as the most happy of those men who were celebrated in former ages; I do not say happy only from the felicity of wisdom, though he

\* Plato in Phædro. Meminit et Plutarch. VIII. Sympos. Suidas in μῆτρα. Fabricius.

possessed this in the highest degree of all men; nor because he abundantly enjoyed the goods of an animal life; nor again on account of his fortune, though this belonged to him in a most eminent degree, for he was supplied with a great abundance of all such things as are called external goods: but I call him happy, because his felicity was perfect, complete in all parts, and composed from each of the preceding particulars. Having then in the first place distributed \* virtues according to their kinds into natural, moral, and political, and also into those of a sublimer rank, which are wholly conversant with purification and contemplation, and are therefore called Cathartic and Theoretic, and also such as are denominated Theurgic, by which we acquire a similitude with some particular divinity; but omitting such as are superior to these, as beyond the reach of man, we shall begin from such as are more natural, and which are first in the progressions of the human soul, though not first in the nature of things.

This blessed man, then, whose praise is the subject of this treatise, naturally possessed, from the hour of his birth, all those physical virtues which fall to the lot of mankind; the traces of which were manifest in the latest period of his life, and appeared to surround and invest his body after the manner of a tenacious shell. In the first place, he was endued with a singular perfection of sensation, which they denominate corporeal prudence; and this was particularly evident in the nobler senses of seeing and hearing, which are indeed given by the gods to men for the purpose of philosophizing, and for the greater convenience of the animal life; and which remained entire to this divine man through the whole of his life. Secondly, he possessed a strength of body which was not affected by cold, and which was neither weakened nor disturbed by any vicious or negligent diet, nor by any endurance of labours, though it was exhausted day and night, while he was employed in prayer, in perusing the works of others, in writing books himself, and in conversing with his familiars; all which he performed with such expedition, that he appeared to study but one thing alone. But a power of this kind may with propriety be called fortitude of body, from the singular strength employed in such exertions.

\* For a full account of the distribution of the virtues according to the Platonists, consult the sentences of Porphyry, and the Prolegomena of Fabricius to this work.

The third corporeal virtue with which he was endued was beauty, which, when compared with temperance, the authors of these appellations have very properly considered as possessing a similitude of nature. For as we consider temperance as consisting in a certain symphony and consent of the powers of the soul, so corporeal beauty is understood to consist in a certain agreement of the organical parts. He was indeed of a most pleasing aspect, not only because he was endued with this excellent proportion of body, but because the flourishing condition of his soul beamed through his corporeal frame like a living light, with splendors too wonderful for language to explain. And indeed he was so beautiful that no painter could accurately describe his resemblance; and all the pictures of him which were circulated, although very beautiful, were far short of the true beauty of the original. But the fourth corporeal virtue which he possessed was health, which they affirm corresponds to justice in the soul; and that this is a certain justice in the disposition of the corporeal parts, as the other in those of the soul. For justice is nothing more than a certain habit, containing the parts of the soul in their proper duty. Hence, that is called health by physicians, which conciliates the jarring elements of the body into union and consent; and which Proclus possessed in such perfection, that he affirmed he was not ill above twice or thrice, in the course of so long a life as seventy-five years. But a sufficient proof of this is evident from hence, that, in his last illness, he was entirely ignorant what the disorders were which invaded his body, on account of the great rarity of their incursions.

Such then were the corporeal goods which Proclus possessed, and which may be called the forerunners, and as it were messengers, of those forms into which we have distributed perfect virtue. But the first powers and progeny of his soul, which he naturally possessed, previous to instruction, and those parts of virtue with which he was adorned, and which Plato reckons the elements of a philosophic nature \*, must excite the wonder of any one who considers their excellent quality. For he was remarkable for his memory and ingenuity; he was of a disposition magnificent, gentle, and friendly; and a com-

\* See the sixth book of his Republic, and the Epinomis.

nion,

nion, as it were, of truth, justice, fortitude and temperance; and his love of truth was so great, that he never admitted any prudent dissimulation, but hated falsehood vehemently. Indeed it is necessary that he who prosecutes truth, with so much earnestness and sincerity, should be extremely desirous of it from his infancy, since truth is the source of every good, both to gods and men. But that he despised corporeal pleasures, and was an eminent lover of temperance, is sufficiently evident from his great propensity to disciplines, and his desire of every kind of studies; for dispositions of this kind never suffer beastly and illiberal pleasure to dwell in the mind, but are able to excite in the soul, from her own internal operations, sincere pleasure and delight. But it is impossible to say how foreign he was from avarice, so that when a boy he despised the wealth of his parents, though very rich, on account of his incredible love towards philosophy. Hence he was far removed from illiberality, and from the care of lesser concerns, as he was most studious of the universe, and of every thing divine and human. But from such a disposition of the rational soul, having acquired true magnanimity, he considered human life as of no account, and, unlike the multitude, viewed nothing dreadful in death. So that he by no means feared all that rout of molestations which appear terrible to others, and this in consequence of that natural affection which it is proper to call by no other name than that of Fortitude alone. But, from all these virtues, I think it must be evident to those who have not experienced his best of dispositions, that he loved equity from a boy; that he was just and mild, and by no means difficult or unjust in his associations or contracts. To us indeed he certainly appeared modest and elegant, neither avaricious nor illiberal, neither arrogant nor timid.

But will it not be superfluous to mention the goodness and fertility of his ingenuity? Especially among those who know and who have heard, that he was full of the most beautiful disciplines, and who are acquainted with the multitude he produced and published to the world, so that he alone seemed to have drank nothing of the cup of oblivion, as he was endued with a power of memory which was never disturbed, and that which belongs to the oblivious, never happened to him. Besides, he never neglected fresh acquisitions, as if possessing a sufficiency

ficiency of disciplines; and as one who is merely delighted with their study. But he was most remote from a nature rustic and horrid, and averse from the Muses, and particularly propense to more cultivated endowments: for on account of his singular urbanity and festivity (without transgressing the bounds of true honesty) in his common associations, sacred feasts, and other actions, he allured and charmed his familiars, and always dismissed them more cheerful and pleased.

His mother, therefore, Marcella, lawfully united to his father Patricius, both of the Lycian nation, and excelling in birth and virtue, produced our philosopher, thus endued from the beginning with all these, and other gifts of nature. And \* Minerva, the tutelar goddess of Byzantium, received him when born, and took care of him as a midwife, she being the cause of his birth in that city: but afterwards she provided for his well-being, when he was numbered among boys and young men; for she appeared once to him in a dream, and exhorted him to the study of philosophy, from whence arose his great propensity to this goddess, as he particularly performed her sacred rites, and cultivated with a greater fury (as I may say) her institutions. Lastly, his parents brought him, when born, into their native country Zanthus, consecrated to Apollo: and I cannot but think that this country happened to him by a certain divine providence; as it was requisite that he, who was to be the prince of all sciences, should be educated under the presiding deity of the Muses. Here, being instituted in the most elegant manners, he pursued moral virtues, and was accustomed to right conduct, and to a declination of its contrary, that which is base.

But at that time the love of the gods, who had attended him from his nativity, manifestly appeared; for being once detained by some disease of body, and it appearing very difficult, and scarcely possible to cure him, there stood at his bed a youth of a more than ordinary

\* We are informed by Fabricius, that the Platonic Olympiodorus in his MS. Commentary on the Alcibiades of Plato, divides the orders of the Gods, into *ὑπερβολικούς*, or super-mundane, which are separate from all connection with body; and into *μητρικούς*, or mundane. And that of these, some are *αἰθέριοι*, or celestial, others *αἰθέριοι*, or etherial, or *πυρί* fiery, others *ἀέριοι*, or aerial, others *ὕδωροι*, or watry, others *χθένοι*, or earthly; and others *ὑποταραχάριοι*, or subterranean. But among the terrestrial, some are *κληματάρχαι*, or governors of climates, others *πολιεῦχαι*, or rulers over cities, and others lastly *κατοικίδιοι*, or governors of houses.

appearance,

appearance, so that even previous to the declaration of his name, he might be considered as \* Telephorus or Apollo: but the god proclaiming who he was, and pronouncing his name, touched the head of Proclus (for he stood reclining his head on Proclus' pillow) and having immediately restored him to health, vanished from his sight. And such was the † divine vision, and the divine benevolence at that time exhibited to our youth.

But having, for a short space of time, in Lycia, applied himself to grammar, he went to Alexandria in Egypt, bringing with him very singular moral virtues, by which he excited towards himself the love of the masters resident in that place. Hence Leonas the rhetorician, who derived (as I think) his lineage from Isaurus, and was illustrious among many of that profession, who were then at Alexandria, not only made him a partaker of his studies, but thought him worthy to become his domestic, and ordered that he should be supplied with food, together with his wife and children, no otherwise than if he had been his || true son. He likewise took care to procure him the notice of the principal men in Egypt, who being wonderfully delighted with the acumen of the youth's ingenuity, and with the elegance and integrity of his morals, reckoned him among their greatest friends. But he was also instructed by Orion the grammarian, whose ancestors discharged the sacerdotal office among the Egyptians, and who had made such a progress in the knowledge of his art, that he composed elaborate books on this subject, which he left not without advantage to posterity. He also went to the schools of the Roman preceptors, and made a great progress in that language; for he was at first led to the study of his father's profession, in which he was illustrious, his employment being the study of law in the royal city. But when it appeared how vehemently the young man was delighted

\* This epithet is likewise ascribed by Onomacritus to the Moon, as may be seen in his hymn to that deity; and the reason of which we have given in our notes to that hymn.

† Divine visions, and extraordinary circumstances, may be fairly allowed to happen to such exalted geniuses as Proclus; but deserve ridicule when ascribed to the vulgar.

|| What glorious times! when it was considered as an extraordinary circumstance for a teacher of rhetoric to treat a noble and wealthy pupil as his domestic. When we compare them with the present, we can only exclaim, *O tempora! O mores!* Philosophy sunk in the ruins of ancient Greece and Rome.

with

## 8 THE LIFE OF PROCLUS.

with the study of rhetoric, as he had not yet touched the writings of the philosophers, he both acquired great glory from his acquisitions, and became the admiration of his fellow pupils and masters, on account of the elegance of his discourse, and his celerity in perceiving; and from his exhibiting more the habit and industry of the master, than that of the scholar.

But while he yet frequented the rhetorical school, the sophist Leonas, made him the companion of his journey to Byzantium: which he undertook for the purpose of gratifying his friend Theodorus, who was at that time præfet of Alexandria, and who was a man both polite and magnificent, and a lover of philosophy. But Proclus, though a youth, followed his master more cheerfully in this journey, lest he should be compelled to interrupt his studies. However, that I may speak more truly, a certain good fortune brought him back to the source of his nativity. For, on his return, his tutelar goddess exhorted him to philosophy, and to visit the Athenian schools. But having first returned to Alexandria, and bid farewell to rhetoric, and the other arts which he had formerly studied, he gave himself up to the discourses of the philosophers then resident at Alexandria. But he frequented \* Olympiodorus, the most illustrious of philosophers, for the sake of imbibing the doctrine of Aristotle; and in mathematical disciplines gave himself to Hero †, a religious man, and one who was eminently skilful in the proper methods of institution. But these men were so delighted with the manners of the youth, that Olympiodorus wished him to espouse his daughter, whom he had taken care to instruct in philosophy, and Hero committed to him all his religion, and made him his constant companion. But having, on a certain time, heard Olympiodorus, a man who was endued with a great power of speaking, and on account of the celerity of his speech, and the gravity of his subjects, was understood by very few of his auditors, as he was departing with the dismissed multitude, he repeated to his companions all that was said, and almost verbatim,

\* Fabricius rightly observes, that this Olympiodorus is not the same with the Philosopher of that name, whose learned commentaries, on certain books of Plato, are extant in manuscript, in various libraries. As in these, not only Proclus himself, but Damascius, who flourished long after Proclus, is celebrated.

† Concerning the various mathematicians of this name, see Fabricius in *Eibliotheca Græca*.

though the discourse was copious; as Ulpianus Gazæus, one of his fellow-disciples informed me, who also consumed not the least part of his life in the study of philosophy. But he likewise learned, with great facility, the writings of Aristotle, pertaining to rational philosophy \*, though the bare reading of them is difficult to those who are engaged in the attempt.

Having therefore, at Alexandria, applied himself to these masters, and enjoyed their confidence in such instruction as they were able to afford, when upon reading together with them a certain author, they appeared to him not to interpret the mind of the philosopher as they ought; conceiving a contempt for these schools, and at the same time being mindful of the exhortation which had been divinely sent to him at Byzantium, he went to Athens, attended by the presiding deities of eloquence and philosophy, and by beneficent dæmons. For that he might preserve the genuine and entire succession † of Plato, he was brought by the gods to the guardian city of philosophy, as the circumstances which happened on his first entrance into the city, and all the divine excitations manifestly evince: for they openly presaged, that this gift was sent from the father Apollo, and was a future suffrage of his succession confirmed by divine events. For when his vessel drove to the Pyræum, and it was told to the citizens, Nicolaus, who afterwards flourished in the rhetorical art, but at that time studied under the masters of Athens, descended to the shore as if to an acquaintance, received him for his guest as if he had been a citizen, and brought him to the city; for Nicolaus was also a Lycian. But Proclus, who perceived himself weary from his journey, sat down at the temple of Socrates, though he did not yet know, nor had heard that Socrates was worshipped in that place, and requested Nicolaus that he would stay there for a short time, and, if possible, procure him some water, as he said that he was exceeding thirsty.

\* The word in the original is *λογικὰ*, which Fabricius rightly conjectures has in this place a more extensive signification than either Logic, or Rhetoric: but I must beg leave to differ from that great critic, in not translating it simply *philosophical*, as I should rather imagine, Marinus intended to confine it to that part of Aristotle's works, which comprehends only logic, rhetoric, and poetry. For the verb *ἰμάσθω*, or to learn, which Marinus uses on this occasion, cannot with propriety be applied to the more abstruse writings of Aristotle.

† Hence Proclus was called, by way of eminence, *διάδοχος Πλατónικος*, or the Platonic Successor.

10           THE LIFE OF PROCLUS.

Immediately Nicolaus, from that very consecrated place, brought him some water; for the fountain belonging to the statue of Socrates was not far distant. But while he was drinking, Nicolaus, for the first time, considering the circumstance: This is an omen, says he, because you have fate in the temple of Socrates, and have there first drank the Attic water. Then Proclus rising, and having paid due reverence to the place, proceeded to the city. But when he came to the tower, the porter who was present at his entrance, and was about to close the gates with bars, said to him, (that I may repeat the words of the man,) "Certainly unless you had come, I should have closed the gates." And what omen could be more manifest, or could less require the interpretations of \* Polletes or Melampodes, or such like diviners?

But Proclus, despising the schools of the rhetoricians, though they very much desired his association, as if he had come for that very purpose, met with the prince of philosophers Syrianus †, the son of Philoxenus. At that time too, Lachares was in the same company, a man much conversant in philosophy, and the companion of Syrianus in his study, but in eloquence he was in as great admiration as Homer in the poetic art. He then was, as I have said, present at the same time. But it was now the evening twilight, and while they were engaged in mutual converse, the sun set, and the moon made her first appearance after her change: wherefore, having saluted the stranger, they endeavoured to dismiss him, as being a young man, from their company, that they might adore the goddess apart. But he not having proceeded far, beheld also the moon appearing from the same house, and laying aside his sandals, in their presence saluted the goddess. Here Lachares, admiring the confidence of the youth,

\* Concerning Polletes, see Suidas; and for Melampodes, consult Fabricius in *Bibliotheca Graeca*.

† This Syrianus was indeed a most excellent philosopher, as we may be convinced from his commentary on the metaphysics of Aristotle, a Latin translation only of which, by one Hieronymus Bagolinus, was published at Venice in 1558. The Greek is extant, according to Fabricius, in many of the Italian libraries, and in the Johannean library at Hamburg. According to Suidas, he writ a commentary on the whole of Homer in six books; on Plato's politics, in four books; and on the consent of Orpheus, Pythagoras, and Plato, with the Chaldean Oracles, in ten books. All these are unfortunately lost; and the liberal few, are by this means deprived of treasures of wisdom, which another philosophical age, in some distant revolution, is alone likely to produce.

said,

faid, speaking to Syrianus, "This is what Plato \* divinely affirms of great geniuses; that they either produce great good, or its contrary." And such, that I may mention a few out of many, were the signatures of divine original, which happened to our philosopher, on his first arrival at Athens.

But Syrianus brought him to the great Plutarch †, the son of Nestor; who, when he saw the young man, not yet twenty years old, and heard of his love and desire of a philosophic life, he was much delighted, and immediately made him a partaker of his philosophic study, though his age almost forbade such an attempt; for he was then very old. He therefore read to Proclus his commentary on Aristotle's books on the soul, and on the Phædo of Plato: and this great man exhorted him to commit to writing what he heard, employing the ambition of the youth as an instrument, by telling him, that if he completed those scholia, they would be reported as the commentaries of Proclus. And as he loved the youth very much on account of his inclination to the best studies, he called him his son, and caused him to reside with him as a domestic. But after he saw his temperance, with regard to animal food, he exhorted him not to abstain from animals entirely, but to use them so far as was necessary to the vital energies of the corporeal part. He likewise gave the same advice to Syrianus, concerning the youth's diet. But he answered the old man, as that divine head (Proclus) informed us: "Suffer him with that frugality to learn what I wish, and then, if he pleases, he may die." Such was the care of his masters respecting him, in every concern. But the old man lived about two years after the arrival of Proclus; and dying, commended the youth to his successor Syrianus, as also his grandson Archiadas. But Syrianus, when he had received Proclus as his pupil, not only much assisted him in learning, but made him his domestic as to other concerns, and the companion of his philosophic life, having found him such an auditor and successor.

\* Socrates, in the 6th book of Plato's Republic, says, that from great geniuses nothing of a middle kind must be expected; but either great good, or great evil.

† The reader will please to take notice, that this great man is not the same with Plutarch the biographer, whose works are so well known; but an Athenian philosopher of a much later period.

as he had for a long time sought for ; and one who was adapted for the reception of a multitude of disciplines, and divine dogmata.

In a shorter space, therefore, than two years, he read, together with Syrianus, all the works of Aristotle, i. e. his logic, ethics, politics, physics, and theological science. But being sufficiently instructed in these, as in certain \* *proteleia*, and small mysteries, Syrianus led him to the sacred discipline of Plato, and this by an orderly progression, and not † according to the oracle, with a transcendent foot. And he was careful that he might survey with him true mysteries, with the eyes of his soul, free from material darkness, and with a speculation of intellect refined and pure. Hence Proclus was employed night and day in vigilant energies, and in writing compendiously what he had heard, employing his own judgment in the selection and order. And in consequence of this unwearied assiduity, he made so great a progress in a short time, that by then he was twenty-eight years of age, he composed a multitude of works ; and among the rest his very learned and elegant commentaries on the *Timæus*. But from an institution of this kind, his manners also received a greater ornament, since as he advanced in science he accumulated virtue.

But he likewise pursued the civil virtues from Aristotle's political writings, and Plato's books concerning laws and a republic. How-

\* Aristotle's philosophy, when compared with the discipline of Plato is, I think, deservedly considered in this place as bearing the relation of the *proteleia* to the *eopoeia* in sacred mysteries. Now the *proteleia*, or things previous to perfection, belong to the initiated, and the mystics ; the former of whom were introduced into some lighter ceremonies only : but the mystics, were permitted to be present with certain preliminary and lesser sacred concerns. On the other hand, the *eopœtae* were admitted into the sanctuary of the greater sacred rites ; and became spectators of the symbols, and more interior ceremonies. Aristotle indeed appears to be every where an enemy to the doctrine of ideas, as understood by Plato ; though they are doubtless the leading stars of all true philosophy. However, the great excellence of his works, considered as an introduction to the divine theology of Plato, deserves the most unbounded commendation. Agreeable to this, Damascius informs us that Isidorus the philosopher, "when he applied himself to the more holy philosophy of Aristotle, and saw that he trusted more to necessary reasons than to his own proper sense, yet did not entirely employ a divine intellect, was but little solicitous about his doctrine : but that when he had tasted of Plato's conceptions, he no longer deigned to behold him in the language of Pindar. But hoping he should obtain his desired end, if he could penetrate into the sanctuary of Plato's mind, he directed to this purpose the whole course of his application." Photii Bibliotheca. p. 1034.

† according to the oracle.] In the original καλὰ τὸ λόγιον, which I wonder Fabricius should translate, *quod aiunt*, as it is usual with the Platonists, to cite the Zoroastrian oracles exactly in these words, instances of which may be found in Proclus on Plato's theology ; and the very words prove themselves to be a part of an oracle, when attentively considered.

ever,

ever, lest he should be conversant only in the contemplation of these, and should attempt nothing actual, since he was hindred from engaging in public affairs himself, from his being employed in more important concerns, he exhorted Archiadas, who was a religious man, to a political life, at the same time instructing him in its nature and virtues, and explaining to him the methods proper to be observed in the discharge of its duties. And in the same manner as they incite those who run in the race, so he exhorted him to preside in common over the whole of his city, and to confer benefits privately upon each according to all the virtues, but particularly in conformity with the laws of justice. But he excited in reality the emulation of himself in Archiadas, when he exhibited to him his own liberality and magnificence with respect to riches, by bestowing gifts at one time on his friends, and at another time on his kindred, guests, and citizens; proving himself by this means not in the least affected with avarice, and conferring not a little to the good of public concerns. But when dying, he bequeathed that part of his possessions which he did not leave to Archiadas, to his own country, and to Athens. But Archiadas became so great a lover of truth, as well from the company of his own associates as from the friendship of Proclus, that he is never mentioned by our acquaintance, without being at the same time celebrated as the religious Archiadas.

But the philosopher also employed himself in civil consultations among public assemblies, where things pertaining to the republic were discussed; giving the most prudent counsels, conferring with governors concerning equity, and not only exhorting them to an impartial distribution of justice, but in a manner compelling them by philosophical authority. For he had a certain public care of the morals of princes, and not only instructed them in the art of temperate government by his discourse, but also by his own example through the whole of his life; since he was, as it were, the exemplar of temperance to the rest. But he gave a specimen of civil fortitude perfectly Herculean: for since at that time there was, as it were, a sea of troubles upon him, and mighty waves of stormy employments were roused by adverse winds against his upright life, he conducted himself, though in danger, with gravity, and an unshaken constancy.

And

And when he was once very much molested by the improbity of some violent men, which was both pernicious and dangerous to himself, he undertook a journey into Asia, which contributed greatly to his own advantage: for as he was not unskilful in the more ancient rites of that place, which he yet preserved, a divine power afforded him this occasion of departure. Hence, as he well knew the whole of their concerns, he taught them more accurately in things pertaining to the gods, if they happened to have neglected any thing through a long interval of time. And while he was engaged in all these employments, and lived in a correspondent manner, he was so concealed from the multitude that he even excelled the Pythagoreans, who keep with unshaken constancy this precept of their founder, *λαθε πιώσας, live concealed.* But having passed a year only in the parts of Lydia, he returned to Athens, under the providential protection of the presiding goddess of philosophy. And thus fortitude was perfected in our philosopher, first by nature, then by custom, and afterwards by science and the consideration of causes. Besides this, he exhibited in another manner his politic habit practically, by composing letters for noblemen; and by this means procuring good to entire cities. But of this I have a sufficient testimony from those on whom they were bestowed, as well Athenians as Argives, and others of different nations.

But he likewise much promoted and increased literary studies, demanding of princes rewards for the preceptors, according to their several deserts. Nor did he undertake this rashly, nor with any interested views, but he compelled them (as he considered it a matter of great moment) to be diligent in their profession, interrogating and discoursing with them respecting every particular: for he was a judge sufficiently instructed in the employments of them all. And if he ever found any one negligent in his profession, he sharply reproved him; so that he appeared very vehement and ambitious, because he was both willing and able to give a just determination on every subject: and he was indeed a lover of glory. But this was not a fault in him, as in most, because it alone regarded virtue and goodness. And, perhaps, without an energy of this kind, nothing great and excellent would ever subsist in the human mind. But he was in this respect

respect vehement: this I will not deny. Yet, at the same time, he was gentle; for he was easily pleased, and demonstrated in a moment that his anger was as pliable as wax. For, almost at the same time, he was (as I may say) wholly transported in reprehension, and with a desire of becoming subservient to their interest, and that he might intercede with princes in their names; being moved with a certain natural conjunction of soul, and, as it were, sympathy of grief.

And here, I very opportunely recollect a peculiar example of his natural sympathy of soul with others: nor do I think the like was ever related of any other man. For, notwithstanding he was unmarried, and had no children (because he was not desirous of such connections, but remained free from them all, though many noble and rich alliances were offered him), yet his care of all his familiars and friends, and of their wives and children, was as great as if he had been some common father, and the cause of their birth; for he bestowed a singular attention on the life of each. And whenever any one of them was detained by any disease, he first earnestly supplicated the gods on their behalf with sacrifices and hymns; afterwards he gave a prompt attendance on the sick person himself, convened the physicians, and urged them to make an immediate trial, if they knew of any thing in their art advantageous to the condition of the diseased; and sometimes he produced some singular advice himself, among the physicians; and thus delivered many from imminent dangers. And the greatness of this blessed man's humanity towards his servants, may be understood by those who desire it, from his will. But of all his familiars, he loved Archiadas and his kindred the most; because, in the first place, their succession was derived from the genus of Plutarch the philosopher; and afterwards on account of that \* Pythagoric friendship which he maintained with Archiadas, as he was both the companion and preceptor of his studies. And this other kind of friendship, differing from the two already mentioned, appears to have

\* Nothing is more celebrated by the ancients than that strict friendship which subsisted among the Pythagoreans; to the exercise of which they were accustomed to admonish each other, *not to divide the god which they contained*, as Jamblichus relates, lib. i. c. 33. De Vita Pythagoræ. Indeed, true friendship can alone subsist in souls, properly enlightened with genuine wisdom and virtue; for it then becomes an union of intellects, and must consequently be immortal and divine.

been

been the most firm and excellent, among these illustrious men. For Archiadas desired nothing, which was not also the wish of Proclus; and on the contrary, the desires of Proclus were the constant wishes of Archiadas.

But having now brought the political virtues, which are inferior to the true ones, to an end, and terminating them in \* friendship, as their proper bound, we shall now pass to the Cathartic differing from the politic virtues. Indeed, the employment of these last consists in purifying the soul, that so being liberated from the body as much as they are able to effect, it may regard human concerns, and possess a certain similitude with divinity; which is the soul's best and most exalted end. Yet they do not all liberate after the same manner, but some more, and others less. Since there are certain political purgations which adorn their possessors, even while connected with body, and reduce them to a better condition; bringing under the dominion of reason, anger and desire, and entirely destroying passion and every false opinion: but the Cathartic virtues, which are superior to these, separate entirely from this truly leaden weight of body, and procure an easy flight from mundane concerns. And in these, indeed, our philosopher was studiously employed during the whole of his life, which was devoted to philosophy; since he both taught by his discourses what they were, and after what manner they were preparatory to felicity, and in a particular manner conformed his life to their institutions; performing every thing which could contribute to the separation of his soul, using both night and day prayers, lustrations, and other purifications, as well according to the Orphic as the Chaldaic institutions: and every month he descended, with great diligence, to the sea; and this sometimes twice or thrice. But he was exercised in these, not only in the vigour of his age, but also towards the close of his life; and these customs he observed perpetually, as if they were certain invariable statutes.

But he used meat and drink, and other necessary pleasures, only so far as was necessary to avoid the molestations of disease; for he was in these by much the most frugal, and particularly loved abstinence

\* Pythagoras, according to Damascius, said, that friendship was the mother of all the political virtues.

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from animal food. And if at any time he was invited to eat it more vehemently, he was so cautious in its use, that he ate it merely after the manner of a taster. But he purified himself every month by the sacred rites, in honour of the mother of the gods, celebrated by the Romans, and prior to them by the Phrygians: he likewise more diligently observed the unfortunate days of the Egyptians than they themselves; and, besides this, fasted on certain days in a peculiar manner, on account of the lunar appearances \*. He likewise instituted a fast on the last day of the month, not having supped the day before. But in what a splendid manner, and with what piety, he celebrated the new moon, and properly observed, with sacrifices, the more illustrious feasts of almost all nations, according to the manner of each country; and how from these he did not, according to the custom of many, take occasion of becoming idle and intemperate, but employed himself in continual prayers, hymns, and the like, his hymns sufficiently evince, which not only celebrate the divinities of the Greeks, but likewise Marna Gazæus, Esculapius Leontechus, Ascalonites, and Theandrites, another god much venerated by the Arabians; together with Isis, worshipped by the Philians; and lastly, all the rest which were the subjects of his devotion. For this sentiment was very familiar to this most religious man, that it was proper a philosopher should not be careful in the observance of the rites and institutions of one particular city, nor of certain nations only, but that he should be the general priest of the universe. And thus was he pure and holy, so far as pertains to the virtue of temperance.

\* A genuine modern will doubtless consider the whole of Proclus' religious conduct as ridiculously superstitious. And so, indeed, at first sight, it appears; but he who has penetrated the depths of ancient wisdom, will find in it more than meets the vulgar ear. The religion of the Heathens, has indeed, for many centuries, been the object of ridicule and contempt: yet the author of the present work is not ashamed to own, that he is a perfect convert to it in every particular, so far as it was understood and illustrated by the Pythagoric and Platonic philosophers. Indeed the theology of the ancient, as well as of the modern vulgar, was no doubt full of absurdity; but that of the ancient philosophers, appears to be worthy of the highest commendations, and the most assiduous cultivation. However, the present prevailing opinions, forbid the defence of such a system; for this must be the business of a more enlightened and philosophic age. Besides, the author is not forgetful of Porphyry's destiny, whose polemical writings were suppressed by the decrees of emperors; and whose arguments in defence of his religion were so very futile and easy of solution, that, as St. Hierom informs us, in his preface on Daniel, Eusebius answered him in twenty-five, and Apollinaris in thirty volumes!

But he declined, as much as possible, pain: and if it ever happened to him, he bore it with gentleness, and diminished it with this view, that his best part might not at the same time be affected with its molestations. And the fortitude of his soul in this respect, was sufficiently evinced in his last illness; for when, at that time, he was oppressed and tormented with the most excruciating pains, he endeavoured to the utmost to mitigate and expel their afflictive invasions. Hence, on such occasions, he often commanded us to repeat certain hymns, which when repeated procured him a remission and cessation of pain. And what is more wonderful, he remembered what he heard of these, though forgetful of almost all human concerns, from the dissolution of his corporeal part continually increasing. For when we began to repeat, he supplied what was unfinished of the hymns, together with many of the Orphic verses; for it was these we were then reciting. Nor was he only thus constant in enduring corporeal evils, but much more so in external unfortunate events, and such as appeared to happen contrary to expectation. So that he would say, concerning particulars of this kind, *So it is, such things are usual;* which seemed to me, or rather at that time appeared to be, worthy of remembrance, and an evident argument of our philosopher's magnanimity. But besides this, he restrained anger as much as possible, so that it might either remain free from all excitation, or that at least reason might not consent to its indulgence, but the irrational faculty alone, contrary to his will, might be moderately and lightly excited. And with respect to venereal concerns, he used them in the natural way, but so as that he might not proceed beyond a very moderate and light phantasy, in their indulgence..

And thus the soul of this blessed man, having collected itself from all parts, and retiring into the depths of its essence, departed after a manner from body; while it yet appeared to be contained in its dark receptacle. For he possessed a prudence, not like that of a civil nature, which is conversant in the administration of fluctuating particulars, but prudence itself, by itself sincere, which is engaged in contemplating, and converting itself into itself, without any longer consenting to a corporeal nature. He likewise possessed a temperance free from evil; and which is not even moderately influenced by perturbations,

sturbations, but is abstracted from all affections. And lastly, he acquired a fortitude, which does not fear a departure from body. But reason and intellect having obtained in him a perfect dominion, and the inferior powers of his soul no longer opposing themselves to purifying justice, his whole life was adorned with the divine irradiations of genuine virtue.

Our philosopher, therefore, having most happily absolved this form of virtues, advancing now, as it were, by the highest and most mystical step, he ascended to the greatest and most consummate or telestic virtues; employing for this purpose, the felicity of his nature, and a sciential institution. Hence, being now purified, and the victor of his nativity, and despising the vain Thrysus-bearers, and boasters of wisdom, he happily penetrated into her profound recesses, and enjoyed the contemplation of the truly blessed spectacles she contains. No longer requiring prolix dissertations, or demonstrations, for the purpose of collecting the science of these, but, with a simple vision and energy of intellect, beholding the exemplar of the divine mind, he obtained a virtue which cannot, with sufficient propriety be called prudence, but is more properly denominated wisdom, or something, if possible, still more venerable and divine. But the philosopher energizing, according to this virtue, easily comprehended all the theology of the Greeks and Barbarians, and whatever is shadowed over by the figments of fables, and placed it in a clear light, for the use of those who are willing and able to pursue its latent signification. But having interpreted divinely every thing of this kind, and shewing the symphony between them all; at the same time, investigating all the writings of the ancients, whatever he found in them of genuine wisdom, and approved by general consent, this he judiciously applied to use; but if he found any thing of a different and dissonant nature, this he entirely rejected, as vicious and false. And whatever he met with contrary to wisdom, though endued with a friendly appearance, this he vigorously subverted by a diligent examination. Nor did he employ less force and perspicuity in his association with other men. For he was a man laborious to a miracle; as he often, in one day absolved five, and sometimes more lectures; and writ besides, many verses, often to the number of seven hundred. Besides this, he went

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to other philosophers, and frequented their company; and again celebrated with them an evening association, ceasing from the labour of writing. And all these employments he executed in such a manner, as not to neglect his nocturnal and vigilant piety to the gods, and assiduously supplicating the sun when rising, when at his meridian, and when he sets.

But he was the parent of many dogmata, which were never known before, both in physics, and in intellectual and more divine concerns. For he first taught, that there was a kind of souls \*, endued with the power of contemplating many forms at once, which he placed, not without great probability, between intellect, collectively, and as it were with one intuition comprehending every thing, and souls, which are alone able to direct their vision to one particular form. And those who are willing to peruse his works will meet with a great variety of dogmata, peculiar to him alone; the relation of which I shall omit, lest I should give a too great extent to my discourse. But he who evolves his writings, will easily perceive that all we have above related of him is most true, and much more if he happens to have known him, to have seen his face, and to have heard him interpreting in the most excellent dissertations, and delivering the Platonic and Socratic dogmata in his yearly schools. Nor did he seem destitute of divine inspiration; for he produced from his wise mouth, words similar to the most white and thick falling snow †; so that his eyes emitted a bright radiance, and the rest of his countenance was resplendent with a divine light. Hence, when on a certain time, one Rufinus, a man of a great name in the republic, who was studious of truth, and in other respects worthy of veneration, came to him when he was teaching and interpreting, he perceived that the head of Proclus was surrounded with a light; and when the philosopher had

\* See Proclus on Plato's Politics, p. 399. Instit. Theolog. num. 196; and the extracts of Ficinus from Proclus's commentary on the first Alcibiades, p. 246. &c.

† Alluding to the beautiful description given of Ulysses, in the 3d book of the Iliad, v. 222.

*Καὶ ἵπται πράσινοι τοικότα χιμείσονται.*

Which is thus elegantly paraphrased by Mr. Pope.

But when he speaks, what elocution flows!  
Soft as the fleeces of descending snows  
The copious accents fall, with easy art;  
Melting they fall, and sink into the heart! &c.

finished

finished his interpretation, Rufinus rising, adored him; and offered to give a public testimony, by oath, of the divine vision which he had observed. And much gold was offered to him, by the same Rufinus, on his return from Asia, having escaped the danger of the war. But Proclus likewise rejected this gift, and was by no means willing to receive it.

But that we may return to our first design, having now discoursed concerning the contemplative wisdom of the philosopher, though in a manner but little suited to its dignity, it remains that we now speak of the justice pertaining to this kind of virtues. For this, unlike that of which we spoke before, is not conversant in distribution, or proportion; but must be equally removed from the kind of self-energizing justice, by which all things are alone directed to the rational soul. For to that, concerning which we now treat, it is alone proper to refer every energy to intellect and the deity, which our philosopher performed in the most exalted manner. For he scarcely rested from his diurnal labours, or refreshed his body with sleep; and perhaps even then was not free from meditation and contemplation. This is certain, that having very speedily roused himself from sleep, as from a certain torpor of the soul, he aspired after the morning, the time of prayer; and left the greater part of the night, should glide from him without advantage, as he was lying alone in his bed, he either composed hymns, or examined and fortified those dogmata which afterwards, in the day time, he committed to writing.

After a similar manner he pursued that temperance which has an affinity with these virtues, and which consists in a conversion of the soul to intellect; so as not to suffer itself to be touched, nor moved with any other concerns. Lastly, he joined fortitude in alliance with these, by a certain perfect method, zealously aspiring after that liberty which is ignorant of all passion, and which he perceived was natural to the divine object of his contemplation. And thus, through the whole of his conduct, he did not lead the life of a man merely good, to which, as Plotinus says, the political virtues may lead, but leaving this far behind him, he endeavoured to change it for one much more perfect and divine, the life of the gods themselves; since, to become similar

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to these, and not to virtuous men, was the great object of his contention.

And thus he had rendered virtues of this kind familiar to himself, while he frequented the philosopher Syrianus, and evolved and studied the commentaries of the ancients. But he received from the mouth of his preceptor certain small seeds, as it were, of the Orphic and Chaldaic theology; because he was prevented from hearing the complete interpretation of his master on the Orphic verses. For Syrianus left to the choice of Proclus and one Domininus \*, a philosopher of the Syrian nation, and who afterwards succeeded Syrianus, the exposition of the Orphic writings, or the oracles. But they were by no means unanimous in their choice; for Domininus preferred the interpretation of the Orphic verses, and Proclus that of the oracles. But our philosopher did not perfect his undertaking, because the death of the great Syrianus happened not long after. Having therefore, as I have said, received the outlines from the mouth of his master, he applied himself with the greatest diligence to the written commentaries of Syrianus upon Orpheus; and being assiduously nourished with the copious lucubrations of Porphyry and Jamblichus on the oracles, and similar writings of the Chaldeans, he arrived, as much as is possible to man, to the top of those highest virtues, which the divine Jamblichus was accustomed to call after a truly divine manner, theurgic. He laboured therefore, not without exquisite judgment, in collecting the expositions of philosophers prior to his time; and contracted into one, other Chaldaic hypotheses, and the most excellent of the commentaries on the divine oracles, completing this great work in the space of five years; concerning which this divine vision appeared to him in his sleep. For he saw the great Plutarch approach to him, affirming that he should live so many years as he had composed tetrads or quaternions on the oracles. Afterwards, having collected the number of these, he found they amounted to seventy. But that this was a divine dream, was sufficiently evinced by the last part of his life; for though he lived seventy-five years, as we have mentioned above, yet he had not the perfect use of his powers, in the five last. But his body, though na-

\* Concerning Domininus, see Photius and Suidas from Damascius in his Life of Isidorus.

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turally of the best constitution, being debilitated by that hard and scarcely tolerable kind of diet which he used, and by so many frequent labours and fastings, began to languish exactly at his seventieth year, so that he then became much more remiss in all his energies than before. Yet, even at this period, and thus affected, he composed orations and hymns: he also writ some things, and conversed with his friends; but his ancient vigour was wanting in each. Hence the memory of the dream excited his wonder, and he every where said, that he had only lived seventy years. But while he laboured under this infirmity of body, a certain youth, named Hegias, rendered him more alert in the business of interpretation. This young man, who already exhibited from his tender years such egregious signs of all the virtues of his ancestors, was one of that golden chain of philosophers, who had formerly appeared to men; and adhered most diligently to Proclus delivering the Platonic and other theologies. But Proclus, at this advanced period, was not moderately rejoiced in communicating with the young man, his own writings, after he understood that he made cubital advances in every kind of disciplines. And thus we have briefly discoursed concerning the apparatus of the philosopher in the Chaldean oracles.

But I, on a certain time, having read with him the Orphic verses, and heard, among his interpretations, not only the recondite theology which is to be found in Jamblichus and Syrianus, but also, among many other divine men, I requested the philosopher that he would not leave these divine verses also without his explanation: but his answer was, That he had often thought of writing commentaries on Orpheus, but that he had been strongly prohibited in more than one dream. For Syrianus appearing to him in his sleep, had deterred him with threats from the design. Having therefore employed other machines, I intreated that at least he would mark what he principally approved of in the books of his master; which when this best of men had performed, in consequence of my persuasions, and had noted some things in the front of each of Syrianus's commentaries, we obtained a collection of all these, and by this means scholia, and commentaries of no small bulk; though to accomplish this on the whole of that divine poetry, and on all the Orphic rhapsodies, was not the intention of Proclus.

But

But since, as we have said, his incredible study of these concerns, procured him a greater and more perfect degree of theurgic virtue, hence he no longer remained in the contemplative order, nor contented with either of the two-fold properties in divine concerns, exercised intellect and thought alone on the more excellent objects of speculation: but he was careful to obtain another kind of virtues more divine than the former, and separated from the politic mode; for he used the Chaldean assemblies and conferences, and their divine and ineffable concealments. And having comprehended these, he learned the manner of pronouncing and expressing them, with their remaining use, from Asclepienia, the daughter of Plutarch: for she alone, at that time, preserved the knowledge of the great Orgies, and of the whole theurgic discipline, delivered to her by her father, who received it from Nestor. Besides this, our philosopher also being purified in an orderly manner in the Chaldean lustrations, was an inspector of the lucid hecatic phasma (or visions) of which he himself makes mention in one of his commentaries. But by moving a certain hecatic sphærule \*, he very opportunely brought down showers of rain, and freed Athens from an unseasonable heat. Besides this, by certain phylacteria, or charms, he stopt an earthquake, and had thoroughly tried the energies of a divining tripod, having learned from certain verses concerning its defect. For when he was in his fortieth year, he thought in a dream, that he repeated to himself, the following verses:

High above æther there with radiance bright,  
A pure immortal splendor wings its flight;  
Whose beams divine with vivid force aspire,  
And leap resounding from a fount of fire.

\* Nicephorus, in his commentary on Synesius de Insomniis, p. 562. informs us, that the hecatic orb, is a golden sphere, which has a sapphire stone included in its middle part, and through its whole extremity, characters and various figures. He adds, that turning this sphere round, they perform invocations, which they call Jyngæ. Thus too, according to Suidas, the magician Julian of Chaldea, and Arnuphis the Egyptian, brought down showers of rain, by a magical power. And by an artifice of this kind, Empedocles was accustomed to restrain the fury of the winds; on which account he was called ἀλεξίηπος, or a chaser of winds.

And

And in the beginning of his forty-second year, he appeared to himself to pronounce these verses, with a loud voice :

I. O! on my soul a sacred fire descends,  
Whose vivid pow'r the intellect extends ;  
From whence far-beaming through dull bodies night,  
It soars to æther deck'd with starry light ;  
And with soft murmurs through the azure round,  
The lucid regions of the gods resound.

Besides, he clearly perceived that he belonged to the Mercurial series; and was persuaded from a dream that he possessed the soul of \* Nicomachus the Pythagorean.

Indeed, if I were willing to be prolix, I could relate many theurgical operations of this blessed man ; from the infinite number of which,

\* No opinion is more celebrated, than that of the metempsychosis of Pythagoras : but perhaps, no doctrine is more generally mistaken. By most of the present day it is exploded as ridiculous ; and the few who retain some veneration for its founder, endeavour to destroy the literal, and to confine it to an allegorical meaning. By some of the ancients this mutation was limited to similar bodies : so that they conceived the human soul might transmigrate into various human bodies, but not into those of brutes ; and this was the opinion of Hierocles, as may be seen in his comment on the Golden Verses. But why may not the human soul become connected with subordinate as well as with superior lives, by a tendency of inclination ? Do not similars love to be united ; and is there not in all kinds of life, something similar and common ? Hence, when the affections of the soul verge to a baser nature, while connected with a human body, these affections, on the dissolution of such a body, become enveloped as it were, in a brutal nature, and the rational eye, in this case, clouded with perturbations, is oppressed by the irrational energies of the brute, and surveys nothing but the dark phantasms of a degraded imagination. But this doctrine is vindicated by Proclus with his usual subtlety, in his admirable commentary on the Timæus, lib. v. p. 329. as follows, " It is usual, says he, to enquire how souls can descend into brute animals. And some, indeed, think that there are certain similitudes of men to brutes, which they call savage lives : for they by no means think it possible that the rational essence can become the soul of a savage animal. On the contrary, others allow it may be sent into brutes, because all souls are of one and the same kind ; so that they may become wolves and panthers, and ichneumons. But true reason, indeed, asserts that the human soul may be lodged in brutes, yet in such a manner, as that it may obtain its own proper life, and that the degraded soul may, as it were, be carried above it, and be bound to the baser nature, by a propensity and similitude of affection. And that this is the only mode of insinuation, we have proved by a multitude of reasons, in our commentaries on the Phædrus. But if it is requisite to take notice, that this is the opinion of Plato, we add, that in his politics, he says, that the soul of Thersites assumed an ape, but not the body of an ape : and in the Phædrus, that the soul descends into a savage life, but not into a savage body ; for life is conjoined with its proper soul. And in this place he says it is changed into a brutal nature : for a brutal nature is not a brutal body, but a brutal life."

I will only relate one in this place of a very wonderful nature. Asclepigenia, the daughter of Archiades and Plutarch, but the wife of Theagenes, from whom I have received many benefits, while she was yet a girl, and was nourished by her parents, fell into a disease, grievous, and incurable by the physicians. And Archiades, who had repos'd in her alone the hope of his race, was deeply afflicted with her condition. But the physicians despairing of her recovery, the father fled to his last anchor, Proclus the philosopher, which he was accustomed to do in affairs of the greatest moment, considering him as his best preserver, and earnestly intreated him to intercede by his prayers with the god for his daughter, whose condition was well known to our philosopher. Proclus therefore, taking with him Pericles \* the Lydian, a man well deserving the name of a philosopher, went to the temple of Esculapius †, that he might pray to the god for the health of the sick virgin: for the city at that time happily possessed this divinity, and rejoiced in the temple of the saviour god, which was as yet free from the destruction of the Christians. As soon, therefore, as Proclus had prayed after the ancient manner, the girl immediately perceived a great change and alleviation of her disease; for the preserving god had easily restored her to health. The sacred rites being finished, Proclus went to Asclepigenia, and found her delivered from the molestations of disease, and in a healthy condition. But this affair, with many others, was performed privately, so that no traces of it remained to succeeding investigators; and the house in which he resided greatly assisted him in this design. For besides his other fortunes, he enjoyed a most convenient dwelling, which his father Syrianus and his grand-father Plutarch (for so he did not hesitate to call him) had once inhabited; and this was situated near the temple of Esculapius, together with that of the celebrated Sophocles, and of Bacchus, which is next to the theatre, and looks towards, or is at least seen from, the lofty towers of Minerva.

But how much Proclus was loved by the philosophic goddess is abundantly evinced by his philosophic life, which he chose through her persuasions, and that with the great success we have hitherto described..

\* Pericles Lydus, a Stoic philosopher.    † Vide Faustan. lib. i. Atticorum, cap. 21. et 20.

But

But she clearly demonstrated her affection to Proclus, by the following circumstance. When her image, which had been so long dedicated in the parthenon, or temple, was taken away by those \* who, without any hesitation, moved out of their places things the most holy, and which ought to be immovable, there appeared to the philosopher in a dream, a woman of a graceful form, who admonished him to build a temple with great expedition, for, says she, *it pleases Miner-va, the presiding deity of philosophy, to dwell with you.* And how familiar he was with Esculapius, besides what we have mentioned above, may be evinced from his perceiving the presence of the same god in his last illness. For while he was between sleeping and waking, he perceived a dragon creeping on his head, which vision occasioned a remission of his disease, and a mitigation of his pain; so that it is probable he would have been restored to perfect health, if his desire and vehement expectation of death had not prevented his recovery, or his no longer bestowing a diligent attention on his body.

But he also related the following circumstances, (worthy of being remembered) but not without tears, through the sympathy of his mind. For, when a young man, he was afraid lest he should be infested with the gout, which was the disease of his father, and which loves to descend from parents to their children. Nor was he afraid, as it seems, without reason; for prior to that which we have already related concerning him, he was tormented with pains of this kind, when another extraordinary circumstance happened to this blessed man. In consequence of a certain person's prescription, whom he had consulted, he applied a plaster to his tormented foot, which an unexpected bird flew away with as he was lying on his bed. And this was certainly a divine and salutary symbol to Proclus, and might have persuaded him not to be afraid of this calamity in future. But he, as I have said, was notwithstanding afflicted with the dread of this disease; he therefore supplicated the healing god concerning this, and intreated him to afford him a more evident token of his will: and after this he saw in his sleep (it is bold indeed to conceive such a circumstance in the mind, but we must dare, nevertheless, nor dread to bring truth

\* He means the Christians.

to light) Esculapius approaching to him, and accurately contemplating his leg, nor disdaining, through his great philanthropy to embrace his knee. Hence, in consequence of this vision, Proclus was, through the whole of his life, free from the apprehensions of this disease, and was never more tortured with this kind of pains.

Again, the god who is worshipped by the Adrotteni, did not less conspicuously evince his great familiarity with this friend of the gods; for, more than once, the god benevolently presented himself to the sight of Proclus, frequenting his temple. And when he was doubtful, and wished to know what god or gods resided and were worshipped in that place, because the inhabitants were of different opinions in this affair, some believing that the temple of Esculapius was there, persuaded of this by various signs, as voices are certainly said to be heard in that place, and a table is consecrated to the gods, and oracles are given concerning the recovery of health, and those who approach are miraculously delivered from the most grievous dangers of life. Others, on the contrary, think that the Dioscuri reside in that place, because there have been those who saw in the way tending towards Adrotta, two young men of the most beautiful form, and riding with great celerity on horseback; at the same time declaring, that they hastened to the temple. They add besides, that the countenance of these was indeed human, but that they immediately gave evidence of a more divine presence; for when the men were arrived at the temple, the above mentioned youths appeared to them, making no enquiry concerning the affair, and occupied in the sacred concerns; but presently after, withdrew themselves from their sight. As the philosopher, therefore, was in doubt concerning these divinities, and did not discredit the relations, having requested the gods who inhabited that place, that they would condescend to manifest who they were; a god appeared to him in a dream, and clearly spoke to him as follows: *What! Hast thou not heard of Jamblichus teaching concerning, and celebrating those two, Machaon and Podalirius?* And besides this, the god condescended to afford so great an instance of benevolence to this blessed man, that he stood after the manner of those who bestow encomiums on others in the theatres, and with a clear voice, and composed habit, extending his right hand, did not hesitate to exclaim, (that I may relate

relate the words of the god,) *Proclus the ornament of the city.* And what could possibly be a greater testimony of this blessed man's friendship with divinity? But he, indeed, in consequence of a most remarkable sympathy, by which he was united with the gods, could never refrain from tears when he mentioned this affair to us, and related the divine encomium of himself.

Indeed, if I were willing to pursue every particular in like manner, and to relate his familiarity with the Hermetic Pan, together with the great benevolence and manifold assistance which that divinity condescended to afford Proclus at Athens, and of that perfectly singular kind of felicity which he obtained from the mother of the gods, and in which he was accustomed especially to triumph and rejoice; I should perhaps seem to many readers, to be rash in my assertions, and to others, the author of things perfectly incredible. For many and great were the daily instances of this goddess's benevolence towards him, in words and actions, which are both innumerable and unheard of, and concerning which, I do not at present retain an accurate remembrance. But if any one is desirous of knowing how great he was in these, he must evolve his book concerning the mother of the gods, from which he will understand, that it was not without divine assistance he produced all the theology of that goddess into light; as likewise, whatever is acted or affirmed concerning the same in the fables of Athis, which he has explained after a philosophical manner, that vulgar ears might afterwards cease to be disturbed, on perceiving the lamentations and other obscure ceremonies with which her mysteries are celebrated.

After having, therefore, run through the theurgic virtue of Proclus, and its energies, and the happy circumstances which befel him, respecting its operations, and having shewn, that he did not less excel in every kind of virtue, and that he was a man such as mortals have not beheld for a long period of time, it remains that we now bring our discourse concerning him to a conclusion: for we are not now at the beginning of our narration, nor does the half of the whole remain to us, according to the proverb, but the whole itself is now perfect and complete. Since having begun from the felicity of the philosopher, and proceeded in its exemplification, our discourse now returns

to

to it again. For we have explained the great goods, and providential exertions, which were granted to this most excellent man from the gods, and have shewn their prompt attention to his prayers, the divine visions which he enjoyed, and the help and solicitude which the gods testified towards him. We have likewise explained his prosperous fate, and propitious fortune, his country, parents, the good habit of his body, his masters, friends, and other external advantages; each of which we have shewn to have been far greater and more splendid in him than in other men; and have likewise diligently enumerated such things as cannot be reckoned amongst outward allurements, but entirely depended on his will, such as are the upright and illustrious deeds of his soul, according to universal virtue. And thus we have summarily demonstrated that his soul arrived in reality at the summit of the most consummate virtue, and was happily established in a perfect life, by human and divine goods of every kind.

But that the lovers of more elegant studies may be able to conjecture from the position of the stars under which he was born, that the condition of his life, was by no means among the last or middling classes, but ranked among the highest and most happy orders, we have thought fit to expose in this place, the following scheme of his nativity \*.

But

○	16° 26'	≡
☽	17 29	II
☿	24 23	8
♃	24 41	8
♂	29 50	‡
♀	— 23	‡
♌	4 42	≡
☿	8 19	¶
Hors.		
Mid.		
Heaven.		
♌	4 42	¶
or the head of the dragon.		
24	33	≡
The new moon preceding his birth.		
8	51	≡

♌ or the head of the dragon.

24 33 ≡

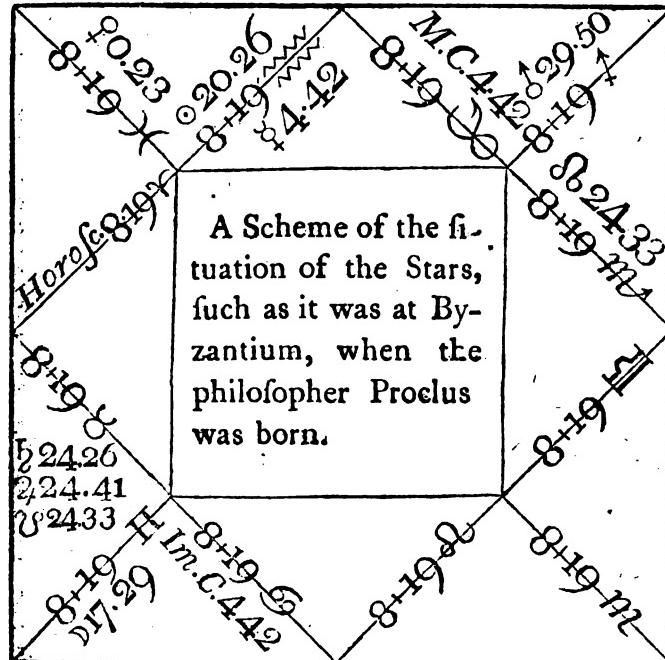
The new moon preceding his birth.

8 51 ≡

\* Proclus was born in the year of Christ 412, on the 6th of the Ides of February. But, for the sake of the astrologers, I have subjoined the following figure from the Prolegomena of Fabricius to this life: and though I am not skilled in the art myself, I am persuaded, from the arguments of Plotinus, that it contains many general truths; but when made subservient to particulars, is liable to great inaccuracy and error. In short, its evidence is wholly of a physiognomic nature; for such is the admirable order and connection of things, that throughout the universe, one thing is signified by another, and wholes are after a manner contained in their parts.

But Proclus departed from this corporeal life, in the one hundred and twenty-fourth year after the government of Julian, on the seventeenth day of the Attic Munichion, or the April of the Romans. Nicagoras, the junior, being at that time the Athenian archon. The dead body was taken care of according to the Athenian rites, as he himself while living had directed. And if that diligent attention which is due to the deceased, was ever paid by any one, it was certainly bestowed by this most blessed man. For he did not neglect any particular of the accustomed ceremonies, but every year, on certain days, visited the sepulchres of the Athenian heroes and philosophers, and besides this sacrificed himself, and not through the medium of another, to the manes of his friends and familiars. And having thus exhibited to each a proper reverence and honour, proceeding into the Academy, he there, in a certain place, pleased one by one the manes of his ancestors and kindred; and shortly after, in another part of the

parts. So that the language of the obscure and profound Heraclitus is perfectly just, when he says, "You must connect the perfect and the imperfect, the agreeing and the disagreeing, the consonant and the dissonant, and out of one all things, and out of all things one."



same

same Academy, he supplicated in common the souls of all philosophers. And these ceremonies being ended, this most excellent man, having chosen a third place, performed sacred rites to all the souls of deceased mankind. The dead body of Proclus, therefore, being disposed of according to his appointment, as we have said, was carried by his friends and buried in the eastern part of the suburbs \*, near to Lycaetus, where also the body of his preceptor Syrianus reposes, who, when he was alive, had requested this of our philosopher, and, in consequence of this, had taken care to procure a twofold receptacle in his sepulchre. But when this most holy man was consulted how he would wish his funeral to be conducted, lest there should be any thing indecent, or without a proper decorum, he desired flutes, with which he was threatened in a dream, and nothing besides. An epigram, consisting of four verses, is inscribed on his tomb, which he composed himself, as follows:

I Proclus, here the debt of nature paid,  
 (My country Lycia) in the dust am laid ;  
 Great Syrianus form'd my early youth,  
 And left me his successor in *the truth*.  
 One common tomb, our earthly part contains,  
 One place our kindred souls,—th' æthereal plains.

Nor were prognostics wanting in the year prior to his decease, such as an eclipse † of the sun, so great that night was produced in the day-time : for the darkness was so profound, that the stars became visible. And this happened when the sun was in Capricorn, in the eastern centre. Besides, the writers of Ephemeride observe that there will be another eclipse of the sun, at the conclusion of the next year. But such like affections of the heavenly bodies are said to portend events on the earth: to us indeed, the eclipse perspicuously signified a privation and, as it were, defection of the illustrious luminary of philosophy.

\* It was formerly the custom of almost all nations, to have their burial places in the suburbs, and not in the city itself.

† This eclipse happened, according to Fabricius, in A. C. 484. 19 Cal. Feb. at sun-rise.

And

And thus much may suffice for the life of the philosopher. It now remains for those who are willing to undertake the labour, to write concerning his disciples and friends. For it appears that many resorted to him from all parts; among which some were only his auditors, but others firm adherents to his doctrine; and, on account of philosophy, his familiars. I could likewise wish that some one, who is more laborious than myself, would give a particular account of his writings. For I have alone delivered these particulars of his life, that I might satisfy my conscience, and might testify that I religiously reverence the soul of Proclus, and his good dæmon. I shall, therefore, add nothing concerning the philosopher's writings, except that I have always heard him prefer his commentaries on the Timæus of Plato beyond all his works. He likewise vehemently approved of his commentaries on the Theætetus. And he used frequently to say, that if he was endued with despotic power, he would alone preserve, of all the writings of the ancients, the Oracles and the Timæus. *For, said he, I would abolish all the rest, and remove them from the present age, because it happens that many are offended, who undertake to read them rashly, and without proper institution.*

## Concerning the Published Writings of P R O C L U S.

MARINUS having neglected to give us an account of the writings of Proclus, I thought it would not be unacceptable to the reader, to present him with a catalogue of his works which are still extant; and the most perfect relation I am able of such as are unfortunately lost. In the execution of this design, I shall follow, for the most part, the accurate Fabricius in the order, and critical account of his works; but shall not hesitate to dissent from him in deciding on their philosophical merit: for it is very rare that philology and philosophy are united in the same person, and coalesce in amicable conjunction.

I. *Four Hymns.* In the preceding life we are informed, by Marinus, that Proclus composed many hymns, in which the divinities, both of Greeks and Barbarians, were celebrated; but unfortunately there are only four preserved, the first of which is to the Sun, the second and third to Venus (which may be found in the Florilegium of Stobus, p. 249. edit. Grotii,) and the fourth to the Muses. They are collected by Fabricius, in the eighth volume of his *Bibliotheca Græca*; and are most happy specimens of philosophical poetry. Indeed, they bear most evident marks of a mind full of divine light, and agitated with the fury of the Muses; and possess all that elegance of composition for which the writings of Proclus are so remarkable. So that it is very strange Gyraldus should ascribe them to a Hierophant of Laodicea, of that name; since, as Fabricius observes, Suidas mentions other writings of this priest, but does not speak of him as the author of any hymns. And if he had, these hymns breathe too much of the spirit and manner of Proclus, to be the production of any other.

2. *Two Books concerning the useful Parts of Learning, contained in Grammar.* Three books on this subject are mentioned by Suidas, and four by Photius; but there are alone extant fragments of two, in the *Bibliotheca* of Photius; which were published separate by Andrew Schottus, at the end of the *syntax* of Apollonius Alexandrinus, Francof.

1590. Hanov. 1615. quarto. It appears from these extracts, that Proclus, in this work, not only treats of every kind of verse, but also of the more celebrated poets. And hence Fabricius conjectures, that the short life of Homer, which Leo Allatius published under the name of Proclus, was taken from the first book of this Chrestomathia.

3. *Eighteen Arguments against the Christians.* The learned Cave, was egregiously mistaken, in supposing that this work, as well as the answer to it by Philoponus, is lost; not considering, probably, that these arguments in defence of the world's eternity are (as Fabricius well observes) directly against one of the Christian dogmata, the creation of the world. These arguments (except the first, which is lost,) are preserved in the answer of Philoponus; the Greek edition of which was published at Venice, in 1535, folio; and in Latin from the version of Joh. Mahatius, at Lyons, 1557. folio. Simplicius, in his commentaries on the first book of Aristotle de Cœlo, and elsewhere, opposes this work of Philoponus; at the same time, suppressing his name. The arguments are, in my opinion, extremely subtle, and unanswerable; and Philoponus, in his refutation, every where discovers, that he has by no means fathomed the depth of our philosopher.

4. *A Commentary on the Timæus of Plato, in five Books.* A most admirable work, composed in the flower of his age, when he was twenty-eight years old, according to Marinus. This invaluable treasure, may be said to contain all the wisdom of the highest antiquity: for Proclus every where cites the most celebrated interpreters on the Timæus, such as Adraustus the Peripatetic; Ælianus and Albinus, Platonists; Aristocles, Clearchus, Crantor, Dercyllides; and especially Jamblichus; likewise Longinus, whom he often refutes; Numenius the Pythagorean, and Origen, (different from Adamantius,) together with Plotinus, Porphyry the Platonic, Ptolemy, Severus, and Theodorus Asinæus; but he always prefers the explications of his master Syrianus, before all the rest. Add too, that he every where conciliates the Orphic writings and the Chaldean oracles with the theology of Plato. And with respect to Orpheus in particular, we are indebted to these sublime commentaries for the greatest part of the fragments of that divine poet. He is perpetually on the wing, and rises gradually

in elevated conceptions, until his mind, like Homer's discord, reaches the heavens. His style is at the same time nervous and diffuse, accurate and elegant. We are astonished with the magnificence of his metaphors, delighted with the copious variety of his matter, and filled with a divine light, by the sacred truths he unfolds. This great work, however, unfortunately, scarcely explains a third part of the *Timæus*; from whence it is probable, as Fabricius well observes, that several books have been lost through the injury of time. It was published in Greek, at Basil, in the year 1556; and is full of mistakes, as is generally the case with the Basil editions of books, so that a more correct edition is greatly to be desired, though at present not much to be expected. On concluding my account of this inestimable work, I find my indignation roused by the following words of Dr. Cudworth, in his *Intellectual System*, p. 306. "Proclus (says he) had some peculiar fancies and whims of his own, and was indeed a confounder of the Platonic theology, and a mingler of much unintelligible stuff with it." I must confess, (and I am neither afraid, nor ashamed of the declaration,) that I never found any thing in Proclus, but what by patient thought, accompanied with a sincere and vehement thirst after truth, I have been able to fathom. Had Dr. Cudworth been endued with these requisites, he would doubtless have had equal success; but without them, the sublimest truths will certainly appear to be *unintelligible stuff*. Besides this consideration is not to be omitted, that a modern priest makes a bad philosopher.

5. *On Plato's Republic.* These commentaries, or rather fragments of commentaries, are extant in Greek, at the end of Proclus on the *Timæus*. Suidas mentions four books of Proclus on Plato's politics; and some of Proclus's dissertations on these books, were found (according to Fabricius) in the library of Lucas Holstenius. The chief design of this work seems to be the unveiling the theological mysteries concealed under the fables of Homer, and other divine poets; which Proclus has accomplished (in my opinion) in a most wonderful manner. That Homer, indeed, every where abounds with Egyptian learning, is obvious to every one; but few are acquainted with the profound wisdom which his fables conceal. The latent meaning of most of these is unfolded in the present invaluable, though imperfect work;

work; and he whose mind is sufficiently enlightened by the ancient philosophy, to comprehend the beauty of these illustrations, will receive an additional delight from the study of Homer, which it is impossible to express. An epitome of this work was published in Latin, by the learned Gesner, 1542. 8vo. under the following title: *Apologiae quædam pro Homero, et Arte Poetica, Fabularumque aliquot Enarrationes ex commentariis Procli Lycii Diadochi philosophi Platonici in libros Platonis de Rep. in quibus plurimæ de Diis Fabulæ non juxta grammaticorum vulgus historicè, physicè aut ethicè tractantur, sed Theologicis, ut Gentiles loquuntur, ex prima Philosophia rationibus explanantur.*

6. *On the first Alcibiades of Plato.* Ficinus translated parts of this work into Latin, and published them under the title of *Procli de Anima ac dæmone, de Sacrificio et Magia*, Venice 1497. and 1516. fol. by Aldus; and in a more simple form at Lyons. Fabricius informs us, that the manuscript commentary of Proclus in Greek, but scarcely explaining the half part of the *Alcibiades*, is to be found in various libraries of France, England, and Italy. Also at Lyons, among the books of Isaac Vossius; and at Hamburgh in the Johannean library. From the specimen given of this work by Ficinus, it appears, like all Proclus's philosophical writings, to be an invaluable treasury of wisdom; and nothing certainly, reflects greater disgrace on a nation than suffering such monuments of ancient learning and wisdom to lie concealed in colleges, covered with dust, and never consulted.

7. *Six Books on Plato's Theology.* A most divine work, in which the philosopher collects into a system the theology dispersed in the writings of Plato, and establishes it by invincible demonstrations. He deduces, in a beautiful and connected series, all the divine orders, from the retreats of ineffable unity; every where connects them by proper mediums, and, after leading us through the long gradation of principles, brings us back again to the original from whence they flowed, and to which they constantly tend. The whole is uncommonly profound and abstruse; and it was not before the third reading, that I could fathom the depth it contains. Fabricius observes, "that it is a subtle and learned work, but from which, you will sooner learn the opinion of Syrianus and Proclus, concerning the deity and divine

concerns, than that of Plato. He adds, that it is usual with the Platonists, even from Plotinus, to unite to the doctrine of Plato, a thousand dogmata, foreign from his philosophy, as if Plato, though he did not perceive after this manner, ought certainly so to perceive." When men mistake their abilities, they always act absurdly, and often dangerously. As a laborious and accurate critic on philological matters, Fabricius merits the highest commendation such attainments can deserve; but when he leaves the beaten road in which nature designed him to walk, and attempts the tractless paths of philosophy, he perpetually stumbles, and often falls on the ground. The wings of philology, like those of the swallow, were never destined for a lofty flight:—it must be the eagle wing of genius, which can alone soar to the sun of philosophy. The Greek and Latin edition of this valuable work, was published at Hamburgh, by Æmilius Portus, 1618, folio.

8. *Theological Institutions*; or, as it may be called, *the Elements of Theology*. This admirable work contains two hundred and ten propositions, disposed in a scientific order, and supported by the firmest demonstrations. They begin from super-essential unity, and proceed gradually through all the beautiful and wonderful progressions of divine causes, ending in the self-moving energies of soul. They possess all the accuracy of Euclid, and all the subtilty and sublimity necessary to a knowledge of the most profound theology; and may be considered as bearing the same relation to the Pythagoric and Platonic wisdom, as Euclid's Elements, to the most abstruse geometry. Patricius, the first Latin translator of this divine work, seems to have been very sensible of the truth of this observation: for he every where carefully distinguishes the propositions from their demonstrations; and adds the word corollary to such consequencies as merit that appellation. His edition was published at Ferraria, 1583. quarto, under the title of *Theological Elements*. The Greek and Latin edition, is subjoined to Proclus's six books on Plato's Theology, Hamburgh 1618. folio.

9. *Two Books concerning Motion*. This useful work, collected, as Fabricius observes, from the third and following books of Aristotle's physics, was published in Greek at Basil, 1531, and with the Latin version of one Justus Velsius, a physician, Basil, 1545. octavo. It was

was likewise translated by Patricius, and is annexed to his version of the Theological Institutions.

10. *An Hypotyposis, or Information concerning Astronomical Hypotheses.* This work, which Fabricius observes is a compendium of Ptolemy's Almagest, was published in Greek, at Basil, 1540. quarto; and in Latin by George Valla, folio, 1541. A part of this work, which treats of the use of the astrolabe, Fabricius informs us, is extant in manuscript, in various libraries. The same accurate critic likewise observes, that a small treatise, inscribed Uranodromus, is extant, under the name of Proclus, in some libraries, as in that of Vindobona, and of Oxford, among the Barocian volumes. The comprehensive variety of Proclus's genius equally demands our admiration and applause.

11. *A small Treatise concerning the Sphere, or Celestial Circles.* This little work is an accurate and elegant introduction to astronomy; and is almost wholly taken from the Isagoge of Geminus Rhodius, on the phænomena. The best editions are the Greek and Latin one published at Paris in 1553, quarto; and that of Bainbridge, professor of astronomy at Oxford, London 1620. quarto.

12. *A Paraphrase in four Books, on the Quadripartite of Ptolemy.* This elegant work must, I should imagine, be an invaluable treasure to the lovers of astrology. It was first published in Greek by Melanchthon; and afterwards in Greek and Latin by Leo Allatius, at Lyons Bat. 1654. octavo.

13. *Four Books, on the first Book of Euclid's Elements.* For an account of this work, see the introduction, and the following sheets, in which it speaks for itself, in an English dress.

14. *A Commentary on Hesiod's Works and Days.* This work contains a valuable moral explanation of this great poet's meaning; and Fabricius justly observes, that he is often assaulted without occasion, by the petulant jeers of that vain man Joh. Tzetzes. The best edition of this work is that of Daniel Heinsius, Lugd. Bat. 1603. quarto.

15. Fabricius informs us, that in some manuscripts, as in the Vindobonensian and Barocian, a small treatise is usually ascribed to Proclus, entitled *Epitolic Characters*; and is prefixed to the Epistles of Phalaris and Brutis, and published under the name of Libanius, in Greek,

40. WRITINGS OF PROCLUS.

Greek, with the version of Casp. Stibilinus, Comelin. 1597, octavo. But it is doubtful whether Proclus is the genuine author of this work: from the title, I should suppose the contrary. And thus much for an account of those writings of Proclus which have escaped the ravages of time, and have been fortunately exposed to public inspection: it now remains that we relate such inestimable works of this philosopher, as are yet preserved in shameful concealment; or are utterly lost in the ruins of antiquity.

Concerning the Unpublished Writings of PROCLUS.

16. *On the Alcibiades of Plato.* See num. 6.

17. *On Plato's Politics.* See num. 5.

18. *On Plato's Parmenides.* A commentary, in seven books; the last of which was not completed by Proclus, but by Damascius. From occasional fragments, which have been published of this commentary, it appears to be a most divine work; and indeed it cannot be otherwise, if we consider it as the production of one of the greatest philosophers, on the most sublime and profound of all Plato's Dialogues. It is dedicated to Asclepiodotus, a physician and philosopher, and is not only extant in Greek MS. in the library of the German emperor, according to Lambecius, lib. vii. p. 41. but also in Latin, from the unpublished version of one Antonius Hermannus Gogava, as the same Lambecius informs us, p. 41. Four books of this work are extant in Greek, in the Bodleian library at Oxford; and it is much to be lamented that Thomson did not publish these, instead of his trifling edition of the Parmenides. Fabricius likewise informs us, that Livius Galantes mentions his having found six of these books in some of the Italian libraries. They are also extant in the Medicæan library of the great Etruscan commander.

19. *On the Cratylus of Plato.* We have already observed, in the dissertation on the Orphic theology, p. 105. what a great treasure of ancient mythology, must be contained in this work; but there is little hope of its ever emerging from the obscurity of public libraries. It is extant

extant in Greek, not only in the Italian libraries, but also among the manuscript books of Isaac Vossius.

20. *Uranodromus*. See above, num. 10.

21. *Ten Doubts concerning Providence, in one Book*. Philoponus mentions this work, in his second book against Proclus on the eternity of the world; and a Latin version of it is extant by one William de Morbekka, in the Johannean library of Hamburgh. Extracts from this translation are preserved by Fabricius, in his Greek Library; and they are in every respect worthy of the genius of Proclus.

22. *Concerning Providence and Fate, and that which is in our Power, one Book*. This work is dedicated to one Theodorus, a mechanist; and is extant in the Latin translation of the same Morbekka, in the Greek Library of Fabricius. The translation is for the most part barbarous, but is, however, sufficiently legible to discover that it is a most valuable treatise, replete with the usual elegance, subtilty, and sublimity of our philosopher.

23. *Concerning the Hypostasis, or Subsistence of Evil*. This book is extant in Latin, in the Johannean library; and fragments of it are preserved by Fabricius, in his Greek library. It is to be regretted, that Fabricius did not preserve the whole in that excellent philological work.

### Concerning the Lost Writings of P R O C L U S.

24. *On the Speech of Diotima, in Plato's Banquet, concerning the Subsistence of the Beautiful*. Fabricius informs us, that this work is distributed into many books; and Holstenius observes, that it is mentioned in a certain scholium of the Medicean copy of Proclus's commentaries on Plato's politics; but it is unfortunately nowhere extant.

25. *On the Philebus of Plato*; as may be inferred from the narration of Damascius in Photius, p. 550; and Suidas in Marinus. For Damascius relates, that Marinus having composed a commentary on this dialogue, on shewing it to Isidorus for his approbation, that philosopher observed, *that those of his master were sufficient*; which words Fabricius, with great propriety, applies to the commentaries of Proclus on the Philebus.

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26. *On the Theætetus of Plato.* This work is praised by Marinus, in the last chapter of the preceding life; and no doubt with great propriety: for this abstruse and sublime dialogue would naturally call forth all the divine fire and elegance of our philosopher.

27. *Commentaries on the Enneads of Plotinus.* This work is mentioned by Gyraldus, in his second dialogue on ancient poets; by Ficinus on Plotinus; by Philip Labbeus, in his account of MS. books, p. 286; and in the notes of Bullialdus to Theo of Smyrna, p. 224. But also in a certain note prefixed to an ancient manuscript of Jamblichus, on the Egyptian mysteries, to this effect: "The philosopher Proclus, commenting on the Enneads of the great Plotinus, says, that it is the divine Jamblichus who answers the epistle of Porphyry." This note is in Greek, in the original, and is (in my opinion) of itself sufficient to prove that such a work was once extant, though now unfortunately lost. How much the want of these commentaries is to be regretted, must be deeply felt by every lover of the Platonic philosophy. For the unequalled profundity, and divine mysteries, contained in the writings of Plotinus, could never be more happily illustrated than by the irradiations of such a genius as Proclus.

28. *Lectures on Aristotle's Book Περὶ Εργασίας, or concerning Interpretation.* This work, it seems, was never published; but Ammonius Hermeas, the disciple of Proclus, has inserted in his valuable commentary on this book all that he could retain in his memory of Proclus's lectures.

29. *Hymns*, not a few, see num. 1.

30. *Chestomathea.* See num. 3.

31. *On the Mother of the Gods*, one book, mentioned by Marinus, in the preceding Life.

32. *On the Theology of Orpheus.* This work is mentioned by Marinus, in the preceding Life, and by Suidas; and its loss must be particularly regretted by all the lovers of recondite theology.

33. *Ten Books, on the Chaldean Oracles.* This most valuable work is mentioned by Marinus, in the preceding Life, and by Proclus himself on Plato's Politics, p. 359. It was doubtless not extant at the time when Psellus and Pletho undertook the illustration of a few of these

these oracles : at least the inconsiderable merit of their commentaries, strongly favours this supposition.

34. *A Commentary on the whole of Homer.* Suidas. A specimen of the great value of this work may be seen in our philosopher's commentaries on Plato's republic. The works of Homer are not only the great fountain of poetry, but likewise of philosophy ; and are no less admirable for inspiring the fury of the Muses than for containing the mysteries of the most recondite theology.

35: *Concerning the Gods, according to Homer.* Had this work been preserved, we should doubtless have been furnished with a defence of the heathen religion, which would have silenced the ignorant clamours of its opponents.

36. *The Symphony or Concord of Orpheus, Pythagoras, and Plato.* Suidas. Proclus, in his published writings, is every where studious of reconciling the doctrines of these great men, and is always successful in this undertaking. Indeed, the same divine genius seems to have irradiated and inspired these wonderful heroes, but in different ways: in Orpheus it was accompanied with the fire of the Muses; in Pythagoras it shone through the mysterious veil of numbers ; and in Plato, combining the preceding modes, it was seen enshrined in awful majesty of thought, clothed with the graces of poetical diction, and resplendent with ineffable light,

37. *Two Books on the Theurgic Discipline.* Suidas. How much Proclus excelled in this art, may be seen in the preceding Life.

38. *Concerning the Oppositions of Aristotle to Plato's Timæus.* This work is mentioned by Proclus in the 3d book of his commentary on the Timæus, p. 226. and seems to have escaped the notice of the accurate Fabricius. Aristotle may, no doubt, in many particulars be reconciled with Plato ; but it is also certain, that in some he is perfectly dissonant. And thus much for the Life and Writings of Proclus.

## C O M M E N T A R I E S

O F

## P R O C L U S.

## B O O K I.

## C H A P. I.

*On the Middle Nature of the Mathematical Essence.*

**I**T is necessary that the mathematical essence should neither be separated from the first nor last genera of things, nor from that which obtains a simplicity of essence; but that it should obtain a middle situation between substances destitute of parts, simple, incomposite and indivisible, and such as are subject to partition, and are terminanted in manifold compositions and various divisions. For since that which subsists in its inherent reasons remains perpetually the same, is firm and durable, and cannot be confuted, it evidently declares it is superior to the forms existing in matter. But that power of progression which apprehends, and which besides uses the dimensions of subjects, and prepares different conclusions from different principles, gives it an order inferior to that nature which is allotted an indivisible essence, perfectly constituted in itself. Hence (as it appears to me) \* Plato also

\* All the ancient theologists, among whom Plato holds a distinguished rank, affirmed that the soul was of a certain middle nature and condition between intelligibles and sensibles: agreeable to which doctrine, Plotinus divinely asserts that she is placed in the horizon, or in the boundary and isthmus, as it were, of eternal and mortal natures; and hence, according to the Magi, she is similar to the moon, one of whose parts is lucid, but the other dark. Now the soul, in con-

also divides the knowledge of things which are, into first, middle, and last substances. And to indivisible natures, indeed, he attributes an

consequence of this middle condition, must necessarily be the receptacle of all middle energies, both vital and gnostic : so that her knowledge is inferior to the indivisible simplicity of intellectual comprehension, but superior to the impulsive perceptions of sense. Hence the mathematical genera and species reside in her essence, as in their proper and natural region ; for they are entirely of a middle nature, as Proclus proves in this and the sixth following chapter. But this doctrine of Plato's, originally derived from Brontinus and Archytas, is thus elegantly explained by that philosopher, in the concluding part of the sixth book of his Republic. "Socrates, know then, they are, as we say, two (the Good itself, and the Sun,) and that the one reigns over the intelligible world, but the other over the visible, not to say the heavens, lest I should deceive you by the name. You comprehend then, these two orders of things, I mean the visible and the intelligible ?—Glauco. I do.—Socrates. Continue this division then, as if it were a line divided into two unequal segments ; and each part again, i. e. the sensible and intelligible, divided after a similar manner, and you will have evidence and obscurity placed by each other. In the visible segment, indeed, one part will contain images. But I call images, in the first place, shadows ; afterwards, the resemblances of things appearing in water, and in dense, smooth, and lucid bodies, and every thing of this kind, if you apprehend me ?—Glauco. I apprehend you. —Socrates. Now conceive that the other section comprehends the things, of which these images are nothing more than similitudes, such as the animals around us, together with plants, and whatever is the work of nature and art.—Glauco. I conceive it.—Socrates. Do you consider this section then, as divided into true and false ? And that the hypothesis of opinion is to the knowledge of science, as a resemblance to its original ?—Glauco. I do, very readily.—Socrates. Now then, consider how the section of the intelligible is to be divided.—Glauco. How ?—Socrates. Thus : one segment is that which the soul enquires after, using the former divisions as images, and compelled to proceed from hypotheses, not to the principle, but to the conclusion. The other is that which employs the cogitative power of the soul, as she proceeds from an hypothesis to a principle no longer supposed, and, neglecting images, advances through their *obscurity* into the *light of ideas themselves*.—Glauco. I do not, in this, sufficiently understand you.—Socrates. But again, for you will more easily understand me from what has been already premised. I think you are not ignorant, that those who are conversant in geometry, arithmetic, and the like, suppose even and odd, together with various figures, and the three species of angles, and other things similar to these, according to each method of proceeding. Now, having established these, as hypotheses sufficiently known, they conceive that no reason is to be required for their position : but beginning from these, they descend through the rest, and arrive at last, at the object of their investigation.—Glauco. This I know perfectly well.—Socrates. This also you know, that they use visible forms, and make them the subject of their discourse, at the same time not directing their intellect to the perception of these, but to the *originals* they resemble ; I mean the square itself, and the diameter itself ; and not to the figures they delineate. And thus, other forms, which are represented by shadows and images in water, are employed by them, merely as resemblances, while they strive to behold that which can be seen by cognition alone.—Glauco. You speak the truth.—Socrates. This is what I called above a species of the intelligible, in the investigation of which, the soul was compelled to use hypotheses ; not ascending to the principle, as incapable of rising above hypotheses, but using the images formed from inferior objects, to a similitude of such as are superior, and which are so conceived and distinguished by opinion, as if they perspicuously contributed to the knowledge of things themselves.—Glauco. I understand indeed, that you are speaking of the circumstances which take place in geometry, and her kindred arts.—Socrates. Understand now, that by the other section of the intelligible, I mean that which reason herself reaches, by her power of demonstrating, when no longer esteeming hypotheses for principles, but receiving them in reality for hypotheses, she uses them as so many steps and handles in her ascent, until she arrives at that which is no longer hypo-

intelligence, which, in a collective manner, and by a certain simple power, divides the objects of intellectual perception; so that being divested of matter, and endued with the greatest purity, it apprehends things themselves, by a certain unifying perception, and excels the other kinds of knowledge. But to divisible essences, and such as are allotted the lowest nature, and to all sensible beings; he attributes opinion, which obtains an obscure and imperfect truth. But to middle essences (and such are mathematical forms), and to things inferior to an indivisible and superior to a divisible nature, he attributes cogitation. For this, indeed, is inferior to intellect, and the supreme science dialectic; but is more perfect than opinion, and more certain and pure. For it advances by a discursive procession, expands the indivisibility of intellect, and unfolds that which was involved in the unity of intellectual apprehension: but it collects things which are divided, and brings them back to mind. Hence, as knowledges differ among themselves, so the objects of knowledge are distinguished by nature. So that intelligible essences having an uniform subsistence,

'hypothetical, the principle of the universe; and afterwards descending, holding by ideas which adhere to the principle, she arrives at the conclusion, employing nothing sensible in her progress, but proceeding through ideas, and in these at last terminating her descent.—Glauco. I understand you, but not so well as I desire: for you seem to me to propose a great undertaking. You endeavour, indeed, to determine that the portion of true being and intelligible, which we speculate by the science of demonstration, is more evident than the discoveries made by the sciences called arts; because in the first hypotheses are principles, and their masters are compelled to employ the eye of cogitation, and not the perceptions of the senses. Yet, because they do not ascend to the principle, but investigate from hypotheses, they seem to you not to have intelligence concerning these, though they are intelligible, through the light of the principle. But you seem to me to call the habit of reasoning on geometrical and the like concerns, cogitation, rather than intelligence, as if cogitation held the middle situation between opinion and intellect.—Socrates. You understand me sufficiently well. And again: with these four proportions take these four corresponding affections of the soul: with the highest intelligence; with the second cogitation; against the third set opinion; and against the fourth assimilation, or imagination. Besides this, establish them in the order of alternate proportion, so that they may partake of evidence, in the same manner as their corresponding objects participate of reality." I have taken the liberty of translating this fine passage differently from both Petvin and Spens; because they have neglected to give the proper meaning of the word *diavōia*, or cogitation, the former translating it *mind*, and the *eye of the mind*, and by this means confounding it with intellect; and the latter calling it *understanding*. But it is certain that Plato, in this place, ranks intellect as the first, on account of the superior evidence of its perceptions; in the next place, cogitation; in the third, opinion; and in the fourth, imagination. However, the reader will please to remember, that by *diavōia*, or cogitation, in the present work, is understood that power of the soul which reasons from premises to conclusions, and whose syllogistic energy, on active subjects, is called prudence; and on such as are speculative, science. But for further information concerning its nature, see the dissertation prefixed to this work, and the following fifth chapter.

evidently

evidently excel all others. But sensibles are entirely excelled by primary essences: and mathematical natures, and whatever falls under cogitation, are allotted a middle order: for they are excelled by the division of intelligibles; but because destitute of matter, they are superior to sensible natures; and by a certain simple power, they are excelled by the first; but by a certain reason are more exalted than the last. Hence they possess notions of an intellectual essence, which are more manifest than sensibles, but which are, at the same time, only the images of an intellectual nature; and they imitate divisibly the indivisible, and, in a multiform manner, the uniform exemplars of things. And, that I may sum up the whole in a few words, they are placed in the vestibules or entrances of primary forms, and disclose their indivisible and prolific subsistence collected into one, but they do not yet excel the division and composition of reasons, and an essence accommodated to the obscurity of images; nor are they capable of passing beyond the various notions of the soul, endued with a discursive power, and of adhering to intellections perfectly simple, and purified from all material imperfection. After this manner then, is the middle nature of mathematical genera and forms to be understood; as filling up the medium between essences entirely indivisible, and such as are divisible about matter.

## C H A P. II.

*Concerning the common Principles of Beings, and of the Mathematical Essence, \* bound and infinite.*

BUT it is necessary that, considering the principles of the whole mathematical essence, we should return to those general principles, which pervade through and produce all things from themselves, I mean *bound* and *infinite*. For from these two after that cause of one, which can neither be explained, nor entirely comprehended, every other thing, as well as the nature of the mathematical disciplines, is constituted. In the former, indeed, producing all things collectively and separately; but in these proceeding in a convenient measure, and receiving a progression in a becoming order; and in some, subsisting among primary, but in others among middle, and in others again among posterior natures. For intelligible genera, by their simplicity of power, are the first participants of *bound* and *infinite*: because, on account of their union and identity, and their firm and stable existence, they are perfected by *bound*: but on account of their division into multitude, their copious power of generation, and their divine diversity and progression, they obtain the nature of *infinite*. But mathematical genera originate, indeed, from *bound* and *infinite*, yet not from primary, intelligible, and occult principles only; but also from those principles which proceed from the first to a secondary order, and which are sufficient to produce the middle ornaments of

\* These two principles, *bound* and *infinite*, will doubtless be considered by the unthinking part of mankind, as nothing more than general terms, and not as the most real of beings. However, an accurate contemplation of the universe, will convince every truly philosophic mind of their reality. For the heavens themselves, by the coherence and order of their parts, evince their participation of *bound*. But by their prolific powers, and the unceasing revolutions of the orbs they contain, they demonstrate their participation of *infinity*. And the finite and perpetually abiding forms with which the world is replete, bear a similitude to *bound*: while, on the contrary, the variety of particulars, their never-ceasing mutation, and the connection of more and less in the communion of forms, represents an image of *infinity*. Add too, that every natural species, by its form is similar to *bound*; but by its matter, to *infinity*. For these two, form and matter, depend on *bound* and *infinity*, and are their ultimate progressions. And each of these, indeed, participates of unity; but form is the measure and bound of matter, and is more *one*. But matter is in capacity all things, because it subsists by an emanation from the first capacity, or the *infinite* itself.

beings, and the variety which is alternately found in their natures. Hence, in these also, the reasons and proportions advance to infinity, but are restrained and confined by that which is the cause of bound. For number rising from the retreats of unity, receives an incessant increase, but that which is received as it stops in its progression, is always finite. Magnitude also suffers an infinite division, yet all the parts which are divided are bounded, and the particles of the whole exist finite in energy. So that without the being of infinity, all magnitudes would be commensurable, and no one would be found but what might either be explained by words, or comprehended by reason (in which indeed geometrical subjects appear to differ from such as are arithmetical;) and numbers would be very little able to evince the prolific power of unity, and all the multiplex and super-particular proportions which they contain. For every number changes its proportion, looking back upon, and diligently enquiring after unity, and a reason prior to itself. But *bound* being taken away, the commensurability and communication of reasons, and one and the same perpetual essence of forms, together with equality, and whatever regards a better co-ordination, would never appear in mathematical anticipations: nor would there be any science of these; nor any firm and certain comprehensions. Hence then, as all other genera of beings require these two principles, so likewise the mathematical essences. But such things as are last in the order of beings, which subsist in matter, and are formed by the plastic hand of nature, are manifestly seen to enjoy these two principles essentially. Infinite as the subject seat of their forms; but bound as that which invests them with reasons, figures, and forms. And hence it is manifest that mathematical essences have the same pre-existent principles with all the other genera of beings.

## C H A P. III.

*What the common Theorems are of the Mathematical Essences.*

BUT as we have contemplated the common principles of things, which are diffused through all the mathematical genera, after the same manner we must consider those common and simple theorems, originating from one science, which contains all mathematical knowledge in one. And we must investigate how they are capable of according with all numbers, magnitudes and motions. But of this kind are all considerations respecting proportions, compositions, divisions, conversions, and alternate changes: also the speculation of every kind of reasons, multiplex, super-particular, super-partient, and the opposite to these: together with the common and universal considerations respecting equal and unequal, not as conversant in figures, or numbers, or motions, but so far as each of these possesses a common nature essentially, and affords a more simple knowledge of itself. But beauty and order are also common to all the mathematical disciplines, together with a passage from things more known, to such as are sought for, and a transition from these to those which are called resolutions and compositions. Besides, a similitude and dissimilitude of reasons are by no means absent from the mathematical genera: for we call some figures similar, and others dissimilar; and the same with respect to numbers. And again, all the considerations which regard powers, agree in like manner to all the mathematical disciplines, as well the powers themselves, as things subject to their dominion: which, indeed, Socrates, in the Republic, dedicates to the Muses, speaking things arduous and sublime, because he had embraced things common to all mathematical reasons, in terminated limits, and had determined them in given numbers, in which the measures both of abundance and sterility appear.

C H A P.

## C H A P. IV.

*How these Common Properties subsist, and by what Science they are considered.*

BUT it is requisite to believe, that these common properties do not primarily subsist in many and divided forms, nor originate from things many and the last: but we ought to place them as things preceding in a certain simplicity and excellence. For the knowledge of these antecedes many knowledges, and supplies them with principles; and the multitude of sciences subsist about this, and are referred to it as their source. Thus the geometrician affirms, that when four magnitudes are proportional, they shall be alternately proportional; and he demonstrates this from principles peculiar to his science, and which the arithmetician never uses. In like manner, the arithmetician affirms, that when four numbers are proportional, they shall be so alternately: and this he evinces from the proper principles of his science. For who is he that knows alternate ratio considered by itself, whether it subsists in magnitudes or in numbers? And the division of composite magnitudes or numbers, and in like manner, the composition of such as are divided? They are surely not the sciences and cognitions of things divisible: but we have no science of things destitute of matter, and which are assigned a more intellectual contemplation; for the knowledge of these is by much prior to science, and from these the common reasons of many sciences are derived. And there is a gradual ascent in cognitions from things more particular to more universal, till we revert to the science of that which *is*, considered as *it is*, abstracted from all secondary properties. For this sublime science does not think it suitable to its dignity, to contemplate the common properties which are essentially inherent in numbers, and are common to all quantities; but it contemplates the one, and firm essence of all the things which *are*. Hence, it is the most capacious of all sciences, and from this all the rest assume their own peculiar principles. For the superior sciences always afford the first suppositions of demonstrations to such as are subordinate. But that which is the most perfect

of all the sciences, distributes from itself principles to all the rest, to some indeed, such as are more universal, but to others, such as are more particular. Hence, Socrates, in the *Theætetus*, mingling the jocose with the serious, compares the sciences which reside in us to doves: but he says they fly away, some in flocks, but others separate from one another. For such, indeed, as are more common and more capacious, comprehend in themselves many such as are more particular: but such as being distributed into forms, touch things subject to knowledge, are distant from one another, and can by no means be copulated together, since they are excited by different primary principles. One science, therefore, precedes all sciences and disciplines, since it knows the common properties which pervade through all the genera of beings, and supplies principles to all the mathematical sciences. And thus far our doctrine concerning dialectic \* is terminated.

## C H A P. V.

*What the Instrument is, which judges of the Mathematical Genera and Species.*

LET us now consider what that instrument is †, adapted to the judgment of mathematical concerns; and let us appoint Plato as our guide in this affair, who, in his *Republic*, divides cognitions separately from such things as are the objects of knowledge; and distributes cognitions in conjunction with things subject to knowledge. For of the things which are, some he ranks among intelligibles, and others among sensibles. And of intelligibles, some are again pure intelligibles, and others subject to cogitation. And of sensibles, some

\* Of human disciplines, those alone deserve to be called sciences which use no hypotheses, which resolve things into their principles, which are conversant with true being, and elevate us to ideas themselves. Dialectic is wholly of this kind (I mean the dialectic of Plato); for this alone uses no suppositions, but, neglecting shadows and images, raises us, by a sublime investigation, to the principle of the universe; and on this account, deserves to be called the very apex of disciplines. But we must not imagine, that by the word dialectic here, is meant logic, or any part of logic, or that method of disputation, by which we fabricate probable reasons; but we must conceive it as signifying a discipline, endued with the greatest acuteness; neglecting all hypotheses, truly soaring to primary causes, and ultimately reposing in their contemplation. Plotinus has given us most happy specimens of this method, in his books *on the genera of being*.

† See note to the first chapter.

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are purely sensibles, but others conjectural. To intelligibles, indeed, which are the first of the four genera, he assigns an intelligible knowledge; but to those which are subject to cogitation, he attributes thought: to sensibles, faith; but to conjecturals, a conjectural or assimilatory power. And he shews, that the assimilatory power has the same proportion to sense as thought to intelligence. For the conjectural power knows the spectres of sensible forms, while they are beheld in water and other bodies, which perspicuously represent their image: since, by their situation in water, they are after a manner, allotted the last seat in the gradations of forms, and truly become the resemblances of resemblances. In like manner, thought beholds the images of intelligibles in a degraded state, fallen from primary simple and indivisible forms, into multitude and division. Hence, a knowledge of this kind, depends on other more ancient hypotheses; but intelligence arrives at that principle which is no longer supposed. If then, mathematical concerns are neither allotted an essence separate from all division and variety, nor that nature which is apprehended by sense, which is obnoxious to many mutations, and is in every proportion divisible, it must be manifest to every one, that they are essentially subject to cogitation: but cogitation presides over these as an instrument adapted to judgment, in the same manner as sense to sensibles, and the assimilatory power to conjecturals. From whence, indeed, Socrates determines that the knowledge of these is more obscure than the first science, but is more evident than the impulsive apprehension of opinion. For in this the mathematical sciences are inferior to intelligence, because they contemplate that which is evolved, and is endowed with a power of progression; but they are superior to opinion, by that stability of reasons which they contain, and which cannot be confuted. And they originate from supposition, through a diminution of the first science; but they contain forms independent of matter, from their possessing a knowledge more perfect than that of sensibles. We have therefore determined an instrument adapted to the judgment of all mathematical concerns, i. e. cogitation, according to the mind of Plato; which places itself indeed above opinion, but is excelled by intelligence.

## C H A P. VI.

*Concerning the Essence of Mathematical Genera and Species \*.*

IT now remains, that we consider what subsistence or essence ought to be assigned to mathematical genera and species? Whether we must deduce their origin and subsistence from sensible objects, or from abstraction, or from a collection of such things as are dispersed by parts into one common definition; or must allow them an existence prior to that of sensibles, as Plato affirms, and as the progression of universal being demonstrates? First then, if we affirm that mathematical species are composed from sensibles; whilst the soul from material triangles or circles, forms in herself the trigonic, or circular species, by a kind of secondary generation; I would ask from whence is derived the great certainty and accuracy of definitions? For it must either proceed from sensibles, or from the soul herself. But from sensibles is impossible, for these, in a continual flow of generation and decay, do not for a moment retain an exact sameness of being; and consequently fall far short of the exactness contained in the definitions themselves. It must therefore proceed from the soul, which, by her immaterial nature, procures perfection from the imperfect, accurate

\* I would particularly recommend this chapter to modern mathematicians, most of whom, I am afraid, have never considered whether or not the subjects of their speculation have any real subsistence: though it is surely an enquiry worthy the earnest attention of every liberal mind. For if the objects of mathematical investigation are merely imaginary, I mean the point without parts, the line without breadth, &c. the science, founded on these false principles, must of course be entirely delusive. Indeed, an absolutely true conclusion, can never flow from an erroneous principle, as from its cause: as the stream must always partake of its source. I mean such a conclusion as is demonstrated by the proper cause,  $\pi\lambda\eta$ ,  $\epsilon$  διότι,  $\alpha\lambda\lambda'$  ἐτι, says Aristotle, in his first Analytics; that is, a syllogism from false principles will not prove the *why*, but only *simplly that it is*; indeed it can only *simplly prove that it is*, to him who admits the false propositions; because he who allows the premises, cannot deny the conclusion, when the syllogism is properly constructed. Thus we may syllogize in the first figure,

Every thing white, is an animal:

Every bird is white:

Therefore, Every bird is an animal.

And the conclusion will be true, though the major and minor terms are false; but then these terms are not the causes of the conclusion, and we have an inference without a proof. In like manner, if mathematical species are delusive and fictitious, the conclusions deduced from them as principles, are merely hypothetical, and not demonstrative.

subtily

## COMMENTARIES OF PROCLUS. 55

subtlety from that which is neither accurate nor subtle, and rekindles the light of ideas from the obscure and unreal objects of sense.

For where shall we find, amongst sensible objects, an indivisible nature, such as that of a point, or a line without the dimension of breadth, or a superficies without depth, or the ever constant proportion of sides, and exact rectitude of angles? For my part, I cannot see where, since all divisible natures are thus mixed and confused together, nothing sincere, nothing free from its contrary, but things every where yielding to separation, as well such as are removed by distance of place, as those which are united together. How then shall we obtain this durable essence for these immovable natures from the ever fluctuating forms of sense? For whatever derives its existence from moveable beings, must of necessity be mutable and frail. And how shall we gain this perfect accuracy for the stable species, from the inaccurate and imperfect? For whatever is the cause of a conception, always immutable, is itself much more stable than its effect. We must therefore admit the soul to be the generator of these mathematical species and reasons. But if she contains them in herself, as first exemplars, she gives them an essential being, so that the generations are nothing else than propagations of species, which had a prior subsistence in herself: and thus we shall speak agreeably to the sentiments of Plato, and discover the true essence of mathematical entities. But if the soul, though she neither possesses nor received the mathematical reasons prior to the energies of sense, yet fabricates this admirable immaterial building, and generates this fair series of speculations; how can she discern whether her productions are stable and constant, or things which the winds may dissipate, and phantoms rather than realities? What standard can she apply as the measure of their truth? Or how, since she is destitute of their essence, can she generate such a variety of reasons? For from such an hypothesis, we make their subsistence fortuitous, not tending to any scientific bound. Mathematical species are therefore the genuine offspring of the soul: nor does she derive from sensible objects the definitions she frames, but rather the first are propagated from the second; they are the energies of soul, which, as it were, pregnant with forms, delivers her immaterial

terial progeny into the dark and fluctuating regions of matter, as evidences of the permanent duration of her species.

Again, if we collect mathematical reasons from externals, why are not demonstrations composed from sensibles, better than the demonstrations of universal and simple species? For we say, in order to the investigation of any thing sought, that the principles and propositions, should be allied to the conclusions. If then, particulars are the causes of universals, and sensibles the sources of reasoning, why does the boundary of demonstration always refer to that which is more universal, and not to that which is partial and particular? And how can we prove that the essence of intelligibles is more allied to demonstration than the essence of sensibles? For thus they speak \*: his knowledge is not legitimate, who demonstrates that the isosceles, the equilateral, or the scalene triangle, have angles equal to two right; but he possesses science, properly so called, who demonstrates this of every triangle simply, or of triangle itself. And again, that universals, for the purpose of demonstration, are superior to particulars; that demonstrations concern things more universal; but that the principles from which demonstrations are composed, have a priority of existence, and a precedency in nature to singulars, and are the causes of the propositions they prove. It is very remote, therefore, from the nature of Apodictical sciences, that from converse with things of posterior origin, and from the dark perceptions of sense, they should falsely collect their indubitable propositions. I add farther, that they who affirm this, make the soul of a baser nature than the material species themselves. For if matter derives from nature beings essential, and participating a high degree of entity and evidence; but the soul, by a posterior energy, receives these from sensible objects, and fashions in herself resemblances and images of posterior origin, contemplating vile essences, and abstracting from matter, the forms inseparable from its nature; do they not make the soul more obscure and indigent

\* Aristotle, in his last Analytics. The reader will please to observe, that the whole force of this nervous, accurate, and elegant reasoning, is directed against Aristotle; who seems unfortunately to have considered, with the moderns, that mathematical species subsist in the soul, by an abstraction from sensibles. See the preceding Dissertation.

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than matter itself? For matter is the receptacle of forms materialized, as the soul is of species immaterialized. But in this case, matter would be the place of primary beings, and the soul of such as are secondary and subordinate: matter and its forms obtaining the lead in being, and existing as the sources of the subsistence of immaterial forms. Lastly, the material forms would have an essential existence, the others only an intentional denomination. How then can the soul, which is the first participant of intellect, and an intellective essence, and which derives from thence consummate knowledge, and a plenitude of life, become the receptacle of the most obscure species, the lowest in the order of things, and participating the most imperfect existence. But this opinion, which has been sufficiently exploded by others, needs no farther confutation.

If then, mathematical species do not subsist by material abstraction, nor by a collection of those common properties inherent in individuals; nor are at all, in their origin, posterior to sensibles, nor derived in any manner from them: it is necessary that the soul should either deduce them from herself, or from intellect; or lastly, from herself and intellect united. But if from herself alone, Whence do the images of intellectual species arise; whence do they derive their middle nature, linking, as it were, the divisible and indivisible essence together, if they do not participate the fullness of entity from primary essences? Lastly, how, upon this hypothesis, are the first exemplars, paradigms, or ideas, which subsist in intellect, the principles of universals? But if they are derived from intellect alone into the soul, how can the soul remain self-operative, and self-motive, if her inherent reasons flow from an external source, and are regulated by its operations? And in what respect does the soul differ from matter, which is all things in mere dormant capacity, but generates nothing appertaining to material species? It remains, therefore, that the soul deduces these species from herself, and intellect; and that she is the absolute consummation of the forms which originate from intellectual exemplars, but which are allotted from themselves a transition to permanent being. The soul, therefore, is by no means to be compared to a smooth tablet, void of all reasons; but she is an ever-written tablet, herself inscribing the characters in herself, of which she

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derives an eternal plenitude from intellect. For soul is a certain subordinate intellect, revolving round an intellect prior to herself, formed to its image, and participating its divine irradiations. If then, this superior intellect is all things intellectually, soul will all things animally; if the first exists as the exemplar, soul will be as its image; if as contracted and united in itself, soul as divisible and expanded. And this is what Plato understood, when in his *Timæus*, he composes the soul of the world from all things, dividing her according to harmonical reasons, and analogies; assigning to her the first principles effective of figures, I mean the right and circular line, and giving an intellectual motion to her inherent circles. All mathematical species, therefore, have a primary subsistence in the soul: so that, before sensible numbers, there are to be found in her inmost recesses, self-moving numbers; vital figures, prior to the apparent, ideal proportions of harmony previous to concordant sounds; and invisible orbs, prior to the bodies which revolve in a circle. So that soul is the prolific abundance of all these, and is another ornament producing herself, and produced from a proper principle, filling herself with life, and at the same time filled from the demiurgus of the universe, in an incorporeal and indistinct manner. When, therefore, she produces and unfolds her latent reasons, she then detects every science and virtue. The essence of soul then consists in these species, nor must we suppose her inherent numbers to be a multitude of units, nor her archytal ideas of divisible forms to be corporeal: but we must conceive all these as subsisting ever vitally, and intellectually, as the exemplars of apparent numbers, figures, reasons and motions. And here we must follow the doctrine of *Timæus*, who derives the origin, and consummates the fabric of the soul, from mathematical forms, and reposes in her nature the causes of every thing which exists. For the seven bounding terms \*, comprehending the principles of all numbers, lines, planes and solids, pre-exist in soul according to cause. And again, the principles of figures are placed in her essence, according to a demigurgical power. And lastly, the first of all motions, which embraces every other motion in its comprehensive

\* Viz. 1, 2, 4, 8, 3, 9, 27. Concerning which, see lib. iii. of Proclus's excellent Commentary on the *Timæus*.

prehensive

prehensive ambit, is co-existent with soul. For the principle of every thing which is moved is a circle, and the circular motion. The mathematical reasons, therefore, which fully consummate the soul, are essential, and self-moving: and the soul, by her cogitative power, diffusing, propagating, and evolving these, from her profound recesses, constitutes all the fair variety of mathematical sciences. Nor will she ever cease to generate, and waken into energy, succeeding species, while she divests her indivisible reasons of their intellectual simplicity. For she previously received all things, after a primary manner; and according to her infinite power, from pre-existent principles, deduces a beautiful series of various speculations.

## C H A P. VII.

*What the Employments and Powers are of the Mathematical Science, and how far they extend themselves in their Energies.*

BUT, after contemplating the essence of mathematical forms, it is necessary we should recur to that one master-science of these, which we have shewn is prior to a multitude of others, and that we should contemplate what its employment is, what are its powers, and how far it advances in its energies. The employment, therefore, of the whole mathematical science, possessing, as we have before said, the power of cognition, must not be placed so high as that of intelligence; which is firmly seated in its own stable essence, is perfect, is contained by itself, and in itself continually verges. Nor must it be situated so low as that of opinion and sense, since these cognitions dwell upon external concerns, energize upon them, and do not possess the causes of the objects of their knowledge. But the mathematical science, receives its commencement, indeed, extrinsically from \* recollection, but ends in the most intimate reasons, residing in the depths of

\* Plato frequently, both in the *Meno* and elsewhere, shews that science is Reminiscence; and I think not without the strongest reason. For since the soul is immaterial, as we have demonstrated in the dissertation to this work, she must be truly immortal, i. e. both *a parte ante*, & *a parte post*. That she must be eternal, indeed, with respect to futurity, if immaterial, is admitted by all; and we may prove, with Aristotle, in his first book de Cœlo, that she is immortal, likewise *a parte ante*, as follows. Every thing without generation, is incorruptible, and every thing incorruptible, is without generation: for that which is without generation, has a necessity

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of the soul; and is excited, indeed, from things posterior, but arrives by gradual advances at the principal essence of forms. Nor is its energy immovable, like that of intelligence, nor is it affected with local motion and alteration, like sense, but it revolves with a vital energy, and runs through the ornament of incorporeal reasons, sometimes advancing from principles to such things as are perfected by principles, but at other times yielding in a retrograde progression from conclusions to their forming principles: and sometimes proceeding from things previously known, to such as are the subject of investigation: but at other times, from things placed in the question, to such as precede in cognition. Besides, it does not excel all inquisition, as if it were perfect from itself, like intellect, nor is it perfected from

necessity of existing infinitely *a parte ante* (from the hypothesis); and therefore, if it possesses a capacity of being destroyed, since there is no greater reason why it should be corrupted now, rather than in some former period, it is endued with a capacity of being destroyed and ceasing to be, in every instant of infinite time, in which it necessarily is. In like manner, that which is incorruptible, has a necessity of existing infinitely *a parte post*; therefore, if it possesses a capacity of being generated, since there is no greater reason why it should be generated now rather than afterwards, it possesses a capacity of being generated, in every instant of time, in which it necessarily is. If then the soul is essentially immortal, with respect to the past and future circulations of time; and if she is replete with forms or ideas of every kind, as we have proved in the dissertation, she must, from her circulating nature, have been for ever conversant in alternately possessing and losing the knowledge of these. Now, the recovery of this knowledge by science, is called by Plato, reminiscence; and is nothing more than a renewed contemplation of those divine forms, so familiar to the soul, before she became involved in the dark vestment of an earthly body. So that we may say, with the elegant Maximus Tyrus, (Differ. 28.) "Reminiscence is similar to that which happens to the corporeal eye, which, though always endued with a power of vision, yet darkness sometimes obstructs its passage, and averts it from the perception of things. Art therefore, approaches, which though it does not give to the eye the power of vision, yet removes its impediments, and affords a free egress to its rays. Conceive now, that our rational soul is such a power of perceiving, which sees and knows the nature of beings. To this the common calamity of bodies happens, that darkness spreading round it, hurries away its aspect, blunts its sharpness, and extinguishes its proper light. Afterwards, the art of reason approaches, which, like a physician, does not bring or afford it a new science, but rouses that which it possesses, though very slender, confused, and unsteady." Hence, since the soul, by her immersion in body, is in a dormant state, until she is roused by science to an exertion of her latent energies; and yet even previous to this awakening, since she contains the vivid sparks, as it were, of all knowledge, which only require to be ventilated by the wings of learning, in order to rekindle the light of ideas, she may be said in this case to know all things as in a dream, and to be ignorant of them with respect to vigilant perceptions. Hence too, we may infer that time does not antecede our essential knowledge of forms, because we possess it from eternity: but it precedes our knowledge with respect to a production of these reasons into perfect energy. I only add, that I would recommend the liberal English reader, to Mr. Sydenham's excellent translation of Plato's Meno, where he will find a familiar and elegant demonstration of the doctrine of Reminiscence.

others,

others, like sense, but it proceeds by enquiry to invention, and ascends from the imperfect to perfection. But it likewise possesses twofold powers, one kind of these deducing principles into multitude, and generating the different paths of contemplation: but the other endued with a power of collecting many transitions into proper suppositions. For since it proposes to itself as principles, as well unity, and multitude, as bound and infinite, and such things as are subject to its comprehension, are allotted a middle order, between forms indivisible and every way divisible; with great propriety (I think) the gnostic powers of the whole science of these are essentially two-fold. One species indeed, hastens to union, and contracts the expansion of multitude: but the other possesses a power of distinguishing things simple into such as are various, more universals into more particulars, and reasons digested in their principle, into things secondary and multifariously multiplied from their principles. For rising higher from its commencement it penetrates even to such things as are the perfections of sensible concerns, is joined with nature, and demonstrates many things together with natural science. Since ascending from inferiors, it accedes in a certain respect proximate to intellectual knowledge, and touches the contemplation of things primary and divine. And hence, in the limits which flow from its essence, it produces the whole mechanic, optic, and catoptric speculation, together with many other sciences which are inwoven and entangled with sensible concerns, and which operate through their assistance. Besides, in its ascensions from corporeal natures, it derives intelligences indivisible and destitute of matter: and with these it perfects its divisible apprehensions, those cognitions which subsist in progressions, and its own genera and forms: it likewise indicates the truth respecting the gods themselves, and in its peculiar treatises exhibits a contemplation of the things which *are*. And thus much concerning the employment and powers of the Mathematical Science.

## C H A P. VIII.

*Concerning the Utility of the Mathematical Science.*

BUT let us now consider the utility of this Science, which extends itself from the most principal to the last cognitions. Timæus, therefore, calls the knowledge of the mathematical disciplines the path of erudition, because, indeed, it has the same proportion to universal science, and the first philosophy, which learning has to virtue. For this last frames our soul to a perfect life, by the possession of worthy manners; but the former prepares our cogitation, and the divine eye of our soul to an elevation from the obscurity of sensible information. Hence, Socrates in the Republic, says, "That the eye of the soul, which is darkened and buried by other studies, can by the mathematical disciplines alone be invigorated, and again excited to the contemplation of that which is, and transferred from resemblances to real beings, from an obscure light to that light which has the power of intelligence, and from a cave, and those bonds which exist in it as the authors of generation, and from material impediments be able to rise to an incorporeal and indivisible essence. For the beauty and order of mathematical reasons, and the firmness and stability of the contemplations they afford, conjoins us with intelligible objects, and perfectly determines us in their essences; which perpetually remain the same, ever shining with divine beauty, and preserving a mutual order without end. But Socrates, in the Phædrus, delivers to us three characters who are elevated from sense, because they fill up and accomplish the primary life of the soul, i. e. the philosopher, the lover, and the musician. But the beginning and path of elevation to the lover, is a progression from apparent beauty, using as excitations the middle forms of beautiful objects. But to the musician, who is allotted the third seat, the way consists in a transition from sensible to invisible harmonies, and to the reasons existing in these. So that to the one, sight is the instrument of reminiscence, and to the other, hearing. But to him who is by nature a philosopher, from whence and by what means is reminiscence the prelude of intellectual knowledge, and an excitation to that which truly is, and to truth itself? For this character

character also, on account of its imperfection, requires a proper principle: for it is allotted a natural virtue, an imperfect eye, and a degraded manner. It must therefore be excited from itself; and he who is of such a nature, rejoices in that which *is*. But to the philosopher, says Plotinus, the mathematical disciplines must be exhibited, that they may accustom him to an incorporeal nature, and that afterwards using these as figures, he may be led to dialectic reasons, and to the contemplation of all the things which *are*. And thus it is manifest, from hence, that the mathematics are of the greatest utility to philosophy. But it is requisite that we should be more explicit, and mention the several particulars to which they conduce, and evince that they prepare the intellectual apprehensions of theology. For whatever to imperfect natures appears difficult and arduous in obtaining the true knowledge of the gods, the mathematical reasons render, by their images, credible, manifest, and certain. Thus, in numbers, they indicate the significations of super-essential properties, but they evince the powers of intellectual figures, in those figures which fall under cogitation. Hence it is, that Plato, by mathematical forms teaches us many and admirable sentences concerning the gods, and the philosophy of the Pythagoreans, using these as veils, conceals from vulgar inspection the discipline of divine sentences. For such is the whole of the *Sacred and Divine Discourse* \*, that of Philolaus in his Bacchics, and the universal method of the Pythagoric narration concerning the Gods. But it especially refers to the contemplation of nature, since it discloses the order of those reasons by which the universe is fabricated, and that proportion which binds, as Timæus says,

\* Concerning this valuable work, entitled ΙΕΡΟΣ ΛΑΓΩΣ, see the Bibliotheca Græca of Fabricius, vol. i. p. 118 and 462. and in the commentary of Syrianus on Aristotle's metaphysics, p. 7, 71, 83, and 108, the reader will find some curious extracts from this celebrated discourse; particularly in p. 82. Syrianus informs us, "that he who consults this work will find all the orders both of Monads and Numbers, without neglecting one, fully celebrated (*ιημενας*)."  
There is no doubt, but that Pythagoras and his disciples concealed the sublimest truths, under the symbols of numbers; of which he who reads and understands the writings of the Platonists will be fully convinced. Hence Proclus, in the third book of his excellent commentary on the Timæus, observes, "that Plato employed mathematical terms for the sake of mystery and concealment, as certain veils, by which the penetralia of truth might be secluded from vulgar inspection, just as the theologians made fables, but the Pythagoreans symbols, subservient to the same purpose: for in images we may speculate their exemplars, and the former afford us the means of access to the latter."

whatever

whatever the world contains, in union and consent; besides, it conciliates in amity things mutually opposing each other, and gives convenience and consent to things mutually disagreeing, and exhibits to our view simple and primary elements, from which the universe is composed, on every side comprehended by commensurability and equality, because it receives convenient figures in its proportions, and numbers proper to every production, and finds out their revolutions and renovations, by which we are enabled to reason concerning the best origin, and the contrary dissolution of particulars. In consequence of this, as it appears to me, Timæus discloses the contemplation concerning the nature of the universe, by mathematical names, adorns the origin of the elements with numbers and figures, referring to these their powers, passions, and energies; and esteeming as well the acuteness as the obtruseness of angles, the levity of sides, or contrary powers, and their multitude and paucity to be the cause of the all-various mutation of the elements. But why may we not say, that it profits much, and in an admirable manner, to that philosophy which is called Politic, as well by measuring the times of actions as affording the various revolutions of the universe, and numbers convenient to things rising into being; I mean the assimilating, and authors of dissimilitude, the prolific too and the perfect, and the contraries to these; together with orderly and elegant ministers of life, and inelegance; and finally, such numbers as procure fertility and sterility. Which, indeed, the speech of the Muses in the Republic \* evinces, placing the universal Geometric Number as the author of better and more debased generations, and as the cause of the indissoluble perseverance of good manners, and of the mutation of the best Republics into such as are remote from reason, and are given to affections. For it is sufficiently evident, that it belongs to the whole mathematical discipline to deliver the science of this number which is called geometrical, and not to one particular science, such as arithmetic, or geometry: since the reasons or proportions of abundance and sterility, permeate through all the mathematical disciplines. Again, it is the means of our institution in moral philosophy which it brings to its ultimate

\* Concerning this Geometric Number, in the 8th book of Plato's Republic, than which Cicero affirms there is nothing more obscure, see the notes of Bullialdus to Theo. p. 292.

perfection,

perfection, and gives order and an elegant life to our manners. Besides this, it delivers to us figures, and modulations and motions convenient to virtue, by which the Athenian guest wishes those to be instituted and perfected, who are destined to pursue moral virtue from their early youth. Add too, that it places before our view the reasons of virtues, in one manner, indeed, in numbers, in another in figures, but differently in musical symphonies; and lastly, it indicates the excess and defect of vices, by which we are enabled to moderate and adorn our manners. Hence it is, that Socrates, in the Gorgias, accusing Calicles of an inordinate and intemperate life, says to him, " You neglect geometry and geometric equality :" but, in the Republic, he finds out the proportion of tyrannic pleasure to a royal interval, according to a plane and solid generation. But we shall learn what great utility is derived to other sciences and arts from the mathematical science, when we consider that it adds order and perfection to contemplative arts; I mean rhetoric, and all such as consist in discourse. But it proposes to the poetic arts, the reasons of poems in the place of an example, because it presides over the measures existing in these. But to the active arts it determines action and motion, by its own abiding and immovable forms. For all arts, as Socrates says, in the Philebus, require arithmetic, mensuration, and statics, either in all, or in some of their operations. But all these are contained in the discourses of the mathematical science, and are terminated according to their diversity. For from this science the divisions of numbers, and the variety of dimensions, and the difference of weights are known. The utility, therefore, of the whole mathematical science to philosophy itself, and to other sciences and arts, may be from hence known to intelligent hearers.

## C H A P. IX.

*A Solution of an Objection raised by some against the Utility of the Mathematical Sciences.*

BUT some, who are prone to contradiction through those who wish to subvert geometry, endeavour to destroy the dignity of this science. One part, indeed, depriving it of ornament and good, because it does not discourse on these. But another part \* affirming that sensible experiments are more useful than the universal objects of its speculation ; I mean, that Geodesia (for instance,) or the mensuration of the earth, is preferable to geometry, and vulgar arithmetic to that arithmetic which is conversant with theorems alone : and that nautical astrology is more useful than that which teaches universally, abstracted from any application to sensible concerns. For we are not, say they, made rich by our knowledge of riches, but by using them ; nor are we happy by the merely understanding felicity, but by living happily. Hence we must confess that those mathematical sciences, which are conversant with cognition, do not profit human life, and confer to action, but those only which are engaged in exercise. For those who are ignorant of the reasons of things, but are exercised in particular and sensible experiments, are in every respect more excellent, for the purposes of human life, than those who are employed in contemplation alone. Against objections then, of this kind, we shall reply, by shewing the beauty of the mathematical disciplines from those arguments by which Aristotle endeavours to persuade us. We must therefore confess that there are three things which especially cause beauty, both in bodies and souls ; I mean, order, convenience, and determination. Since corporeal baseness, indeed, arises from material inordination, deformity, and inconvenience, and from the do-

\* I am sorry to say, that this part of the enemies to pure geometry and arithmetic, are at the present time very numerous ; conceptions of utility in these sciences, extending no farther than the sordid purposes of a mere animal life. But surely, if intellect is a part of our composition, and the noblest part too, there must be an object of its contemplation ; and this, which is no other than truth in the most exalted sense, must be the most noble and useful subject of speculation to every rational being.

minion

minion of the indefinite in the composite body. But the baseness of the soul originates from its irrationality, and inordinate motion, and from its being in a state of discord with reason, and not receiving from thence its proper limitation. Hence, beauty exists even in contraries, by means of order, convenience and determination. But we may behold these in a more eminent degree in the mathematical science; order, indeed, in the perpetual exhibition of things posterior and more various, from such as are primary and more simple; for things subsequent are always annexed to their precedents, the latter ranking as principles, and the former as the first suppositions of things consequent: but convenience is evinced in the mutual consonance of things demonstrated, and in the relation of all of them to intellect, since intellect is the common measure of all science, from which it receives its principles, and to which it converts the learner: but determination is perceived in its perpetually abiding and immoveable reasons, for the objects of its knowledge are not, at times, subject to variation, like those of opinion and sense, but present themselves for ever the same, and are bounded by intellectual forms. If such then, are the principal requisites of beauty, it is evident, that in these sciences that illustrious ornament and gracefulness is found. For how is it possible this should not be the case with a science receiving a supernal illumination from intellect, to which it continually advances, hastening to transfer us from the obscure light of sensible information? With respect to the second objection, we think it proper to judge of its utility, without regarding the conveniences and necessities of human life. For otherwise, we must confess that contemplative virtue is also useless, which separates itself from human concerns, which it is very little desirous to look down upon and understand. Indeed Socrates, in the Theætetus, affirming this concerning noblemen endued with the prophetic power, says, "that it withdraws them from all regard to human life, and raises their thoughts, properly liberated, from all necessity and use, to the very summit of all true being. The mathematical science, therefore, must be considered as desirable for its own sake, and for the contemplation it affords, and not on account of the utility it administers to human concerns. But if it is necessary to refer the utility it produces to something different

from itself, it must be referred to intellectual knowledge. For it leads us to this, and prepares the eye of the soul for the knowledge of universals, removing and obliterating the impediments arising from the senses, and from corporeal involution. As therefore we call the whole of purgative virtue useful, or the contrary, not regarding the use of the sensible life, but of that which is contemplative, so indeed it is requisite to refer the end of mathematics to intellect, and universal wisdom. Hence its energy is worthy our study, both on its own account, and on account of an intellectual life. But it appears, as Aristotle \* says, that this science is desirable of itself to its votaries, because though no reward is proposed to its enquirers, yet the mathematical contemplation receives, in a small time, an abundant increase. Besides, this is farther evident from hence, that all men are willingly employed in its pursuit, and wish to dwell on its speculations, omitting every other concern; even those who have, with their lips, as it were, but just touched its utility. And hence it follows, that they who despise the knowledge of the mathematical disciplines, have very little tasted of the pleasures they contain. The mathematics, therefore, are not to be despised because their speculative parts do not immediately confer to human utility, (for the ultimate limits of its progressions, and whatever operates with matter, consider a use of this kind;) but on the contrary we should admire its immateriality, and the good which it contains, considered by itself alone. For when mankind were entirely disengaged from the care of necessary concerns, they converted themselves to the investigation of the mathematical disciplines; and this, indeed, with the greatest propriety. Since affairs familiar to human life in its most imperfect state, and which are immediately connected with its origin, first of all employed the studies of mankind: but, in the second place, those concerns succeeded which separate the soul from generation; and restore its memory of that which IS. After this manner, then, we are engaged in necessities, before things honourable for their own sakes, on account of their intrinsic dignity and worth; and in things related to sense, before such as are apprehended by the nobler energies of mind.

\* In the 13th book of his Metaphysics, cap. iii.

For every origin and life of the soul which is converted into herself, is naturally adapted to proceed from the imperfect to the perfect. And thus much against those who despise the mathematical science. //

## C H A P. X.

*A Solution of another Objection of certain Platonists, against the Utility of the Mathematical Sciences.*

BUT, perhaps, some of our own family will here rise up against us, and, proposing Plato as a witness, will endeavour to provoke ruder understandings into a contemptuous disregard of the mathematical disciplines. For they will say, that this philosopher entirely excludes (in his Republic) the mathematical knowledge from the choir of the sciences, and that he accuses it as being ignorant of its own principles, that its very principle is to itself unknown; and its ends and mediums composed from things of which it is ignorant. To these objections they may likewise add whatever other reproaches are there urged by Socrates against this contemplation. In answer then, to the objections of our friends, we shall recall into their memory, that Plato himself perspicuously asserts the mathematical science to be the purgation of the soul, and that it is endued with a power of leading it on high; because, like the Homeric Minerva, it removes the darkness of a sensible nature from the intellectual light of thought, which is better worth saving than ten thousand corporeal eyes, and which not only participates of a mercurial gift, (preserving us from the incantations and delusions of this material abode, which is similar to the fascinating realms of Circe,) but also of the more divine arts of Minerva. He likewise every where calls it by the name of science, and asserts that it is the cause of the greatest felicity to those who are exercised in its contemplation. But I will briefly explain why, in the Republic he takes from it the surname of science: for my present discourse is addressed to the learned. Plato, indeed, in most places, calls all the knowledge (as I may say) of universals by the name of science, opposing it in a division to sense which apprehends only particulars, whether such a mode of cognition is accomplished by art

or

or experience. And in this sense, as it appears to me in the *Civil Dialogue*, and in the *Sophista*, he seems to use the name of science; placing likewise the illustrious Sophistic science, which Socrates in the *Gorgias*, says, is a certain experience: also, the adulatory, and many others, which are experiences, but not true sciences. But again, dividing this knowledge of universals into that which knows causes, and into that which understands without a cause, he thinks that the one should be called science, but the other experience. And hence, to arts he sometimes attributes the name of science, but to experience never. For how (says he in the *Banquet*) can a thing which possesses no reason be science? All knowledge, therefore, which contains the reason and cause of the things known, is a certain science. Again, therefore, he divides this science which is endued with a power from the cause of knowing, by the peculiarity of its subjects, and he places one, conjectural of things divisible; but the other of such as subsist by themselves, and are ever knowable after the same manner. And according to this division he separates from science, medicine, and every faculty which is conversant with material concerns. But he calls mathematical knowledge, and whatever possesses a power of contemplating eternal objects, by the name of science. Lastly, dividing this science, which we distinguished from arts, he considers one part as void of supposition; but the other as flowing from supposition. And that the one which is void of supposition, has a power of knowing universals: that it rises to good, and the supreme cause of all; and that it considers good as the end of its elevation: but that the other, which previously fabricates for itself definite and determinate principles, from which it evinces things consequent to such principles, does not tend to the principle, but to the conclusion. And hence he asserts, that mathematical knowledge, because it makes use of supposition, falls short of that science which is without supposition, and is perfect. For there is one true science, by means of which we are disposed to know all the things *which are*, and from which also principles emerge to all sciences; to some, indeed, constituted more proximately, but to others more remotely. We must not say, therefore, that Plato expels mathematical knowledge from the number of the sciences, but that he asserts it to be the second

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cond from that one science, which possesses the supreme seat of all: nor must we affirm, that he accuses it as ignorant of its own principles, but that receiving these from the master science dialectic, and possessing them without any demonstration, it demonstrates from these its consequent propositions. For, indeed, he sometimes allows the soul, which is constituted from mathematical reasons, to be the principle of motion: and sometimes he affirms, that it receives its motion from genera which are subject to intelligence. And these variations accord among themselves. For to such things as are moved by another, the soul is a certain cause of motion, but it is not the cause of every motion. After the same manner, the mathematical science is indeed the second from the first of all sciences, and, with reference to it, imperfect: but it is, nevertheless, a science, not as being free from supposition, but as knowing the peculiar reasons resident in the soul, and as bringing the causes of conclusions, and containing the reason of such things as are subject to its knowledge. And thus much for the opinion of Plato respecting mathematics.

## C H A P. XI.

BUT let us now consider what are the things which may be required of a mathematician, and how any one may rightly judge concerning his distinguishing peculiarities. For \* Aristotle indeed, says, that he who is simply learned in all disciplines, is adapted to judge of all: but that he who is alone skilled in the mathematical sciences, can alone determine concerning the magnitude of reasons inherent in these: It is requisite, therefore, that we should previously assume the terms of judging, and that we should know, in the first place, in what things it is proper to demonstrate generally, and in what to regard the peculiarities of singulars. For many of the same properties reside in things differing in species, as two right angles in all triangles: but many have indeed the same predicament, yet differ in their individuals in a common species, as similitude in figures and numbers. But one demonstration is not to be sought for by the

\* In. I. De Partib. Animalium, et in primo Ethic. cap. iii.

mathem-

mathematician in these, for the principles of figures and numbers are not the same, but differ in their subject genus. And if the essential accident is one, the demonstration will also be one \*: for the possession of two right angles is the same in all triangles, and that general something to which this pertains is the same in all, I mean triangle, and a triangular reason. In the same manner, likewise, the possession of external angles to four right ones, not only pertains to triangles, but also to all right-lined figures; and the demonstration, so far as they are right-lined, agrees in all. For every reason brings with it, at the same time, a certain property and passion, of which all participate through that reason, whether triangular, or rectilinear, or universally figure. But the second limit by which a mathematician is to be judged, is, if he demonstrates according to his subject-matter, and renders necessary reasons, and such as cannot be confuted, but are at the same time neither probable, nor replenished with a similitude of truth. For, says Aristotle, it is just the same to require demonstrations from a rhetorician, and to assent to a mathematician disputing probably; since every one, endued with science and art, ought to render reasons adapted to the subjects of his investigation. In like manner also, Plato in the Timæus, requires credible reasons of the natural philosopher, as one who is employed in the resemblances of truth: but of him who discourses concerning intelligibles, and a stable essence, he demands reasons which can neither be confuted nor moved. For subjects every where cause a difference in sciences and arts, since, if some of them are immoveable, others are conversant with motion; and some are more simple, but others more composite; and some are intelligibles, but others sensibles. Hence we must not require the same certainty from every part of the mathematical science. For if one part, after a manner, borders upon sensibles, but another part is the knowledge of intelligible subjects, they cannot both be equally certain, but one must inherit a higher degree of evidence than the other. And hence it is, that we call arithmetic more certain than the science of harmony. Nor must we think it just that mathematics and other sciences should use the same demonstrations; for their subjects afford them no small variety. In

\* See more concerning this in the Dissertation.

the third place, we must affirm, that he who rightly judges mathematical reasons, must consider sameness and difference, what subsists by itself, and what is accidental, what proportion is, and every consideration of a similar kind. For almost all errors of this sort happen to those who think they demonstrate mathematically, when at the same time they by no means demonstrate, since they either demonstrate the same thing as if different in each species, or that which is different as if it were the same: or when they regard that which is accidental, as if it were an essential property; or that which subsists by itself, as if it were accidental. For instance, when they endeavour to demonstrate that the circumference of a circle is more beautiful than a right line, or an equilateral than an isosceles triangle. For the determination of these does not belong to the mathematician, but to the first philosopher alone. Lastly, in the fourth place, we must affirm, that since the mathematical science obtains a middle situation between intelligibles and sensibles, and exhibits in itself many images of divine concerns, and many exemplars of natural reasons, we may behold in it three kinds of demonstration \*, one approaching nearer to intellect, the second more accommodated to cogitation, and the third bordering on opinion. For it is requisite that demonstrations should differ according to the varieties of problems, and receive a division correspondent to the genera of beings, since the mathematical science is connected with all these, and adapts its reasons to the universality of things. And thus much for a discussion of the subject proposed.

\* Since number is prior to magnitude, the demonstrations of arithmetic must be more intellectual, but those of geometry more accommodated to the rational power. And when either arithmetic or geometry is applied to sensible concerns, the demonstrations, from the nature of the subjects, must participate of the obscurity of opinion. If this is the case, a true mathematician will value those parts of his science most, which participate most of evidence; and will consider them as degraded, when applied to the common purposes of life.

## C H A P. XII.

*What and how many the Species of the whole Mathematical Science are, according to the Opinion of the Pythagoreans.*

BUT after these considerations, it is requisite to determine concerning the parts of the mathematical science, what, and how many they are. For it is just, after speculating its whole and entire genus, to consider the differences of its more particular sciences, according to their species. The Pythagoreans \*, therefore, thought that the whole mathematical science should receive a fourfold distribution, attributing one of its parts to the how-many, but the other to the how-much ; and they assigned to each of these parts a twofold division. For they said, that discrete quantity, or the how-many, either subsists by itself, or must be considered with relation to some other; but that continued quantity, or the how-much, is either stable or in motion. Hence they affirmed, that arithmetic contemplates that discrete quantity which subsists by itself, but music that which is related to another ; and that geometry considers continued quantity so far as it is immoveable ; but spherics contemplates continued quantity as moving from itself, in consequence of its union with a self-motive nature. They affirmed besides, that these two sciences, discrete and continued quantity, did not consider either magnitude or multitude absolutely, but that alone which in each of these is definite from the participation of bound. For sciences alone speculate the definite, rejecting as vain the comprehension of infinite quantity. But when these wise men assigned this distribution, we must

\* This division of the mathematical science, according to the Pythagoreans, which is nearly coincident with that of Plato, is blamed by Dr. Barrow in his Mathematical Lectures, p. 15. as being confined within too narrow limits : and the reason he assigns for so partial a division, is, "because, in Plato's time, others were either not yet invented, or not sufficiently cultivated, or at least were not yet received into the number of the mathematical sciences." But I must beg leave to differ from this most illustrious mathematician in this affair ; and to assert that the reason of so confined a distribution (as it is conceived by the moderns) arose from the exalted conceptions these wise men entertained of the mathematical sciences, which they considered as so many preludes to the knowledge of divinity, when properly pursued ; but they reckoned them degraded and perverted, when they became mixed with sensible objects, and were applied to the common purposes of life.

not suppose they understood that discrete quantity which is found in sensible natures, nor that continued quantity which subsists about the fluctuating order of bodies. For, I think, the contemplation of these pertains to the natural and not to the mathematical science. But because the demiurgus of the universe, employed the union, division, and identity of general natures, together with difference, station, and motion, for the purpose of completing the essence of the soul, and composed it from these genera, as Timæus informs us, we must affirm, that cogitation, abiding according to its diversity, its division of reasons, and its multitude, and understanding itself to be both one and many, proposes indeed to itself, and produces numbers, together with an arithmetical knowledge of these: but it provides for itself music according to an union of its multitude, and a communication and junction with itself; and hence it is that arithmetic excels music in antiquity; since, according to the narration of Plato, the demiurgus first divided the soul, and afterwards collected it in harmonical proportions. Again, thought establishing its energy according to the stability which it contains, draws from its inmost retreats geometry, together with one essential figure, and the demiurgical principles of all figures \*: but, according to its inherent motion, it produces the spherical science. For it is moved also by circles, but abides perpetually the same from the causes of circles. Hence, likewise, geometry precedes spherics, in the same manner as station is prior to motion. But because cogitation itself produces these sciences, not by looking back upon its convolution of forms, endued with an infinite power, but upon the inclosure of *bound* according to its definite genera; hence they say, that the mathematical sciences take away infinite from multitude and magnitude, and are only conversant about finite quantity. Indeed, intellect has placed in cogitation all the principles both of multitude and magnitude. For since it wholly consists, with reference to itself, of similar parts, and is one and indivisible, and again divisible, educating the ornament of forms, it participates of *bound* and *infinite*, from intelligible essences themselves. But it understands, indeed, from its participation of bound, and gene-

\* That is, a right and circular line.

rates vital energies, and various reasons from the nature of infinite. The intellec[t]ions, therefore, of thought, constitute these sciences according to the *bound* which they contain, and not according to an infinity of life; since they bring with them an image of intellect, but not of life. Such then is the opinion of the Pythagoreans, and the division of the four mathematical sciences.

## C H A P. XIII.

*Another Division of the Mathematical Science, according to Geminus.*

**A**GAIN, some think (among whom is Geminus) that the mathematical science is to be divided in a different manner from the preceding. And they consider that one of its parts is conversant with intelligibles only, but the other with sensibles, upon which it borders; denominating as intelligibles whatever inspections the soul rouses into energy by herself, when separating herself from material forms. And of that which is conversant with intelligibles they establish two, by far the first and most principal parts, arithmetic and geometry: but of that which unfolds its office and employment in sensibles, they appoint six parts, mechanics, astrology, optics, geodæsia, canonics, and logistics, or the art of reckoning. But they do not think that the military art, or tactics, should be called any one part of mathematics, according to the opinion of some \*; but they consider it as using at one time the art of reckoning, as in the numbering of legions; but at another time geodæsia, as in dividing and measuring the spaces filled by a field of camps. As, say they, neither the art of writing, nor the art of healing, are any part of mathematics, though frequently both the historian and physician use mathematical theorems. This is the case with historians indeed, when relating the situation of climates, or collecting the magnitudes and dimensions of

\* I am afraid there are few in the present day, who do not consider tactics as one of the most principal parts of mathematics; and who would not fail to cite, in defence of their opinion, that great reformer of philosophy, as he is called, Lord Bacon, commanding pursuits which come home to men's businesses and bosoms. Indeed, if what is lowest in the true order of things, and best administers to the vilest part of human nature, is to have the preference, their opinion is right, and Lord Bacon is a philosopher!

cities,

cities, or their compass and circuit: but with physicians, when elucidating by ways of this kind, many things in their art. For Hippocrates himself shews the utility derived to medicine from astrology, and almost all who speak of opportune times and places. By the same reason he also, who accommodates his work to tactics, uses indeed mathematical theorems, yet is not on this account a mathematician, although he is sometimes willing that a numerous camp should exhibit a very small multitude, and forms his army according to a circular figure; but sometimes in a quadraangular, quinquangular, or some other multangular figure, when he desires it to appear numerous. But since these are the species of the whole mathematical science, geometry is again divided into the contemplation of planes, and the dimension of solids, which is called stereometry. For there is not any peculiar treatise about points and lines, because no figure can be produced from these without planes or solids. For geometry treats of nothing else in every one of its parts, than that it may constitute either planes or solids: or that when constituted, it may compare and divide them among themselves. In like manner, arithmetic is distributed into the contemplation of linear, plane, and solid numbers. For it considers the species of numbers separate from sensible connections, proceeding from unity, and the origin of plane numbers; I mean of the similar, dissimilar, and solid, even to the third increase. But geodæsia, and the art of reckoning, are divided similarly to arithmetic and geometry, as they do not discourse concerning intelligible numbers or figures, but of such as are sensible alone. For neither is it the office of geodesia to measure the cylinder or the cone, but material masses as if they were cones, and wells as if they were cylinders. Neither does it accomplish this purpose by intelligible right lines, but by such as are sensible, sometimes indeed by a more certain means, as by the solar rays: but at other times by grosser ones, as by a line and perpendicular. In like manner, the reckoner does not survey the passions of numbers by themselves, but as they are resident in sensible objects. From whence he also imposes a name upon these derived from the things which he reckons, calling them *μηλίται*, & *φιαλίται*. Besides this, he does not admit of any least, like the arithmetician, who receives that minimum, as a genus of relation.

relation. For some one man is considered by him as the measure of the whole multitude of men, as unity also is the common measure of all numbers. Again, optics and canonics are produced from geometry and arithmetic. And optics uses the visual rays which are constituted by the rays of the eyes, as lines and angles. But it is divided into that which is properly called optics (because it renders the cause of those appearances, which are accustomed to present themselves to us different from their reality, on account of the different situations and distances of visible objects, as the coincidence of parallel lines, or the appearance of quadrangles as if they were circles); and into universal catoptrics, which is conversant about various and manifold refractions, and is connected with imaginative or conjectural knowledge: as also into that which is called sciography \*, or the delineation of shadows, which shews how appearances in images may seem neither inelegant nor deformed, on account of the distances and altitudes of the things designed. But canonics (music) or the regular art, considers the apparent reasons of harmonies, finding out the sections of rules, every where using the assistance of sense, and, as Plato says, seeming to prefer the testimony of the ears to intellect itself. But to the parts we have hitherto enumerated, mechanics must be added, as it is a certain part of the whole science, and of the knowledge of sensible objects, and of things united with matter. But under this exists the art effective of instruments, which is called (*οργανωποιητικη*) I mean of those instruments proper for the purposes of war: such, indeed, as Archimedes is reported to have constructed, resisting the besiegers of sea and land; and that which is effective of miracles, and which is called (*θαυματοποιητικη*.) One part of this constructs with the greatest artifice pneumatic engines, such as Ctesibius and Heron fabricated: but another operates with weights, the motion of which is reckoned to be the cause of inequilibrium; but their station of equilibrity, as Timæus also has determined: and again, another part imitates animate foldings and motions by strings and ropes. Again, under mechanics is placed the knowledge of equilibriums, and of such instruments as are called centroponderants: also (*σφαιροποια*) or the

\* By this is to be understood the art now called Perspective: from whence it is evident that this art was not unknown to the ancients, though it is questioned by the moderns.

art effective of spheres, imitating the celestial revolutions, such as Archimedes fabricated; and lastly, every thing endued with a power of moving matter. But the last of all is astrology, which treats of the mundane motions, of the magnitudes of the celestial bodies, their figures and illuminations, their distances from the earth, and every thing of this kind; assuming many things indeed to itself from sense; but communicating much with the natural speculation. One part of this is gnomonics, which is exercised in settling the dimension of hororary gnomons: but the other is metheoroscopies, which finds out the differences of elevations, and the distances of the stars, and also teaches many other and various astrological theorems. The third part is dioptrics, which ascertains by dioptric instruments of this kind the distances of the sun and moon, and of the five other stars. And such is the account of the parts of the mathematical science, delivered by the ancients, and transmitted to our memory by the informing hand of time.

## C H A P. XIV.

*How Dialectic is the Top of the Mathematical Sciences, and what their Conjunction is, according to Plato.*

LET us again consider after what manner Plato, in his Republic, calls dialectic the top of the mathematical disciplines; and what their conjunction is, according to the tradition of the author of the Epinomis \*. And in order to this we must assert, that as intellect is superior to cogitation, supplying it with supernal principles, and from itself giving perfection to cogitation; in the same manner dialectic also, being the purest part of philosophy, excels in simplicity the mathematical disciplines, to which it is proximate, and with which it is conjoined. Indeed it embraces the complete circle of

\* From hence it appears, that it is doubtful whether Plato is the author of the dialogue called Epinomis; and I think it may with great propriety be questioned. For though it bears evident marks of high antiquity, and is replete with genuine wisdom, it does not seem to be perfectly after Plato's manner; nor to contain that great depth of thought with which the writings of this philosopher abound. Fabricius (in his Bibliotheca Græca, lib. i. p. 27.) wonders that Suidas should ascribe this work to a philosopher who distributed Plato's laws into twelve books, because it was an usual opinion; from whence it seems, that accurate critic had not attended to the present passage.

these.

these sciences, to which it elevates from itself various energies, endued with a power of causing perfection, judgment, and intelligence. And these energies consist in resolving, dividing, defining, and demonstrating; by which mathematics itself, receiving assistance and perfection, invents some things by resolution, but others by composition: and some things it explains by division, others by definition: but collects other subjects of its investigation by demonstration; accommodating, indeed, these ways to its subjects, but using each of them for the purpose of beholding its middle enquiries. From whence indeed, both the resolutions, definitions, divisions, and demonstrations which it contains, are peculiar, and adapted to its nature, and revolve according to the mode of mathematical cognition. Not undeservedly, therefore, is dialectic the vertex as it were, and summit of mathematics. Since it perfects all which mathematics contains of intelligence; renders its certainty free from reprehension, preserves the stability of its immoveable essence, and refers what it contains destitute of matter and pure to the simplicity of intellect, and a nature separated from material connections. Besides, it distinguishes the first principles of these sciences, by definitions: exhibits the separations of genera and forms contained under the genera themselves: and besides this, teaches the compositions, which, from principles, produce things consequent to principles: and the resolutions which rise and mount up to things first, and to principles themselves. But with respect to what remains, proportion itself is not to be considered (as Eratosthenes thought it was) as the conjunction of the mathematical disciplines. Since proportion is said to be, and indeed is one of those things common to the mathematics. But in short, many other things besides proportion regard all the mathematical disciplines, which are essentially inherent in the common nature of the mathematics. But as it appears to me, we should say, that there is one proximate conjunction of these, and of the whole mathematical science, which especially embraces in itself, in a more simple manner, the principles of all sciences; which considers their community and difference; teaches whatever is found in these the same; together with what things are inherent in a many, and what in a few. So that

that to those who aptly learn there is a reversion from many other sciences to this alone\*. But dialectic is a conjunction of the mathematical disciplines superior to the preceding; which Plato, as I have already observed, calls in his Republic their vertex: for, indeed, it perfects the whole of mathematics, brings it back to intellect by its powers, shews it to be a true science, and causes it to be certain and obnoxious to no reproof. But, intellect obtains the third order between these conjunctions, which comprehends in itself uniformly all the dialectic powers, contracts their variety by its simplicity, their partition by its indivisible knowledge, and their multitude by its occult union. Hence, intellect itself congregates indeed the involutions and deviations of the dialectic paths, into an intelligible essence, but it collects supernally all the progression of mathematical discourses: and it is the best end both of the elevating power of the soul, and of the energy consisting in cognition. And such are the sentiments declared by me on the present enquiry.

## C H A P. XV.

*From whence the Name Mathematics originated.*

A GAIN, from whence shall we say this name of mathematics, and mathematical disciplines, was assigned by the ancients, and what apt reason can we render of its position? Indeed, it appears to me, that such an appellation of a science which respects cogitative reasons, was not, like most names, invented by indifferent persons, but (as the truth of the case is, and according to report) by the Pythagoreans alone. And this, when they perceived, that whatever is called mathesis or discipline, is nothing more than reminiscence; which does not approach the soul extrinsically, like the images which rising from sensible objects are formed in the phantasy: nor is it adventitious and foreign, like the knowledge consisting in opinion, but it

\* This proximate conjunction of the mathematical sciences, which Proclus considers as subordinate to dialectic, seems to differ from that vertex of science in this, that the former merely embraces the principles of all science, but the latter comprehends the universal genera of being, and speculates the principle of all.

is excited, indeed, from apparent objects, and is perfected within, by thought intimately converted to itself. And when they likewise perceived that though reminiscence might be shewn from many particulars, yet it was evinced in a more eminent manner (as Plato also says \*) from the mathematical disciplines. For if any one, says he, is led into the descriptions, he will there easily prove that discipline is reminiscence. From whence Socrates also, in the Meno, shews by this method of arguing, that learning is nothing else than the soul's recollection of her inherent reasons. And this, because that which recollects, is alone the cogitative part of the soul; but this perfects her essence in the reasons of the mathematical disciplines, the sciences of which she previously received into herself, though she does not always energize on their fair variety. Indeed, she contains them all essentially and occultly; but she produces each of them when she is freed from the impediments originating from sense. For sense unites her with divisible objects: the phantasy fills her with forming motions, and appetite bends her to an indulgent and luxurious life. But every thing divisible is an obstacle to our self-conversion. And whatever invests with form, disturbs and offends that knowledge which is destitute of form. And whatever is obnoxious to perturbations is an impediment to that energy which is unimpaired by affections. When, therefore, we have removed all these from the cogitative power, then shall we be able to understand by thought itself, the reasons which thought contains: then shall we become scientific in energy; and unfold our essential knowledge. But whilst we are captive and bound, and winking with the eye of the soul, we cannot by any means attain to a perfection convenient to our nature. Such then is mathesis or discipline: a reminiscence of the eternal reasons contained in the soul. And the mathematical or disciplinative science, is on this account particularly denominated that knowledge which especially confers to our reminiscence of these essential reasons. Hence, the business and office of this science †, is apparent from its name.

\* In the Meno.

† This is certainly the true or philosophical employment of the mathematical science; for by this means we shall be enabled to ascend from sense to intellect, and rekindle in the soul that divine light of truth, which, previous to such an energy, was buried in the obscurity of a corporeal

name. For its duty is to move the inherent knowledge of the soul; to awaken its intelligence; to purify its cogitation; to call forth its essential forms from their dormant retreats; to remove that oblivion and ignorance, which are congenial with our birth; and to dissolve the bonds arising from our union with an irrational nature. It plainly leads us to a similitude of that divinity who presides over this science, who manifests intellectual gifts, and fills the universe with divine reasons; who elevates souls to intellect, wakens them as from a profound sleep, converts them by enquiry to themselves; and by a certain obstetric art, and invention of pure intellect, brings them to a blessed life. To whom indeed, dedicating the present work, we here conclude our contemplation of the mathematical science.

corporeal nature. But by a contrary process, I mean, by applying mathematical speculations, to experimental purposes, we shall blind the liberal eye of the soul, and leave nothing in its stead but the darkness of corporeal vision, and the phantoms of a degraded imagination.

# B O O K II.

## C H A P. I.

*What Part Geometry is of Mathematics, and what the Matter is of which it consists.*

**I**N the preceding discourses we have considered those common properties which respect the whole of the mathematical science; and this we have done agreeable to the doctrine of Plato; at the same time collecting such particulars as pertain to our present design. But consequent to this it is requisite that we should discourse on geometry itself, and on the proposed institution of the elements, for the sake of which we have undertaken the whole of the present work. That geometry then, is a part of the whole of mathematics, and that it obtains the second place after arithmetic, since it is perfected and bounded by this, (for whatever in geometry may be expressed and known, is determined by arithmetical reasons) has been asserted by the ancients, and requires no long discussion in the present enquiry. But we also may be able to relate our opinion on this particular, if we consider what place, and what essence its subject matter \* is allotted

\* The design of the present chapter is to prove that the figures which are the subjects of geometric speculation, do not subsist in external and sensible matter, but in the receptacle of imagination, or the matter of the phantasy. And this our philosopher proves with his usual elegance, subtlety, and depth. Indeed, it must be evident to every attentive observer, that sensible figures fall far short of that accuracy and perfection which are required in geometrical definitions: for there is no sensible circle perfectly round, since the point from which it is described is not without parts; and, as Vossius well observes, (de Mathem. p. 4.) there is not any sphere in the nature of things, that only touches in a point, for with some part of its superficies it always touches the subjected plane in a line, as Aristotle shews Protagoras to have objected against the geometers. Nor must we say, with that great mathematician Dr. Barrow, in his Mathematical Lectures, page 76, "that all imaginable geometrical figures, are really inherent in every particle of matter, in the utmost perfection, though not apparent to sense; just as the effigies of Cæsar lies hid in the unhewn marble, and is no new thing made by the statuary, but only is discovered and brought to sight by his workmanship, i. e. by removing the parts of matter by which it is overshadowed and involved. Which made Michael Angelus, the most famous carver, say, *that sculpture was nothing but a purgation from things superfluous. For take all that is superfluous,* (says he) *from the wood or stone, and the rest will be the figure you intend.* So, if the hand of an angel (at least the power of God) should think fit to polish any particle

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allotted among the universality of things. For from a proper survey of this, the power of the science which knows this subject matter, the utility arising from it, and the good acquired by its learners, will immediately appear. Indeed, some one may doubt in what genus of things he ought to place geometrical matter, so as not to deviate from the truth it contains. For if the figures concerning which geometry discourses, exist in sensible natures, and cannot be separated from the dark receptacle of matter; how can we assert that geometry frees us from sensible objects, that it brings us to an incorporeal essence, that it accustoms us to an inspection of intelligibles, and prepares us for intellectual energy? Where shall we ever survey among sensible objects a point without parts, or a line destitute of breadth, or a superficies without profundity, or the equality of lines from the centre to the circumference; or the multangles, and all the figures of many bases, concerning which geometry informs us? Lastly, after what manner can the reasons of such a science remain free from all possible confutation; since, indeed, sensible forms and figures are susceptible of the more and the less, are all moveable and mutable, and are full of material variety; among which equality subsists mixt and confused with its contrary inequality, and into which things without parts have proceeded into partition, and interval, darkened with the shades of matter, and lost in its infinite folds? But if the subjects of geometry are removed from matter, are pure forms, and are separated from

particle of matter, without vacuity, a spherical superficies would appear to the eyes, of a figure exactly round; not as created anew, but as unveiled and laid open from the disguises and covers of its circumjacent matter." For this would be giving a perfection to sensible matter, which it is naturally incapable of receiving: since external body is essentially full of pores and irregularities, which must eternally prevent its receiving the accuracy of geometrical body, though polished by the hand of an angel. Besides, what polishing would ever produce a point without parts, and a line without breadth? For though body may be reduced to the greatest exility, it will not by this means ever pass into an incorporeal nature, and desert its triple dimension. Since external matter, therefore, is by no means the receptacle of geometrical figures, they must necessarily reside in the catoptric matter of the phantasy, where they subsist with an accuracy sufficient for the energies of this science. It is true, indeed, that even in the purer matter of imagination, the point does not appear perfectly impenetrable, nor the line without latitude: but then the magnitude of the point, and the breadth of the line is indefinite, and they are, at the same time, unattended with the qualities of body, and exhibit to the eye of thought, magnitude alone. Hence, the figures in the phantasy, are the proper recipients of that universal, which is the object of geometrical speculation, and represent, as in a mirror, the participated subsistence of those vital and immaterial forms which essentially reside in the soul.

sensible

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sensible objects: they will be all of them, without doubt, void of parts, incorporeal, and destitute of magnitude. For extension, tumor, <sup>mass</sup> and interval, approach to forms, on account of the material receptacle in which they are involved, and which receives things destitute of parts, distributed into parts; things void of dimension, extended into dimension; and immoveable natures accompanied with motion. How then, if this is the case, shall we cut a right line, triangle, and circle? How can we speak of the diversities of angles, and the increments and decrements of triangular and quadrangular figures? Or how exhibit the contacts of circles or right lines? For all these evince that the geometric matter consists of parts, and does not reside among indivisible reasons. Such then are the doubts concerning the matter of geometry, to which we may add, that Plato considers the forms of geometry as placed in cogitation; and grants, that we advance from sensibles to forms of this kind, and that we rise from sensibles to intellect, though (as we have previously observed) the reasons subsisting in cogitation are indivisible; are separated by no interval, and subsist according to the peculiarity of the soul. But if reasons are to be rendered agreeable to things themselves, and to the doctrine of Plato, the following division must be adopted. \* Every universal,

and

\* This division is elegantly explained by Ammonius, (in Porphy. p. 12.) as follows, "Conceive a seal-ring, which has the image of some particular person, for instance, of Achilles, engraved in its seal, and let there be many portions of wax, which are impressed by the ring. Afterwards conceive that some one approaches, and perceives all the portions of wax, stamped with the impression of this one ring, and keeps the impression of the ring in his mind: the seal engraved in the ring, represents the universal, prior to the many: the impression in the portions of wax, the universal in the many: but that which remains in the intelligence of the beholder, may be called the universal, after and posterior to the many. The same must we conceive in genera and species. For that best and most excellent artificer of the world, possesses within himself the forms and exemplars of all things: so that in the fabrication of man, he looks back upon the form of man resident in his essence, and fashions all the rest according to its exemplar. But if any one should oppose this doctrine, and assert that the forms of things do not reside with their artificer, let him attend to the following arguments. The artificer either knows, or is ignorant of that which he produces: but he who is ignorant will never produce any thing. For who will attempt to do that, which he is ignorant how to perform? since he cannot act from an irrational power like nature, whose operations are not attended with animation. But if he produces any thing by a certain reason, he must possess a knowledge of every thing which he produces. If, therefore, it is not impious to assert, that the operations of the Deity, like those of men, are attended with knowledge, it is evident that the forms of things must reside in his essence: but forms are in the demiurgus, like the seal in the ring; and these forms are said to be prior to the many, and separated from matter. But the species man, is contained

and one thing containing many, is either naturally disposed to be thought of in particulars, or to appear such, because it possesses its existence in these; is inseparable from them; is disposed and distributed in them; and together with these is either moved, or firmly and immovably abides. Or it is adapted to subsist prior to many, and to possess a power of generating multitude, affording to many things images from itself, being furnished with a nature destitute of parts, from the essences which it participates, and raising various participations to secondary natures: or it is disposed to be formed by thought, from the many, to possess a generating existence, and to reside in the last place in the many. For, according to these three modes of subsistence, we shall find, I think, that some subsist before the many, others in the many, and others from the relation and predication which they possess to these. But, that I may absolve all in one word, universal forms being threefold, we shall consider the differences of that form which many participate, which exists in many, and fills particular natures according to its subject matter. Besides this, establishing a twofold order of participants, one subsisting in sensible objects, but the other in the phantasy, (since matter is twofold; one indeed, of things united with sense, but the other of such as fall under the inspection of phantasy, as Aristotle asserts, in a certain place \*) we must allow that the universal, which is distributed in the many, is likewise twofold. The one, indeed, sensible, as being that which sensible objects participate; but the other imaginative, as that which subsists in the *many* of the phantasy. For the phantasy, on account of its forming motion, and because it subsists with, and in body, always receives impressions which are both divided and figured. So

contained in each particular man, like the impression of the seal in the wax, and is said to subsist in the many, without a separation from matter. And when we behold particular men, and perceive the same form and effigy in each, that form seated in our soul, is said to be after the many, and to have a posterior generation: just as we observed in him, who beheld many seals impressed in the wax from one and the same ring. And this one, posterior to the many, may be separated from body, when it is conceived as not inherent in body, but in the soul: but is incapable of a real separation from its subject." We must here, however, observe, that when Ammonius speaks of the knowledge of the Deity, it must be conceived as far superior to ours. For he possesses a nature more true than all essence, and a perception clearer than all knowledge. And as he produced all things by his unity, so by an ineffable unity of apprehension, he knows the universality of things.

\* In lib. vii. Metaphys. 35 & 39.

that

Piculus does  
not write here

that whatever is known by it, is allotted a correspondent existence: on which account, Aristotle \* does not hesitate to call it passive intellect. But if it is intellect, why is it not impassive, and destitute of matter? And if it operates with passion, how can it with propriety be called intellect? For impassivity, indeed, properly belongs to intellect and an intelligent nature: but passivity is very remote from such an essence: But (unless I am deceived) Aristotle being willing to explain its middle nature between cognitions the most primary, and such as are the last, calls it at the same time intellect, because similar to primary cognitions, and passive from that alliance which it possesses with such as are posterior. For first cognitions are indeed destitute of figures and forms; comprehending in themselves, intelligible natures, energizing about themselves, united with the objects of knowledge, and free from all extrinsical impression and passion. But last cognitions exercise themselves through the medium of instruments, are rather passions than energies, admit extrinsical knowledge, and move themselves together with their various subjects. For such (says Plato) are the sensations which arise from violent passions. But the phantasy, obtaining a middle centre in the order of cognitions, is excited, indeed, by itself, and produces that which falls under cogitation: but because it is not separate from body, it deduces into partition, interval, and figure, the objects of its knowledge, from the indivisibility of an intellectual life. Hence, whatever it knows, is a certain impression and form of intelligence. For it understands the circle, together with its interval, void, indeed, of external matter, but possessing intelligible matter. On this account, like sensible matter, it does not contain one circle only: for we behold in its receptacle, distance, together with the more and the less, and a multitude of circles and triangles. If then an universal nature is distributed in sensible circles, since each of these completes a circular figure, and they are all mutually similar, subsisting in one reason, but differing in magnitudes or subjects: in like manner, there is a common something in the circles, which subsist in the receptacle of the phantasy, of which all its circles participate, and according to which they all

\* In lib. iii. de Anima, tex. 20.

possess

possess the same form; but in the phantasy they possess but one difference only, that of magnitude. For when you imagine many circles about the same centre, they all of them exist in one immaterial subject and life, which is inseparable from a simple body, which, by the possession of interval, exceeds an essence destitute of parts; but they differ in magnitude and parvitude, and because they are contained and contain. Hence, that universal is two-fold, which is understood as subsisting in the many: one, indeed, in sensible forms; but the other in such as are imaginative. And the reason of a circular and triangular figure, and of figure universal, is twofold. The one subsisting in intelligible, but the other in sensible matter. But prior to these is the reason which resides in cogitation, and that which is seated in nature herself. The former being the author of imaginative circles, and of the one form which they contain; but the other, of such as are sensible. For there are circles existing in the heavens, and universally those produced by nature, the reason of which does not fall under a cogitative distribution. For in incorporeal causes, things possessing interval, are distinguished by no intervals: such as are endued with parts, subsist without parts: and magnitudes without the diffusion of magnitude, as on the contrary in corporeal causes, things without parts subsist divisibly, and such as are void of magnitude with the extension of magnitude. Hence, the circle resident in cogitation, is one, simple and free from interval: and magnitude itself is there destitute of magnitude; and figure expressed by no figure: for such are reasons separate from matter. But the circle subsisting in the phantasy, is divisible, figured, endued with interval, not one only, but one and many, nor form alone, but distributed with form. And the circle, in sensible objects, is composite, distant with magnitude, diminished by a certain reason, full of ineptitude, and very remote from the purity of immaterial natures. We must therefore say, that geometry, when it asserts any thing of circle and diameter, and of the passions and affections which regard the circle; as of contacts, divisions, and the like: neither teaches nor discourses concerning sensible forms, (since it endeavours to separate us from these), nor yet concerning the form resident in cogitation, (for here the circle is one, but geometry discourses of many, proposing

something of each, and contemplating the same of all: and here it is indivisible, but the geometric circle is divisible); but we must confess, that it considers universal itself; yet as distributed in imaginative circles. And that it beholds, indeed, one circle \*: and by the medium of another, contemplates the circle resident in the depths of cogitation: but by another, different from the preceding, fabricates the fair variety of its demonstrations. For since cogitation is endued with reasons, but cannot behold them contractedly, separated from material figure; it distributes and removes them, and draws them forth seated in the shadowy bosom of the phantasy, and placed in the vestibules of primary forms; revolving in it, or together with it, the knowledge of these: loving, indeed, a separation from sensibles, but finding imaginative matter proper for the reception of its universal forms. Hence, its intellect does not subsist without the phantasy. And the compositions and divisions of figures are imaginative; and their knowledge is the way which leads us to that essence pursued by cogitation: but cogitation itself, does not yet arrive at this stable essence, while it looks abroad to externals, contemplates its internal forms according to these, uses the impressions of reasons, and is moved from itself to external and material forms. But if it should ever be able to return to itself, when it has contracted intervals and impressions, and beholds multitude without impression, and subsisting uniformly; then it will excellently perceive geometrical reasons, void of division and interval, essential and vital, of which there is a copious variety. And this energy will be the best end of the geometric study; and truly the employment of a Mercurial gift, bringing it back as from a certain Calypso, and her detaining charms, to a more intellectual knowledge; and freeing it from those *forming* apprehensions with which the mirror of the phantasy is replete. Indeed, it is requisite that a true geometrician should be employed in this meditation, and should establish, as his proper end, the excitation and transition from the phantasy to cogitation alone; and that he should

\* That is, geometry first speculates the circle delineated on paper, or in the dust: but by the medium of the circular figure in the phantasy, contemplates the circle resident in cogitation; and by that universal, or circular reason, participated in the circle of the phantasy, frames its demonstrations.

accomplish this by separating himself from intervals, and the passive intellect to that energy which cogitation contains. For by this means he will perceive all things without an interval, the circle and diameter without a part, the polygons in the circle, all in all, and yet every one separate and a-part. Since, on this account, we exhibit also in the phantasy, both circles inscribed in polygons, and polygons in cireles ; imitating the alternate exhibition of reasons destitute of parts. Hence, therefore, we describe the constitutions, the origin, divisions, positions, and applications of figures: because we use the phantasy, and distances of this kind proceeding from its material nature; since form itself is immoveable, without generation, indivisible, and free from every subject. But whatever form contains occultly, and in an indistinct manner, is produced into the phantasy subsisting with intervals, divisibly and expanded. And that which, indeed, produces the forms of geometric speculation, is cogitation: but that from whence they are produced, is the form resident in cogitation: and that in which the produced figure resides is what is called the passive intellect. Which folds itself about the imparibility of true intellect, separates from itself the power of pure intelligence free from interval; conforms itself according to all formless species, and becomes perfectly every thing from which cogitation itself, and our indivisible reason consists. And thus much concerning the geometric matter, as we are not ignorant of whatever Porphyry the Philosopher has observed in his miscellanies, and whatever many of the Platonists describe. But we think that the present discussions are more agreeable to geometric dissertations, and to Plato himself, who subjects to geometry the objects of cogitation: For these mutually agree among themselves; because the causes, indeed, of geometrical forms, by which cogitation produces demonstrations, pre-exist in demonstration itself: but the particular figures which are divided and compounded, are situated in the receptacle of the phantasy.

## C H A P. II.

*What kind of Science Geometry is.*

**B**UT let us now speak of that science which possesses a power of contemplating the universal forms participated by imaginative matter. Geometry, therefore, is endued with the knowledge of magnitudes and figures, and of the terms and reasons subsisting in these; together with the passions, various positions and motions which are contingent about these. For it proceeds, indeed, from an impenetrable point, but descends even to solids, and finds out their multiform diversities. And again, runs back from things more composite, to things more simple, and to the principles of these: since it uses compositions and resolutions, always beginning from suppositions, and assuming its principles from a previous science; but employing all the dialectic ways. In principles, by the divisions of forms from their genera, and by defining its orations. But in things posterior to principles, by demonstrations and resolutions. As likewise, it exhibits things more various, proceeding from such as are more simple, and returning to them again. Besides this, it separately discourses of its subjects; separately of its axioms; from which it rises to demonstrations; and separately of essential accidents, which it shews likewise are resident in its subjects. For every science has, indeed, a genus, about which it is conversant, and whose passions it proposes to consider: and besides this, principles, which it uses in demonstrations; and essential accidents. Axioms, indeed, are common to all sciences (though each employs them in its peculiar subject matter), but genus and essential accident vary according to the sciential variety. The subjects of geometry are therefore, indeed, triangles, quadrangles, circles, and universally figures and magnitudes, and the boundaries of these. But its essential accidents are divisions, ratios, contacts, equalities, applications, excesses, defects, and the like. But its petitions and axioms, by which it demonstrates every particular are, this, to draw a right line from any point to any point; and that, if from equals you take away equals, the remainders will be

be equal ; together with the petitions and axioms consequent to these. Hence, not every problem nor thing sought is geometrical, but such only as flow from geometric principles. And he who is reproved and convicted from these, is convinced as a geometrician. But whoever is convinced from principles different from these, is not a geometrician, but is foreign from the geometric contemplation. But the objects of the non-geometric investigation, are of two kinds. For the thing sought for, is either from entirely different principles, as we say that a musical enquiry is foreign from geometry, because it emanates from other suppositions, and not from the principles of geometry : or it is such as uses, indeed, geometrical principles, but at the same time perversely, as if any one should say, that parallels coincide. And on this account, geometry also exhibits to us instruments of judging, by which we may know what things are consequent to its principles, and what those are which fall from the truth of its principles : for some things attend geometrical, but others arithmetical principles. And why should we speak of others, since they are far distant from these ? For one science is more certain than another (as Aristotle says \*) that, indeed, which emanates from more simple suppositions, than that which uses more various principles ; and that which tells the *why*, than that which knows only the simple existence of a thing ; and that which is conversant about intelligibles, than that which touches and is employed about sensibles. And according to these definitions of certainty, arithmetic is, indeed, more certain than geometry, since its principles excel by their simplicity. For unity is void of position, with which a point is endued. And a point, indeed, when it receives position, is the principle of geometry : but unity, of arithmetic. But geometry is more certain than spherics; and arithmetic, than music. For these render universally the causes of those theorems, which are contained under them. Again, geometry is more certain than mechanics, optics, and catoptrics. Because these discourse only on sensible objects. The principles, therefore, of geometry and arithmetic, differ, indeed, from the principles of other sciences ; but the hypotheses of these two,

\* In his first Analytics, t. 42. See the Dissertation to this work.

alternately

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alternately differ and agree according to the difference we have already described. Hence, also, with respect to the theorems which are demonstrated in these sciences, some are, indeed, common to them, but others peculiar. For the theorem which says, *every proportion may be expressed*, alone belongs to arithmetic; but by no means to geometry; since this last science contains things which cannot be expressed \*. That theorem also, which affirms, that *the gnomons of quadrangles are terminated according to the least* †, is the property of arithmetic: for in geometry, a minimum cannot be given. But those things are peculiar to geometry, which are conversant about positions; for numbers have no position: which respect contacts; for contact is found in continued quantities: and which are conversant about ineffable proportions; for where division proceeds to infinity, there also that which is ineffable is found ‡. But things common to both these sciences, are such as respect divisions, which Euclid treats of in the second book; except that proposition which divides a right line into extreme and mean proportion §. Again, of these common theorems, some, indeed, are transferred from geometry into arithmetic; but others, on the contrary, from arithmetic into geometry: and others similarly accord with both, which are derived into them from the whole mathematical science. For the permutation, indeed,

• Such as the proportion of the diagonal of a square to its side; and that of the diameter of a circle, to the periphery.

† The gnomons, from which square numbers are produced, are odd numbers in a natural series from unity, i. e. 1, 3, 5, 7, 9, 11, &c. for these, added to each other continually, produce square numbers *ad infinitum*. But these gnomons continually decrease from the highest, and are at length terminated by indivisible unity.

‡ This doctrine of ineffable quantities, or such whose proportion cannot be expressed, is largely and accurately discussed by Euclid, in the tenth book of his Elements: but its study is neglected by modern mathematicians, because it is of no use, that is, because it contributes to nothing mechanical.

§ This proposition is the 11th of the second book: at least, the method of dividing a line into extreme and mean proportion, is immediately deduced from it; which is done by Euclid, in the 30th, of the sixth book. Thus, Euclid shews (11. 2.) how to divide the line (A      G      B) A B, so that the rectangle under the whole A B, and the segment G B, may be equal to the square made from A G: for when this is done, it follows, that as A B is to A G, so is A G to G B; as is well known. But this proposition, as Dr. Barrow observes, cannot be explained by numbers; because there is not any number which can be so divided, that the product from the whole into one part, may be equal to the square from the other part.

conversions, compositions, and divisions of ratios are, after this manner, common to both. But such things as are commensurable, arithmetic first beholds; but afterwards geometry, imitating arithmetic. From whence, also, it determines such things to be commensurables of this kind, which have the same mutual ratio to one another, as number to number; because commensurability principally subsists in numbers. For where number is, there also that which is commensurable is found; and where commensurable is, there also number. Lastly, geometry first inspects triangles and quadrangles: but, arithmetic, receiving these from geometry, considers them according to proportion. For in numbers, figures reside in a causal manner. Being excited, therefore, from effects, we pass to their causes, which are contained in numbers. And at one time, we indifferently behold the same accidents, as when every polygon is resolved by us into triangles \*: but, at another time, we are content with what is nearest to the truth, as when we find in geometry one quadrangle the double of another, but not finding this in numbers, we say that one square is double of another, except by a deficiency of unity. As for in-

\* All polygonous figures, may, it is well known, be resolved into triangles; and this is no less true of polygonous numbers, as the following observations evince. All number originates from indivisible unity, which corresponds to a point: and it is either linear, corresponding to a line; or superficial, which corresponds to a superficies; or solid, which imitates a geometrical solid. After unity, therefore, the first of linear numbers is the duad; just as every finite line is allotted two extremities. The triad is the first of superficial numbers; as the triangle of geometrical figures. And the tetrad, is the first of solids; because a triangular pyramid, is the first among solid numbers, as well as among solid figures. As, therefore, the monad is assimilated to the point, so the duad to the line, the triad to the superficies, and the tetrad to the solid. Now, of superficial numbers, some are triangles, others squares, others pentagons, hexagons, heptagons, &c. Triangular numbers are generated from the continual addition of numbers in a natural series, beginning from unity. Thus, if the numbers 1, 2, 3, 4, 5, &c. be added to each other continually, they will produce the triangular numbers 1, 3, 6, 10, 15, &c. and if every triangular number be added to its preceding number, it will produce a square number. Thus 3 added to 1 makes 4; 6 added to 3 is equal to 9; 10 added to 6 is equal to 16; and so of the rest. Pentagons, are produced from the junction of triangular and square numbers, as follows. Let there be a series of triangular numbers 1, 3, 6, 10, 15, &c.

And of squares 1, 4, 9, 16, 25, &c.

Then the second square number, added to the first triangle, will produce the first pentagon from unity, i. e. 5. The third square added to the second triangle, will produce the second pentagon, i. e. 12; and so of the rest, by a similar addition. In like manner, the second pentagon, added to the first triangle, will form the first hexagon from unity; the third pentagon and the second triangle, will form the second hexagon, &c. And, by a similar proceeding, all the other polygons may be obtained.

stance,

stance, the square from 7, is double the square from 5, wanting one. But we have produced our discussion to this length, for the purpose of evincing the communion and difference in the principles of these two sciences. Since it belongs to a geometrician to survey from what common principles common theorems are divided; and from what principles such as are peculiar proceed; and thus to distinguish between the geometrical, and non-geometrical, referring each of them to different sciences.

## C H A P. III.

*From whence the whole of Geometry originated, how far it proceeds, and in what its Utility consists.*

BUT, beginning still higher, let us contemplate the whole of geometry, from whence it originated, and how far it proceeds in its energies: for thus we shall properly perceive the ornament which it contains. Indeed, it is necessary to understand that it is extended through the universality of things: that it accommodates its animadversions \* to all beings; and contains in itself the forms of all things: that, according to its supreme part, and which is endued with the highest power of intelligence, it surveys true beings; and teaches by images the properties of divine ornaments, and the powers of intellectual forms: for it contains the reasons of these also in its peculiar contemplations. And it exhibits what figures are convenient to the god, to primary essences, and to the natures of souls. But, according to its middle cognitions, it evolves cogitative reasons; explains and beholds the variety which they contain; exhibits their existence, and inherent passions; as also, their communities and di-

\* Intellections are universally correspondent to their objects, and participate of evidence or the contrary, in proportion as their subjects are lucid or obscure. Hence, Porphyry, in his sentences, justly observes, that "we do not understand in a similar manner with all the powers of the soul, but according to the particular essence of each. For with the intellect we understand intellectually; and with the soul, rationally: our knowledge of plants is according to a seminal conception; our understanding of bodies is imaginative; and our intellection of the divinely solitary principle of the universe, who is above all things, is in a manner superior to intellectual perception, and by a super-essential energy." *Αφεμαι τρόπος τὰ Νοητά,* (10.) So that, in consequence of this reasoning, the speculations of geometry are then most true, when most abstracted from sensible and material natures.

versities.

versities. From which, indeed, it comprehends, in terminated bounds, the imaginative formations of figures, and reduces them to the essential substance of reasons. But, according to the third propagations of cogitative intelligence, it considers nature, and delivers the manner in which the forms of sensible elements, and the powers which they contain, are previously received according to cause, in the reasons themselves. For it possesses, indeed, the images of universal intelligible genera; but the exemplars of such as are sensible: and completes its own essence, according to such things as are subject to cogitation. And through these, as through proper mediums, it ascends and descends to those universals which truly are, and to sensible forms which are in a state of perpetual formation. But always geometrically philosophising concerning the things which are, it comprehends in all the proportions of virtues, the images of intellectual, animal, and natural concerns. And it delivers, in an orderly manner, all the ornaments of republics: and exhibits in itself their various mutations. Such then are its energies arising from a certain immaterial power of cognition: but when it touches upon matter, it produces from itself a multitude of sciences; such as geodæsia, mechanics, and perspective: by which it procures the greatest benefit to the life of mortals. For it constructs by these sciences, war-instruments, and the bulwarks of cities; and makes known the circuits of mountains, and the situations of places. Lastly, it instructs us in measures: at one time of the diversified ways of the earth; and at another, of the restless paths of the deep. Add too, that it constructs balances and scales, by which it renders to cities a sure equality according to the invariable standard of number. Likewise, it clearly expresses, by images, the order of the whole orb of the earth; and by these, manifests many things incredible to mankind, and renders them credible to all. Such, indeed, as Hiero of Syracuse is reported to have said of Archimedes \*, when he had fabricated a ship furnished with three sails, which he had prepared to send to Ptolemy king of Egypt. For when all the Syracusians together, were unable to draw this ship, Archimedes enabled Hiero

\* See Plutarch, in the life of Marcellus.

to draw it himself, without any assistance from others. But he, being astonished, said, From this day, Archimedes shall be believed in whatever he shall affirm. They also report, that Gelo said the same, when Archimedes discovered the weight of the several materials from which his crown was composed, without dissolving their union. And such are the narrations which many of the ancients have delivered to our memory, who were willing to speak in praise of the mathematics: and, on this account, we have placed before the reader, for the present, a few out of the many, as not foreign from our design of exhibiting the knowledge and utility of geometry.

## C H A P. IV.

*On the Origin of Geometry, and its Inventors.*

*Act. 457*  
BUT let us now explain the origin of geometry, as existing in the present age of the world. For the demoniacal Aristotle \* observes, that the same opinions often subsist among men, according to certain orderly revolutions of the world: and that sciences did not receive their first constitution in our times, nor in those periods which are known to us from historical tradition, but have appeared and vanished again in other revolutions of the universe; nor is it possible to say how often this has happened in past ages, and will again take place in the future circulations of time. But, because the origin of arts and sciences is to be considered according to the present revolution of the universe, we must affirm, in conformity with the most general tradition, that geometry was first invented by the Egyptians, deriving its origin from the mensuration of their fields: since this, indeed, was necessary to them, on account of the inundation of the Nile washing away the boundaries of land belonging to each. Nor ought it to seem wonderful, that the invention of this as well as of other sciences, shculd receive its commencement from convenience and opportunity. Since whatever is carried in the circle of genera-

\* In lib. i. de Cælo, tex. 22. et lib. i. Meteo. cap. 3. Aristotle was called demoniacal by the Platonic philosophers, in consequence of the encomium bestowed on him by his master, Plato, "That he was the dæmon of nature." Indeed, his great knowledge in things subject to the dominion of nature, well deserved this encomium; and the epithet *divine*, has been universally ascribed to Plato, from his profound knowledge of the intelligible world.

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tion, proceeds from the imperfect to the perfect. A transition, therefore, is not undeservedly made from sense to consideration, and from this to the nobler energies of intellect \*. Hence, as the certain knowledge of numbers received its origin among the Phoenicians, on account of merchandise and commerce, so geometry was found out among the Egyptians from the distribution of land. When Thales, therefore, first went into Egypt, he transferred this knowledge from thence into Greece: and he invented many things himself, and communicated to his successors the principles of many. Some of which were, indeed, more universal, but others extended to sensibles.

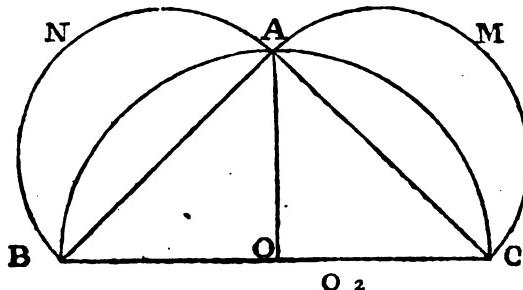
Mamercus. After him Ameristus, the brother of Stesichorus the poet, is celebrated as one who touched upon, and tasted the study of geometry, and who is mentioned by Hippias the Elean, as restoring the glory of geometry. But after these, Pythagoras changed that philosophy, which is conversant about geometry itself, into the form of a liberal doctrine, considering its principles in a more exalted manner; and investigating its theorems immaterially and intellectually; who likewise invented a treatise of such things as cannot be explained † in geometry, and discovered the constitution of the mundane figures. After him, Anaxagoras the Clazomenian succeeded, who undertook many things pertaining to geometry. And Oenopides the Chian, was somewhat junior to Anaxagoras, and whom Plato mentions in his Rivals, as one who obtained mathematical glory. To these, succeeded Hippocrates, the Chian, who invented the quadrature of the lunula ‡,

and

\* Εἰς τὸν, is wanting in the original, but is supplied by the excellent translation of Barocius.

† Ἀλόγων, in the printed Greek, which Fabricius, in his *Bibliotheca Græca*, vol. i. page 395, is of opinion, should be read ἀλόγων; but I have rendered the word according to the translation of Barocius, who is likely to have obtained the true reading, from the variety of manuscripts which he consulted.

‡ The quadrature of the Lunula is as follows.



Let

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and Theodorus the Cyrenean, both of them eminent in geometrical knowledge. For the first of these, Hippocrates composed geometrical elements: but Plato, who was posterior to these, caused as well geometry itself, as the other mathematical disciplines, to receive a remarkable addition, on account of the great study he bestowed in their investigation. This he himself manifests, and his books, replete with mathematical discourses, evince: to which we may add, that he every where excites whatever in them is wonderful, and extends to philosophy. But in his time also lived Leodamas the Thasian, Archytas the Tarentine, and Theætetus the Athenian; by whom theorems were increased, and advanced to a more skilful constitution. But Neoclides was junior to Leodamas, and his disciple was Leon; who added many things to those thought of by former geometers. So that Leon also constructed elements more accurate, both on account of their multitude, and on account of the use which they exhibit: and besides this, he discovered a method of determining when a problem, whose investigation is sought for, is possible, and when it is impossible. But Eudoxus the Cnidian, who was somewhat junior to Leon, and the companion of Plato, first of all rendered the multitude of those theorems which are called universals more abundant; and to three proportions added three others; and things relative to a section, which received their commencement from Plato, he diffused into a richer multitude, employing also resolutions in the prosecution of these. Again, Amyclas the Heracleotean, one of Plato's familiars, and Menæchmus, the disciple, indeed, of Eudoxus, but conversant with Plato, and his brother Dinostratus, rendered the whole of geometry as yet more perfect. But Theudius, the Magnian, appears

Let  $A B C$  be a right-angled triangle, and  $B A C$  a semi-circle on the diameter  $B C$ :  $B N A$  a semi-circle described on the diameter  $A B$ ;  $A M C$  a semi-circle described on the diameter  $A C$ . Then the semi-circle  $B A C$  is equal to the semi-circles  $B N A$ , and  $A M C$  together: (because circles are to each other as the squares of their diameters, 31, 6.) If, therefore, you take away the two spaces  $B A$ ,  $A C$  common on both sides, there will remain the two lunulas  $B N A$ ,  $A M C$ , bounded on both sides with circular lines, equal to the right-angled triangle  $B A C$ . And if the line  $B A$ , be equal to the line  $A C$ , and you let fall a perpendicular to the hypotenuse  $B C$ , the triangle  $B A O$  will be equal to the lunular space  $B N A$ , and the triangle  $C O A$  will be equal to the lunula  $C M A$ . Those who are curious, may see a long account of an attempt of Hippocrates to square the circle, by the invention of the lunulas, in Simplicius on Aristotle's Physics, lib. i.

to have excelled, as well in mathematical disciplines, as in the rest of philosophy. For he constructed elements egregiously, and rendered many particulars more universal. Besides, Cyzicus the Athenian, flourished at the same period, and became illustrious in other mathematical disciplines, but especially in geometry. These, therefore, resorted by turns to the Academy, and employed themselves in proposing common questions. But Hermotimus, the Colophonian, rendered more abundant what was formerly published by Eudoxus and Theætetus, and invented a multitude of elements, and wrote concerning some geometrical places. But Philippus the Mendæan \*, a disciple of Plato, and by him inflamed in the mathematical disciplines, both composed questions, according to the institutions of Plato, and proposed as the object of his enquiry whatever he thought conduced to the Platonic philosophy. And thus far historians produce the perfection of this science. But Euclid was not much junior to these, who collected elements, and constructed many of those things which were invented by Eudoxus; and perfected many which were discovered by Theætetus. Besides, he reduced to invincible demonstrations, such things as were exhibited by others with a weaker arm. But he lived in the times of the first Ptolemy: for Archimedes mentions Euclid, in his first book, and also in others. Besides, they relate that Euclid was asked by Ptolomy, whether there was any shorter way to the attainment of geometry than by his elementary institution, and that he answered, there was no other royal path which led to geometry. Euclid, therefore, was junior to the familiars of Plato, but more ancient than Eratosthenes and Archimedes (for these lived at one and the same time, according to the tradition of Eratosthenes) but he was of the Platonic sect, and familiar with its philosophy: and from hence he appointed the constitution of those figures which are called Platonic †, as the end of his elementary institutions.

\* So Barocius reads, but Fabricius Μεδεαῖος.

† i. e. The five regular bodies, the pyramid, cube, octaedron, dodecaedron and icosaedron; concerning which, and their application to the theory of the universe, see Kepler's admirable work, *De Harmonia Mundi*.

## C H A P. V.

Fasc. 68, 23

*What Mathematical Volumes Euclid composed.*

THERE are, therefore, many other mathematical volumes of this man, full of admirable diligence, and skilful consideration: for such are his Optics \*, and Catoptrics: and such also, are his elementary institutions, which conduce to the attainment of music †; and his book concerning divisions ‡. But his geometrical institution of the Elements is especially admirable, on account of the order and election of those theorems and problems, which are distributed through the Elements. For he does not assume all which might be said, but that only which could be delivered in an elementary order. Besides this, he exhibits modes of syllogisms of every kind; some, indeed, receiving credibility from causes, but others proceeding from certain signs; but all of them invincible and sure, and accommodated to science. But, besides these, he employs all the dialectic ways, dividing, indeed, in the inventions of forms; but defining in essential reasons: and again, demonstrating in the progressions from principles to things sought, but resolving in the reversions from things sought to principles. Besides this, we may view in his geometrical elements, the various species of conversions, as well of such as are simple as of such as are more composite. And again, what wholes may be converted with wholes: what wholes with parts; and on the other hand, what as parts with parts §. Besides this, we must say, that in the continuation of inventions, the dispositions and order of things preceding and following, and in the power with which he treats

\* It may be doubted whether the optics and catoptrics, ascribed to Euclid in the editions of his works are genuine: for Savil, and Dr. Gregory, think them scarcely worthy so great a man.

† There are two excellent editions of this work, one by Meibomius, in his collection of ancient authors on harmony; and the other by Dr. Gregory, in his collection of Euclid's works.

‡ This work is most probably lost. See Dr. Gregory's Euclid.

§ All this is shewn by Proclus in the following Commentaries; and is surely most admirable and worthy the investigation of every liberal mind; but I am afraid modern mathematicians very little regard such knowledge, because it cannot be applied to practical and mechanical purposes.

every

every particular, he is not deceived, as if falling from science, and carried to its contrary, falsehood and ignorance. But because we may imagine many things as adhering to truth, and which are consequent to principles producing science, which nevertheless tend to that error which flows from the principles, and which deceives ruder minds, he has also delivered methods of the perspicacious prudence belonging to these. From the possession of which, we may exercise those in the invention of fallacies, who undertake this inspection, and may preserve ourselves from all deception. And this book, by which he procures us this preparation, is inscribed *προειδαρίος*, or, concerning fallacies \*. Because he enumerates in order their various modes, and in each exercises our cogitation with various theorems. And he compares truth with falsehood, and adapts the confutation of deception to experience itself. This book, therefore, contains a purgative and exercising power. But the institution of his elementary, skilful contemplation of geometrical concerns, possesses an invincible and perfect narration.

## C H A P. VI.

*Concerning the Purport of Geometry.*

*True Latin 70, 19*  
 BUT, perhaps, some one may enquire in what the design of this treatise consists? To this I answer, that its design is to be distinguished as well according to the objects of enquiry, as according to the learner. And, indeed, regarding the subject, we must affirm, that all the discourse of geometry is concerning the mundane figures. Because it begins from such things as are simple, but ends in the variety of their constitution. And, indeed, it constitutes each of them separately, but at the same time delivers their inscriptions in a sphere, and the proportions which they contain. On which account some have thought, that the design of each of the books is to be referred to the world; and they have delivered to our memory, the utility which they afford us in the contemplation of the universe.

\* This work is unfortunately lost.

But

But distinguishing the design with respect to the learner, we must affirm, that its purpose is the institution of elements; and the perfection of the learners cogitative powers in universal geometry. For beginning from these, we are enabled to understand the other parts of this science, and to comprehend the variety which they contain. And, indeed, without these, the discipline of the rest, is to us impossible and incomprehensible. For such theorems as are most principal and simple, and are most allied to first suppositions, are here collected in a becoming order. And the demonstrations of other mathematicians, use these as most known, and advance from these in their most complicated progressions. For thus Archimedes, in what he has writ concerning the sphere and cylinder, and Apollonius, and the rest of mathematicians, use, as evident principles, the things exhibited in this treatise. Its purpose, therefore, is the institution of learners in the whole geometric science, and to deliver the determinate constitutions of the mundane figures.

## C H A P. VII.

*From whence the Name of Elementary Institution originated, and why Euclid is called the Institutor of Elements.*

**B**UT what gave rise to the name of elementary institution, and of element itself, from which elementary institution was derived? To this we shall reply, by observing, that of theorems some are usually called elements, but others elementary, and others again are determined beyond the power of these. Hence, an element is that whose consideration passes to the science of other things, and from which we derive a solution of the doubts incident to the particular science we investigate. For as there are certain first principles of speech, most simple and indivisible, which we denominate elements, and from which all discourse is composed; so there are certain principal theorems of the whole of geometry, denominated elements, which have the respect of principles to the following theorems; which regard all the subsequent propositions, and afford the demonstrations of many accidents essential to the subjects of geometric speculation.

But

But things elementary are such as extend themselves to a multitude of propositions, and possess a certain simplicity and sweetnes, yet are not of the same dignity with elements; because their contemplation is not common to all the science to which they belong, as is the case in the following theorem, that in triangles, perpendiculars, drawn from their angles to their sides, coincide in one point \*. Lastly, whatever neither possesses a knowledge extended into multitude, nor exhibits any thing skilful and elegant, falls beyond the elementary power. Again, an element, as Menæchmus says, may have a twofold definition. For that which confirms, is an element of that which is confirmed; as the first proposition of Euclid with respect to the second, and the fourth with regard to the fifth. And thus, indeed, many things may be mutually called elements one of another; for they are mutually confirmed. Thus, because the external angles of right-lined figures, are equal to four right angles, the multitude of internal ones equal to right angles; and, on the contrary, that from this is exhibited †. Besides, an element is otherwise called that into which, because it is more simple, a composite is dissolved. But it must be observed, that every element cannot be called the element of every thing: but such as are more principal are the elements of such as are constituted in the reason of the thing effected; as petitions are the elements of theorems. And, according to this signification of an element, Euclid's elements are constructed. Some, indeed, of that geometry which is conversant about planes; but others of stereometry. In the same manner, likewise, in arithmetic and astronomy, many have composed elementary institutions. But it is difficult, in each science, to chuse and conveniently ordain elements, from which all the peculiarities of that science originate, and into which they may be resolved. And among those who have undertaken this employment, some have been able to collect more, but others fewer elements. And some, indeed, have used shorter demonstrations; but others have extended their treatise to an infinite length. And some have omitted the method by an impossibility; but others

\* Because this is true only in isosceles and equilateral triangles.

† This follows from the 32d proposition of the first book of Euclid; and is demonstrated by Dr. Barrow, in his scholium to that proposition.

that by proportion; and others, again, have attempted preparations against arguments destroying principles. So that many methods of elementary institution have been invented by particular writers on this subject. But it is requisite that this treatise should entirely remove every thing superfluous, because it is an impediment to science. But every thing should be chosen, which contains and concludes the thing proposed; for this is most convenient and useful in science. The greatest care, likewise, should be paid to clearness and brevity; for the contraries to these, disturb our cogitation. Lastly, it should vindicate to itself, the universal comprehension of theorems, in their proper bounds: for such things as divide learning into particular fragments, produce an incomprehensible knowledge. But in all these modes, any one may easily find, that the elementary institution of Euclid excels the institutions of others. For its utility, indeed, especially confers to the contemplation of primary figures: but the transition from things more simple to such as are more various, and also that perception, which from axioms possesses the beginning of knowledge, produces clearness, and an orderly tradition: and the migration from first and principal theorems to the objects of enquiry, effects the universality of demonstration. For whatever he seems to omit, may either be known by the same ways, as the construction of a scalene and isosceles triangle \*: or because they are difficult, and capable of infinite variety, they are far remote from the election of elements, such as the doctrine of perturbate proportions, which Apollonius has copiously handled: or, lastly, because they may be easily constructed from the things delivered, as from causes, such as many species of angles and lines. For these, indeed, were omitted by Euclid, and are largely discoursed of by others, and are known from simple propositions. And thus much concerning the universal elementary institution of geometry.

\* The method of constructing these is shewn by our philosopher, in his comment on the first proposition, as will appear in the second volume of this work.

## C H A P. VIII.

*Concerning the Order of Geometrical Discourses.*

BUT let us now explain the universal order of the discourses contained in geometry. Because then, we assert that this science consists from hypothesis\*, and demonstrates its consequent propositions from definite principles (for one science only, I mean the first philosophy, is without supposition, but all the rest assume their principles from this) it is necessary that he who constructs the geometrical institution of elements, should separately deliver the principles of the science, and separately the conclusions which flow from those principles; and that he should render no reason concerning the nature or truth of the principles, but should confirm by reasons, the things consequent to these geometric principles. For no science demonstrates its own principles, nor discourses concerning them; but procures to itself a belief of their reality, and they become more evident to the particular science to which they belong than the things derived from them as their source. And these, indeed, science knows by themselves; but their consequents, through the medium of these. For thus, also, the natural philosopher propagates his reasons from a definite principle, supposing the existence of motion. Thus too, the physician, and he who is skilled in any of the other sciences and arts. For if any one mingles principles, and things flowing from principles into one and the same, he disturbs the whole order of knowledge, and conglutinates things which can never mutually agree; since a principle, and its emanating consequent, are naturally distinct from each other. In the first place, therefore (as I have said), principles in the geometric institution are to be distinguished from their consequents, which is performed by Euclid in each of his books; who, before every treatise, exhibits the common principles of this science; and afterwards divides these common principles into hypotheses, petitions, and axioms. For all these mutually differ; nor is an axiom, petition,

\* The reader will please to observe, that the definitions are, indeed, hypotheses, according to the doctrine of Plato, as may be seen in the note to chap. i. book I. of this work.

and hypothesis the same, according to the demoniacal Aristotle; but when that which is assumed in the order of a principle, is indeed known to the learner, and credible by itself, it is an axiom: such as, that things equal to the same, are mutually equal to each other. But when any one, hearing another speak concerning that of which he has no self-evident knowledge, gives his assent to its assumption, this is hypothesis. For that a circle is a figure of such a particular kind, we presume (not according to any common conception) without any preceding doctrine. But when, again, that which is asserted was neither known, nor admitted by the learner, yet is assumed, then (says he) we call it petition; as the assumption that all right angles are equal. But the truth of this is evinced by those who study to treat of some petition, as of that which cannot by itself be admitted by any one. And thus, according to the doctrine of Aristotle \*, are axiom, petition, and supposition distinguished. But oftentimes, some denominate all these hypotheses, in the same manner as the Stoicks call every simple enunciation an axiom. So that, according to their opinion, hypotheses also will be axioms; but, according to the opinion of others, axioms will be called suppositions. Again, such things as flow from principles are divided into problems and theorems. The first, indeed, containing the origin, sections, ablations, or additions of figures, and all the affections with which they are conversant; but the other exhibiting the accidents essential to each figure. For, as things effective of science, participate of contemplation, in the same manner things contemplative previously assume problems in the place of operations. But formerly some of the ancient mathematicians thought that all geometrical propositions should be called theorems, as the followers of Speusippus and Amphinomus, believing, that to contemplative sciences, the appellation of theorems is more proper than that of problems; especially since they discourse concerning eternal and immutable objects. For origin does not subsist among things eternal: on which account, problems cannot have any place in these sciences; since they enunciate origin, and the production of that which formerly had no existence, as the construction of an equi-

\* In his last Analytics. See the preceding Dissertation.

## COMMENTARIES OF PROCLUS. 109

Iateral triangle, or the description of a square on a given right line, or the position of a right line at a given point. It is better, therefore (say they), to assert that all propositions are of the speculative kind: but that we perceive their origin, not by production, but by knowledge, receiving things eternal as if they were generated; and on this account we ought to conceive all those theorematically, but not problematically. But others, on the contrary, think that all should be called problems: as those mathematicians who have followsd Menæchmus. But that the office of problems is two-fold, sometimes, indeed, to procure the thing sought; but at other times when they have received the determinate object of enquiry, to see, either what it is, or of what kind it is, or what affection it possesses, or what its relation is to another. And, indeed, the assertions of each are right; for the followers of Speusippus well perceive. Since the problems of geometry are not of the same kind, with such as are mechanical. For these are sensibles, and are endued with origin, and mutation of every kind. And, on the other hand, those who follow Menæchmus do not dissent from truth: since the inventions of theorems cannot by any means take place without an approach into matter; I mean intelligible matter. Reasons, therefore, proceeding into this, and giving form to its formless nature, are not undeservedly said to be assimilated to generations. For we say that the motion of our cogitation, and the production of its inherent reasons, is the origin of the figures situated in the phantasy, and of the affections with which they are conversant: for there constructions and sections, positions and applications, additions and ablations, exist: but every thing resident in cogitation, subsists without origin and mutation. There are, therefore, both geometrical problems and theorems. But, because contemplation abounds in geometry, as production in mechanics, all problems participate of contemplation; but every thing contemplative is not problematical. For demonstrations are entirely the work of contemplation; but every thing in geometry posterior to the principles, is assumed by demonstration. Hence, a theorem is more common: but all theorems do not require problems; for there are some which posseſs from themselves the demonstration of the thing sought. But others, distinguishing a theorem from a problem, say, that

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that indeed every problem receives whatever is predicated of its matter, together with its own opposite: but that every theorem receives, indeed, its symptom predicate, but not its opposite. But I call the matter of these, that genus which is the subject of enquiry; as for instance, a triangle, quadrangle, or a circle: but the symptom predicate, that which is denominated an essential accident, as equality, or section, or position, or some other affection of this kind. When, therefore, any one proposes to inscribe an equilateral triangle in a circle, he proposes a problem: for it is possible to inscribe one that is not equilateral. But when any one asserts that the angles at the base of an isosceles triangle are equal, we must affirm that he proposes a theorem; for it is not possible that the angles at the base of an isosceles triangle should be unequal to each other. On which account, if any one forming problematically, should say that he wishes to inscribe a right angle in a semi-circle, he must be considered as ignorant of geometry; since every angle in a semi-circle is necessarily a right one. Hence, propositions which have an universal symptom, attending the whole matter, must be called theorems; but those in which the symptom is not universal, and does not attend its subject, must be considered as problems. As to bisect a given terminated right line, or to cut it into equal parts: for it is possible to cut it into unequal parts. To bisect every rectilinear angle, or divide it into equal parts; for a division may be given into unequal parts. On a given right line to describe a quadrangle; for a figure that is not quadrangular may be described. And, in short, all of this kind belong to the problematical order. But the followers of Zenodotus, who was familiar with the doctrine of Oenopides, but the disciple of Andron, distinguish a theorem from a problem, so far as a theorem enquires what the symptom is which is predicated of the matter it contains; but a problem enquires what that is, the existence of which is granted. From whence the followers of Posidonius define a theorem a proposition, by which it is enquired whether a thing exists or not; but a problem, a proposition, in which it is enquired what a thing is, or the manner of its existence. And they say that we ought to form the contemplating proposition by enunciating, as that every triangle has two sides greater than the remaining one, and that

## COMMENTS OF PROCLUS. 117

the angles at the base of every isosceles triangle are equal: but we must form the problematical proposition, as if enquiring whether a triangle is to be constructed upon this right line. For there is a difference, say they, absolutely and indefinitely, to enquire whether the thing proposed is from a given point to erect a right line at right angles to a given line, and to behold what the perpendicular is: And thus, from what has been said, it is manifest there is some difference between a problem and a theorem. But that the elementary institution of Euclid, also, consists partly of problems, and partly of theorems, will be manifest from considering the several propositions: Since, in the conclusion of his demonstrations, he sometimes adds (which was to be shewn) sometimes (which was to be done) the latter sentence being the mark or symbol of problems, and the former of theorems. For although, as we have said, demonstration takes place in problems, yet it is often for the sake of generation; for we assume demonstration in order to shew, that what was commanded is accomplished: but sometimes it is worthy by itself, since the nature of the thing sought after may be brought into the midst. But you will find Euclid sometimes combining theorems with problems, and using them alternately, as in the first book; but sometimes abounding with the one and not the other. For the fourth book is wholly problematical; but the fifth is entirely composed from theorems. And thus much concerning the order of geometrical propositions.

## C H A P. IX.

*Concerning the Design of the first Book,—its Division,—and a previous Admonition to the Reader.*

*Fr. 6em 81, 23*  
**B**UT, after these considerations, when we have determined the design of the first book, and have exhibited its division, we shall enter upon the treatise of the definitions. The design, then, of this book, is to deliver the principles of the contemplation of right lines. For though a circle, and its consideration, is more excellent than the essence and knowledge of right lines, yet the doctrine concerning these is more adapted to us, who are hastening to transfer our cogitation

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tation from more imperfect and sensible natures, to such as are intelligible. For, indeed, right lined figures are proper to sensibles, but a circle to intelligibles. Because that which is simple, uniform, and definite, is proper to the nature of the things which are: but that which is various, and which increases indefinitely from the number of its containing sides, regards the fluctuating essence of sensible particulars. Hence, in this book, the first and most principal of right lined figures are delivered; I mean the triangle and parallelogram. For in these, as under their proper genus, the causes of the elements are contained: viz. the isosceles and scalene, and those which are formed from these, the equilateral triangle, and the quadrangle, from which the four figures of the elements are composed. We shall find, therefore, as well the origin of the equilateral triangle as of the quadrangle; of the last, indeed, upon, but of the first from a given right line. [\*An equilateral triangle, therefore, is the proximate cause of the three elements, fire, air, and water: but a quadrangle is annexed to earth. And lastly, the design of the first book is adapted to the whole treatise, and confers to the universal knowledge of the mundane elements. Besides, it instructs learners in the science concerning right-lined figures; since it rightly invents, and accurately collects, the first principles of these.

But this book is divided into three greatest parts, of which the first declares the origin and properties of triangles, as well according to angles, as also according to sides. Besides, it makes mutual comparisons of these, and beholds every one by itself. For receiving one triangle, sometimes it considers the angles from the sides; but sometimes the sides from the angles: and this according to equality and inequality. And supposing two triangles, it discovers the same property again, by various methods. But the second part combines the contemplation of parallelograms, describing their properties and generations. And the third part shews the communication of triangles

\* That part of this work enclosed within the brackets, is wanting in the original; which I have restored from the excellent version of Barocius. The philosophical reader, therefore, of the original, who may not have Barocius in his possession, will, I hope, be pleased, to see so great a vacancy supplied; especially, as it contains the beginning of the commentary on the definition of a point.

and

and parallelograms, both in symptoms and mutual comparisons. For it shews that triangles and parallelograms constituted on the same and on equal bases, are affected with the same passions ; and by complication, when both stand upon one base : and again, after what manner a parallelogram may be made equal to a triangle ; and lastly, concerning the proportion which in right angled triangles, the square made from the side subtending, has to the squares containing the right angle. And such is the division of the first Book.

But, previous to our enquiry into each of these parts, we think it requisite to admonish the reader, that he must not require of us, those small assumptions, and cases, and whatever else there may be of that kind, which has been divulged by our predecessors. For we are fatiated with these, and shall, therefore, but rarely adopt them in our discourse. But whatever has a more difficult contemplation, and regards universal philosophy, of this we shall make a particular relation : imitating the Pythagoreans, with whom this ænigma was common, “ a \* figure and a step : but not a figure and three oboli.” shewing by this, that it is requisite to pursue that philosophy which ascends every theorem by a step, and raises the soul on high ; but does not suffer it to remain among sensibles, to fill up the use attendant on mortals, and, consulting for this, to neglect the elevation which rises from hence to an intelligible essence.

\* I do not find this ænigma among the Pythagoric symbols which are extant ; so that it is probably no where mentioned but in the present work. And I am sorry to add, that a *figure and three oboli*, is too much the general cry of the present times.

## DEFINITIONS.

## DEFINITION I.

A POINT is that which has no PARTS.

THAT geometry, according to the transition which takes place from things more composite to such as are more simple, runs from body, which is diffused into distance by three dimensions, to a superficies by which it is bounded; but from superficies to a line, the boundary of superficies; and from a line to a point destitute of all dimension, has been often said, and is perfectly manifest. But because these terms, in many places, on account of their simplicity, appear to be more excellent than the nature of composites; but in many, as when they subsist in things which they terminate, they are similar to accidents, it is necessary to determine in what genera of beings each of these may be beheld \*. I say then, that such things as are destitute of matter, which subsist in separate reasons, and in those forms which are placed under themselves, are always allotted a subsistence of more simple essences, superior to the subsistence of such as are more composite. On this account, both in intellect, and in the ornaments, as well of the middle kind as among those peculiar to the soul, and in natures themselves, the terms which proximately vivify bodies, excel according to essence the things which are terminated; and are more impenetrable, more uniform, and more primary than these. For in immaterial forms, unity is more perfect than multitude; that which

\* The present Comment, and indeed most of the following, eminently evinces the truth of Kepler's observation, in his excellent work, *De Harmonia Mundi*, p. 118. For, speaking of our author's composition in the present work, which he every where admires and defends, he remarks as follows, "oratio fluit ipsi torrentis instar, ripas inundans, et cæca dubitationum vada gurgitesque occultans, dum mens plena majestatis tantarum rerum, luctatur in angustiis linguae, et conclusio nunquam sibi ipsi verborum copiâ satisfaciens, propositionum simplicitatem excedit." But Kepler was skilled in the Platonic philosophy, and appears to have been no less acquainted with the great depth of our author's mind than with the magnificence and sublimity of his language. Perhaps Kepler is the only instance among the moderns, of the philosophical and mathematical genius being united in the same person.

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is imparible, than that which is endued with unbounded progression; and that which terminates, than that which receives bound from another. But such things as are indigent of matter, and abide in others, and degenerate from the perfection of their essence, which are scattered about subjects, and have an unnatural union, are allotted more composite reasons, prior to such as are more simple. Hence, things which appear in the phantasy invested with form, and the matter of the figures which the phantasy contains, and whatever in sensibles is generated by nature, have, in a preceding order, the reasons of the things terminated; but the reasons which terminate, in a following and adventitious rank \*.] For lest that which is distributed into three dimensions, should be extended into infinite magnitude, either according to intelligence or sense, it was every way terminated by superficies. And lest a plane superficies should conceal itself in an infinite progression, a line approaching opposed its diffusion, and gave bound to its indefinite extension. And, in like manner, a point limited the progressions of a line; composite natures deriving their subsistence from such as are simple. For this also is again manifest, that in separate forms the reasons of terms subsist in themselves, but not in those which are terminated; and abiding such as they are in reality, possess a power of constituting secondary natures. But, in inseparable forms they give themselves up to things which are terminated, reside in them, become, as it were, their parts, and are replenished with baser natures. On which account, that which is imparible is there endued with a partible essence, and that which is void of latitude is diffused into breadth. And terms are no longer able to preserve their simplicity and purity. For since they abide in another, they necessarily change their own nature into the matter of their containing subject. Matter, indeed, disturbs the perfection of these, and causes the reason of a plane to become a profound plane; but obscuring the one dimension of a line, causes it to be every way partible; and gives corporeity to the indivisibility of a point, and separates it together with the natures which it terminates. For all these reasons falling into matter, the one kind from cogitation into intelli-

\* That is, the reason of a triangular figure (for instance) in the phantasy, or triangle itself, is superior to the triangular nature participated in that figure.

gible matter, but the other from nature into that which is sensible, are replenished with their containing subjects; and depart from their own simplicity, into foreign compositions and intervals. But here a doubt arises how all these, existing in intellect and soul in an impartible manner, and without any dimension, are distributed into matter, some indeed, principally, but others on account of its nature? Shall we say that there is a certain order in immaterial forms, so that some are allotted the first, some the middle, and others the last place; and that of forms some are more uniform, but that others are more multiplied; and that some have their powers collected together, but others tending into interval; and that some, again, border upon bound, but that others are proximate to infinity? For though all participate of these two principles, yet some originate from bound, but others from infinity, of which they more largely participate. Hence, a point is entirely impartible, since it subsists according to bound, yet it occultly contains an infinite power, by which it produces every interval, and the progression of all intervals, unfolds its infinite power. But body, and the reason of body, participates more of an infinite nature; on which account it is among the number of things terminated by another, and divisible in infinitum, according to all dimensions. But the mediums between these, according to the distance of the extremes, are either among the number of things which have an abundance of bound; or among such as have an affluence of infinity: on which account they both terminate and are terminated. For, indeed, so far as they consist from bound, they are able to terminate others; but so far as they participate of infinity, they are indigent of termination from others. Hence, since a point is also a bound, it preserves its proper power in participation: but since it likewise contains infinity occultly, and is compelled to be every where present with the natures which it terminates, it resides with them infinitely. And, because among immaterial forms there was a certain infinite power capable of producing things distant from each other by intervals, a point is present with its participants in capacity. For infinity in intelligibles is the primary cause and prolific power of the universe; but in material natures it is imperfect, and is alone all things in dormant capacity. And in short, those forms which, on account of their simplicity

simplicity and imparibility, hold a superior rank among principles, preserve, indeed, (in conformity to their nature,) their own property in their participations, but become worse than more composite reasons. For matter is able to participate these more clearly, and to be prepared for their reception, rather than that of the most simple causes of beings. On which account, the vestiges of separate principles descend into matter; but the participations of those in a second and third order, become more conspicuous. Hence, matter participates more of the cause of body, than of a plane; and of this more than the form of a line; and of this still more than that of a point, which contains all these, and is the boundary of them all. For the reason of a point presides over this whole series, unites and contains all partible natures, terminates their progressions, produces them all by its infinite power, and comprehends them in its indivisible bound. • On which account also, in the images of immaterial forms, some are the boundaries of others; but a point is the limit of them all. But that we must not think with the Stoicks, that these boundaries of bodies alone subsist from cogitation; but that there are certain natures of this kind among beings, which previously contain the demiurgical reasons of things, we shall be enabled to remember, if we regard the whole world, the convolutions of its parts, the centres of those convolutions, and the axes which penetrate through the whole of these revolving circles. For the centres subsist in energy, since they contain the spheres, preserve them in their proper state, unite their intervals, and bind and establish to themselves the powers which they possess. But the axes themselves being in an immoveable position, evolve the spheres, give them a circular motion, and a revolution round their own abiding nature. And the poles of the spheres, which both terminate the axes, and bind in themselves the other convolutions, do they not perspicuously evince, that points are endued with demiurgical and capacious powers, that they are perfective of every thing distant by intervals, and are the sources of union, and an unceasing motion? From whence, indeed, Plato \* also says, that they have an adamantine subsistence; shewing by this, the immutable, eternal, and stable power

\* In the tenth book of his Republic.

of their essence, ever preserving itself in the same uniform mode of existence. He adds too, that the whole spindle of the Fates, is turned about these, and leaps round their coercive union. But other more recondite and abstruse discourses affirm, that the demiurgus presides over the world; seated in the poles, and, by his divine love, converting the universe to himself. But the Pythagoreans thought that the pole should be called the Seal of Rhea \*; because the zoogonic, or vivific goddess, pours through these into the universe, an inexplicable and efficacious power. And the centre they called the prison of Jupiter; because, since Jupiter has placed a demiurgical guard in the bosom of the world, he has firmly established it in the midst. For, indeed, the centre abiding, the universe possesses its immovable ornament, and unceasing convolution: and the gods who preside over the poles, obtain a power collective of divisible natures, and unific of such as are multiplied: and those who are allotted the government of the axes, restrain and eternally evolve their perpetual convolutions. And, if it is lawful to offer our own opinion on this subject, the centres and poles of all the spheres are the symbols of the conciliating gods, shadowing forth their imperceptible and unifying composition. But the axes express the coherencies of the universal ornaments; and are endued with a power of comprehending the mundane integrities and periods, in the same manner as their presiding deities, of such as are intellectual. But the spheres themselves are images of the gods, called perfectors of works, copulating the principle with the end, and excelling all figures in simplicity, similitude, and perfection. But we have been thus prolix, that we might evince the power of imparables, and of the terms which the world contains, and that so far as they bear an image of primary and most principal causes, they are allotted the most excellent order in the universe. For centres and poles are not of the same kind with things which are terminated; but they subsist in energy, and possess an essence, and perfect power, which pervades through all partible natures. But many beholding those terms which imperfectly subsist in terminated essences, consider them as endued with a slender subsistence; and some indeed say, that

\* See the Hymn to the Mother of the Gods, in my translation of the Orphic Initiations.

they

they are alone separated from sensibles by thought ; but others, that they have an essence no where but in our thoughts. However, since the forms of all these are found both in the nature of intellect, in the ornaments of soul, in the nature of things, and in inferior bodies, let us consider how, according to the order they contain, they subsist in the genera of beings. And indeed, all of them pre-exist in intellect, but in an impartible and uniform manner : so that they all subsist according to one form, the reason of a point, which exists occultly and impartibly. But they all subsist in soul according to the form of a line : on which account Timæus also composes the soul from right and circular lines : for every circle is a line alone \*. But they all subsist in natures, according to the reason of a plane ; and on this account, Plato commands us to manifest those natural reasons, which are endued with a power of constituting bodies by a plane. And the resolution of bodies into planes leads us to the proximate cause of appearances. Lastly, they all subsist in bodies, but in a corporeal manner ; since all forms have their being in these, according to the partible nature of bodies. Hence, all of them appear every where, and each according to its proper order ; and diversity arises from predominating power. The point, indeed, is every where impartible, and when that which is divisible into parts, excels according to the diminution of beings, it vindicates to itself, an illustrious subsistence of partible natures. And sometimes the point is entirely superior, according to the excellence of cause ; but sometimes it is connected with divisibles, and sometimes it is allotted in them an adventitious existence ; and, as if swallowed up by the partition of the lowest natures, loses its own proper impartibility. As, therefore, with respect to the monad, one † is the mother of number, but the other is as

\* The philosopher here seems to contradict what he asserts in the end of his comment on the 13th Definition : for there he asserts, that the circle is a certain plane space. Perhaps he may be reconciled, by considering, that as the circle subsists most according to bound, when we speculate its essence in this respect we may define it according to the circumference, which is the cause of its bound. But when we consider it as participating of infinity also, though not in so eminent a degree, and view it from its emanations from the centre as well as in its reg·essions, we may define it a plane space.

† That is, the essential one of the soul is the mother of number ; but that which subsists in opinion is nothing more than the receptacle of the former ; just as matter is the seat of all forms. For a farther account of the subsistence of numbers, see the first section of the preceding Dissertation.

matter

matter spread under, and the receptacle of numbers ; and each of them is a principle, (yet neither of them is number), but in a different respect : in the same manner a point also, is partly the parent and author of magnitudes ; but is partly a principle in another respect, and not according to a generative cause. But is a point, then, the only im-partible ? Or may we affirm this of the now in time, and of unity in numbers ? Shall we not say, that to the philosopher, indeed, dis-coursing concerning the universality of things, it is proper to behold every thing, however falling under distribution ; but that to him who is endued with the science of particulars, who produces his contem-plation from certain definite principles, and runs back even to these, but very little scrutinizes the progressions of beings, it is requisite to attempt, consider, and treat concerning that im-partible nature alone, which regards his first principles ; and to behold that simplicity which presides over all the particular subjects of his knowledge ? In conse-quence of this reasoning, therefore, a point alone, according to the geometric matter, is destitute of partition ; but unity according to that which is arithmetical. And the reason of a point, however in some other respects it may be imperfect, yet is perfect in the present science. For, indeed, the physician also says, that the elements of bodies are fire and water, and things similar to these ; and as far as to these the resolution of bodies proceeds. But the natural philosopher passes on to more simple elements ; and the one defines an element simple as to sense, but the other simple as to reason ; and both of them properly as to their peculiar science. We must not, therefore, think that the definition of a point is faulty, nor determine it as imperfect ; for so far as pertains to the geometric matter, and its principles, it is suf-ficiently delivered. This alone, indeed, is wanting to its completion, that the definition does not clearly say, *that which is im-partible with me is a point ; and my principle, and that which I contain as most simple, is nothing else than this.* And after this manner it is proper to hear the geometrician addressing us. Euclid, therefore, from a negation of parts, declares to us a principle, leading to the theory of its whole subject nature. For negative discourses are proper to principles, as Parmenides teaches us, who delivers the doctrine concerning the first and last cause, by negations alone. Since every principle consists of

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an essence different from its flowing consequents; and the negations of these exhibit to us the property of their source. For that it is, indeed, the cause of these, yet at the same time has nothing in common with these, becomes perspicuous from a doctrine of this kind. But here a doubt may arise, how, since the phantasy receives all things invested with forms, and in a partible manner, the geometrician beholds in it the point destitute of parts? For it is not because they are reasons existing in cogitation, but the phantasy receives the resemblances of intellectual and divine forms according to its own proper nature, exhibiting in its shadowy bosom the forms of formless natures, and clothing with figure things entirely free from the affections of figure. To this ambiguity we must say, that the species of imaginative motion is neither alone partible, nor impartible; but that it proceeds from the imparible to the partible, and from the formless nature to that which is expressed by form. For if it was partible alone, it could not preserve in itself many impressions of forms, since the subsequent would obscure the pre-existent figures: for no body can contain at once, and according to the same situation, a multitude of figures; but the former will be blotted out by the succession of the latter. But if it was alone imparible, it would not be inferior to cogitation, and to soul, which surveys all things in an imparible manner. Hence, it is necessary that it should indeed begin from an imparible according to its motion, and from thence draw forth the folded and scattered form of every thing falling under cogitation, and penetrating to its shadowy receptacle: but, that it should at length end in form, figure, and interval. And if it be allotted a nature of this kind, it will, after a certain manner, contain an imparible essence: and a point, according to this, must be said to have its principal subsistence: for the form of a line is contracted in the phantasy according to this. Hence, because it possesses a twofold power, imparible and partible, it will indeed contain a point in an imparible, and intervals in a partible manner. But as the Pythagoreans define a point to be unity having position, let us consider what they mean. That numbers, indeed, are more immaterial and more pure than magnitudes, and that the principle of numbers is more simple than the principle of magnitudes, is manifest to every one: but when they say that a

point is unity endued with position, they appear to me to evince that unity and number subsist in opinion: I mean monadic number \*. On which account, every number, as the pentad and the heptad, is one in every soul, and not many; and they are destitute of figure and adventitious form. But a point openly presents itself in the phantasy, subsists, as it were, in place, and is material according to intelligible matter. Unity, therefore, has no position, so far as it is immaterial, and free from all interval and place: but a point has position, so far as it appears seated in the bosom of the phantasy, and has a material subsistence. But unity is still more simple than a point, on account of the community of principles. Since a point exceeds unity according to position; but appositions in incorporeals produce diminutions of those natures, by which the appositions are received.

## DEFINITION II.

A Line is a Length without Breadth.

**A** Line obtains the second place in the Definitions, as it is by far the first and most simple interval, which the geometrician calls a length, adding also without breadth; since a line, in respect of a superficies, ranks as a principle. For he defines a point, as it is the principle of all magnitudes, by negation alone; but a line, as well by affirmation as by negation. Hence it is a length, and by this exceeds the imparibility of a point; but it is without breadth, because it is separated from other dimensions. For, indeed, every thing which is void of breadth, is also destitute of bulk, but the contrary is not true, that every thing void of bulk is also destitute of breadth. Since, therefore, he has removed breadth from a line, he has also removed at the same time bulk. On which account he does not add, that a line also has no thickness, because this property is consequent to the notion of being without breadth. But it is defined by others in various ways: for some call it the flux of a point, but others a magnitude contained by one interval. And this definition,

\* That is, number composed from units.

indeed,

indeed, is perfect, and sufficiently explains the essence of a line; but that which calls it the flux of a point, appears to manifest its nature from its producing cause; and does not express every line, but alone that which is immaterial. For this is produced by a point, which though impartible itself, is the cause of being to partible natures. But the flux of a point, shews its progression and prolific power, approaching to every interval, receiving no detriment, perpetually abiding the same, and affording essence to all partible magnitudes. However, these observations are known, and manifest to every one. But we shall recall into our memory, discourses more Pythagorical, which determine a point as analogous to unity, a line to the duad, a superficies to the triad, and body to the tetrad. [\* Yet when we compare those which receive interval together, we shall find a line monadic; but a superficies dyadic, and a solid body triadic.] From whence also, Aristotle † says, that body is perfected by the ternary number. And, indeed, this is not wonderful, that a point, on account of its impartibility, should be assimilated to unity; but that things subsequent to a point, should subsist according to numbers proceeding from unity, and should preserve the same proportion to a point, as numbers to unity; and that every one should participate of its proximate superior, and have the same proportion to its kindred, and following degree, as the superior to this, which is the immediate consequent. [‡ For example, that a line has the order of the duad with respect to the point, but of unity to a superficies; and that this last has the relation of a triad to the point, but of the duad to a solid.] And on this account, body is tetradiic, with respect to a point, but triadic as to a line. Each order, therefore, has its proportion; but the order of the Pythagoreans is the more principal, which receives its commencement from an exalted source, and follows the nature of beings. For a point is indeed twofold; since it either subsists by itself, or in a line; in which last respect also, since as a boundary it is alone and one, neither having a whole nor parts, it imitates the supreme.

\* This sentence within the brackets, is wholly omitted in the printed Greek.

† In i. De Cælo.

‡ This sentence within the brackets, which is very imperfect in the Greek, I have supplied from the excellent translation of Barocius. In the Greek there is nothing more than  $\lambda\delta\gamma\omega\ \delta\epsilon\pi\eta\tau\eta\ \gamma\delta\mu\mu\eta\ \delta\omega\delta\epsilon\varsigma\ \omega\pi\varsigma\ \tau\delta\varsigma\pi\varsigma$ .

nature of beings. On which account too, it was placed in a correspondent proportion to unity. \* For as the oracle says, *Unity is there first, where the paternal unity abides*. But a line is the first endued with parts and a whole, and it is monadic because it is distant by one interval only; and dyadic on account of its progression: for if it be infinite, it participates of the indefinite duad; but if finite, it requires two terms, from whence and to what place; since, on account of these it imitates totality, and is allotted an order among totals. For unity, according to the oracle, is extended †, and generates two; and this produces a progression into longitude, together with that which is distant extendedly, and with one interval, and the matter of the duad. But superficies, since it is both a triad and duad, as also the receptacle of the primary figures, and that which receives the first form and species, is in a certain respect similar to the triadic nature, which first terminates beings; and to the duad, by which they are divided and dispersed. But a solid, since it has a triple distance, and is distinguished by the tetrad, which is endued with a power of comprehending all reasons, is reduced to that order in which the distinction of corporeal ornaments appears; as also the division of the universe into three parts, together with the tetrads property, which is generative and female. And these observations, indeed, might be more largely discussed, but for the present, must be omitted. Again, the discourse of the Pythagoreans, not undeservedly, calls a line, which is the second in order, and is constituted according to the first motion from an imparible nature, dyadic. And that a point is posterior to unity, a line to the duad, and a superficies to the triad, Parmenides himself shews, by first of all taking away multitude from one by negation, and afterwards the whole. Because, if multitude is before that which is a whole, number also will be prior to that which is continuous, and the duad to the line, and unity to the point: since the epithet *not many*, belongs to unity which generates multitude, but

\* In the Greek, γαρ η μονη; ιντι περιτος, οντε παλικη μονης ιτι φησι το λεγιον. The latter part only of this oracle, is to be found in all the printed editions of the Zoroastrian oracles; though it is wonderful how this omission could escape the notice of so many able critics, and learned men. It seems probable, from hence, that it is only to be found perfect in the present work.

† The word ταπαν, is omitted in the Greek.

to

to the point, the term *not a whole*, is proper, because it produces a whole; for this is said to have no part. And these things are affirmed of a line, while we more accurately contemplate its nature. But we should also admit the followers of Apollonius, who say, that we obtain a notion of a line, when we are ordered to measure the lengths alone, either of ways or walls; for we do not then subjoin either breadth or bulk, but only make one distance the object of our consideration. In the same manner we perceive superficies, when we measure fields; and a solid, when we take the dimensions of wells. For then, collecting all the distances together, we say, that the space of the well is so much, according to length, breadth, and depth. But a line may become the object of our sensation, if we behold the divisions of lucid places from those which are dark, and survey the moon when dichotomized: for this medium has no distance with respect to latitude; but is endued with longitude, which is extended together with the light and shadow.

## DEFINITION III.

But the Extremities of a Line are Points.

EVERY composite receives its bound from that which is simple, and every thing partible from that which is impartible; and the images of these openly present themselves in mathematical principles. For when it is said that a line is terminated by points, it seems manifestly to make it of itself infinite, because, on account of its proper progression, it has no extremity. As, therefore, the duad is terminated by unity, and reduces its own intolerable boldness under bound, when it is restrained in its comprehensive embrace: so a line also is limited by the points which it contains. For, since it is similar to the duad, it participates of a point having the relation of unity, according to the nature of the duad. Indeed, in imaginative, as well as in sensible forms, the points themselves terminate the lines in which they reside. But in immaterial forms, the reason of the imparible point pre-exists separate and apart; but when proceeding from thence by far the first of all, by determining itself with interval, moving itself, and flowing in.

in infinite progression, and imitating the indefinite duad, it is restrained indeed, by its proper principle, is united by its power, and on every side seized by its coercive bound. Hence it is, at the same time, both infinite and finite: infinite, indeed, according to its progression; but finite according to its participation of a terminating cause. So that, when it approaches to this cause, it is detained in its comprehension, and is terminated according to its union. Hence too, in the images of incorporeal forms, a point is said to terminate a line, by occupying its beginning and end. Bound, therefore, in immaterials, is separated from that which is bounded: but here it is twofold; for it subsists in that which is terminated. And this affords a wonderful symptom, that forms, indeed, abiding in themselves, precede their participants according to cause; but when giving themselves up to their subordinate natures, subsist according to their diversified properties: since they are multiplied and distributed together with these, and receive the division of their subjects. Besides, this also must be previously received concerning a line, that our geometrician uses it in a threefold acceptation. As terminated on both sides, and finite; as in the problem \* which says, Upon a given terminated right line to construct an equilateral triangle. And as partly infinite and partly finite; as in the problem which commands us from three right lines, which are equal to three given right lines, to construct a triangle; for in the construction of the problem, he says, Let there be placed a certain right line, on one part finite, but on the other part infinite. And again, a line is received by Euclid as on both sides infinite; as in the problem which says, Upon a given infinite right line, from a given point, which is not in that line, to let fall a perpendicular. But, besides this, the following doubts, since they are worthy of solution, must

\* This and the following problems, are the 1st, 22d, and 12th propositions of the first book. But in the two last, instead of the word ἀπειρόν, or infinite, which is the term employed by Euclid, Mr. Simson, in his edition of the Elements, uses the word unlimited. But it is no unusual thing with this great geometrician, to alter the words of Euclid, when they convey a philosophical meaning; as we shall plainly evince in the course of these Commentaries. He certainly deserves the greatest praise for his zealous attachment to the ancient geometry: but he would (in my opinion) have deserved still more, had he been acquainted with the Greek philosophy; and fathomed the depth of Proclus; for then he would never have attempted to restore Euclid's Elements, by depriving them of some very considerable beauties.

not

not be omitted. How are points called the extremities of a line? and of what line, since they can neither be the bounds of one that is infinite, nor of every finite? For there is a certain line, which is both finite, and has not points for its extremities. And such is a circular line, which returns into itself, and is not bounded by points, like a right line. And such also is the ellipsis, or line like a shield. Is it therefore requisite to behold a line, considered as a line? for we must receive a certain circumference, which is terminated by points, and a part of the elliptic line; having, in like manner, its extremities bounded by points. But every circular and elliptic line, assumes to itself another certain property, by which it is not line alone, but is also endowed with a power of perfecting figure \*. Lines, themselves, therefore, have their extremities terminated by points; but those which are effective of such like figures, return into themselves. And, indeed, if you conceive them to be described, you will also find how they are bounded by points; but if you receive them already described, and connect the end with the beginning, you can no longer behold their extremes.

## DEFINITION IV.

A Right Line, is that which is equally situated between its *bounding Points*.

PLATO, establishing two most simple and principal species of lines, the right and the circular, composes all the rest from the mixture of these; I mean such as are called curve lines, some of which are formed from planes; but others subsist about solids; and whatever species of curve lines are produced by the sections of solids. And it seems, indeed, that a point (if it be lawful so to speak) bears an image of the one itself, according to Plato: for unity has no part, as he also shews in the Parmenides. But, because after unity itself there are three hypostases, or substances, bound, infinite, and that which is

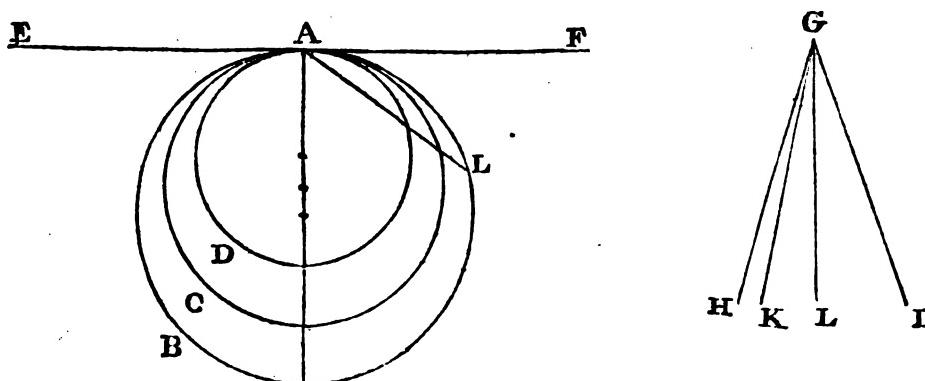
\* This is doubtless the reason why the proportion between a right and circular line, cannot be exactly obtained in numbers: for on this hypothesis, they must be incommensurable quantities; because the one contains a property essentially different from the other.

mixed

mixed from these, the species of lines, angles, and figures, which subsist in the nature of things originate from thence. And, indeed, a circumference and a circular angle, and a circle among plane figures, and a sphere among solids, are analogous to *bound*. But a right line corresponds to *infinity*, according to all these; for it properly belongs to all, if it is beheld as existing in each. But that which is mixed in all these, is analogous to the mixt which subsists among intelligibles. For lines are mixed, as those which are called spirals. And angles, as the semi-circular and cornicular \*. And plane figures, as segments and apsides; but solids, as cones and cylinders, and others of that kind. Bound, therefore, infinite, and that which is mixed,

are

\* The cornicular angle is that which is made from the periphery of a circle and its tangent;



that is, the angle comprehended by the arch LA, and the right line FA, which Euclid in (16. 3-) proves to be less than any right-lined angle. And from this admirable proposition it follows, by a legitimate consequence, that any quantity may be continually and infinitely increased, but another infinitely diminished; and yet the augment of the first, how great soever it may be, shall always be less than the decrement of the second: which Cardan demonstrates as follows. Let there be proposed an angle of contact BA E, and an acute angle HG I. Now if there be other lesser circles described AC, AD, the angle of contact will be evidently increased. And if between the right lines GH, GI, there fall other right lines GK, GL, the acute angle shall be continually diminished: yet the angle of contact, however increased, is always less than the acute angle, however diminished. Sir Isaac Newton likewise observes, in his Treatise on Fluxions, that there are angles of contact made by other curve lines, and their tangents infinitely less than those made by a circle and right line; all which is demonstrably certain: yet, such is the force of prejudice, that Mr. Simson is of opinion, with Vieta, that this part of the 16th proposition is adulterated; and that the space made by a circular line and its tangent, is no angle. At least his words, in the note upon this proposition, will bear such a construction. Peletarius was likewise of the same opinion; but is elaborately confuted by the excellent Clavius, as may be seen in his comment on this proposition. But all the difficulties and paradoxes in this affair, may be easily solved and admitted, if we consider, with our philosopher

are participated by all these. But Aristotle \* likewise assents to Plato; for every species of lines, says he, is either right or circular, or mixed from these two. From whence also there are three motions, one according to a right line; the other circular; and the third mixed. But some oppose this division, and say that there are not two simple lines alone, but that there is a certain third line given, i. e. a helix or spiral, which is described about a cylinder †, when, whilst a right line is moved round the superficies of the cylinder, a point in the line is carried along with an equal celerity. For by this means, a helix, or circumvolute line, is produced, which adapts all the parts of itself to all, according to a similitude of parts, as Apollonius shews in his book concerning the Cochlea; which passion, among all spirals, agrees to this alone. For the parts of a plane helix are dissimilar among themselves; as also of those which are described about a cone and sphere. But the cylindric spiral alone, consists of similar parts in the same manner as a right and circular line. Are there, then, three simple lines, and not two only? To which doubt we reply, that a helix of this kind is, indeed, of similar parts, as Apollonius teaches, but is by no means simple; since among natural productions, gold and silver are composed of similar parts, but are not simple bodies. But the generation of the cylindric helix evinces that its mixture is from things simple; for it originates while a right line is circularly moved round the axis of the cylinder, a point at the same time flowing along in the right line. Two simple motions, therefore, compose its nature; and, on this account, it is among the number of mixt lines, and not among such as are simple: for that which is composed from dissimilars is not simple, but mixt. Hence, Geminus, with great propriety, when he admits that some simple lines may be produced from many motions, does not grant that every such line is mixt; but that alone, which arises from dissimilar motions. For if you conceive

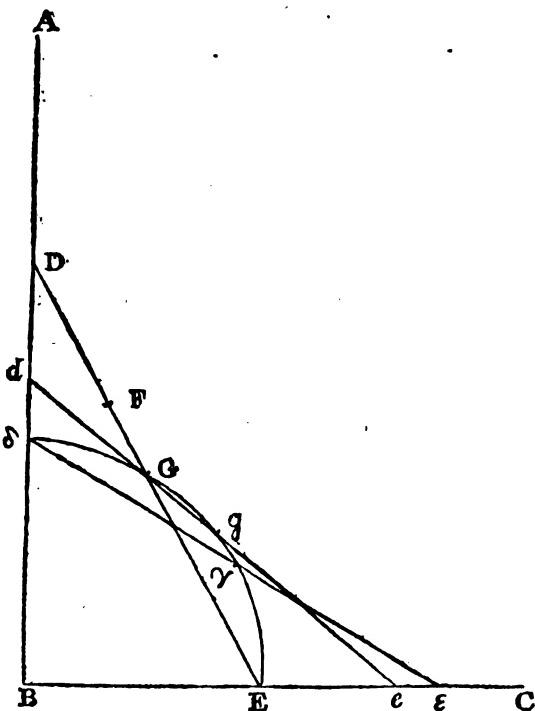
sopher, that the essence of an angle does not subsist in either quantity, quality, or inclination, taken singly, but in the aggregate of them all. For if we regard the inclination of a circular line to its tangent, we shall find it possest the property, by which Euclid defines an angle: if we respect its participation of quantity, we shall find it capable of being augmented and diminished; and if we regard it as possessing a peculiar quality, we shall account for its being incommensurable with every right-lined angle. See the Comment on the 8th Definition.

\* In i. De Cælo.

† It is from this cylindric spiral that the screw is formed.

a square, and two motions which are performed with an equal celerity, one according to the length, but the other according to the breadth; a right line or the diameter will be produced; but the right line will not, on this account, be mixed: for no other line precedes it, formed by a simple motion, as we asserted of the cylindric helix. Nor yet, if you suppose a right line, moving in a right angle, and by a bisection to describe a circle \*, is the circular line, on this account, produced with mixture: for the extremities of that which is moved after this manner, since they are equally moved, will describe a right line; and the bisection, since it is unequally devolved, will delineate a circle; but the other points will describe an ellipsis. On which account, the

\* The present very obscure passage, may be explained by the following figure. Let ABC,



be a right angle, and DE the line to be moved, which is bisected in G. Now, conceive it to be moved along the lines AB, BC, in such a manner, that the point D may always remain in AB, and the point E in BC. Then, when the line DE, is in the situations  $d\epsilon$ ,  $\delta\epsilon$ , the point G, shall be in  $g\gamma$ , and these points G,  $g\gamma$ , shall be in a circle. And any other point F in the line DE, will, at the same time, describe an ellipsis; the greater axis being in the line AB, when the point F is between D and G; and in the line BC, when the point F is between G and E.

generation

generation of a circular line is the consequence of that inequality of lation arising from the bisection ; because a right line was supposed to be moved in a right angle, but not in a natural manner. And thus much concerning the generation of lines. But it seems, that of the two simple lines, the right and the circular, the right line is the more simple; for in this, dissimilitude cannot be conceived, even in opinion. But in the circular line, the concave and the convex, indicate dissimilitude. And a right line, indeed, does not infer a circumference according to thought; but a circumference brings with it a right line, though not according to its generation, yet with respect to its centre. But what if it should be said that a circumference requires a right line to its construction ! For if either extreme of a right line remains fixt, but the other is moved, it will doubtless describe a circle, whose centre will be the abiding extreme of the right line. Shall we say that the generator of the circle is the point which is carried about the abiding point, but not the right line itself ? For the line only determines the distance, but the point composes the circular line, while it is moved in a circular manner: but of this enough. Again, a circumference appears to be proximate to bound, and to have the same proportion to other lines, as bound to the universality of things. For it is finite, and is alone among simple lines perfective of figure. But a right line is proximate to infinity ; for its capacity of infinite extension never fails : and as all the rest are produced from bound and infinite, in the same manner from the circular and right line, every mixt genus of lines is composed, as well of planes as of those which consist in solid bodies. And on this account, the soul also \* previously assumed into herself the right and circular according to her essence, that she might moderate all the co-ordination of infinite, and all the nature of bound, which the world contains. By a right line, indeed, constituting the progression of these principles into the universe ; but by a circular line, their return to their original source : and by the one, producing all things into multitude ; but by the other, collecting them into one. And not only the soul, but he also who produced the soul, and endued her with these powers, contains in himself both these primary causes.

\* That is, the soul of the world.

For when he previously assumed the beginning, middle, and end of all things, he terminated right lines (says Plato \*), by a circular progression according to nature. And proceeding to all things by provident energies, and returning to himself, he established himself, says Timæus, after his own peculiar manner. But a right line is the mark or symbol of a providence, indeclinable, incapable of perversion, immaculate, never-failing, omnipotent, and present to all beings, and to every part of the universe. But a circumference, and that which environs, is the symbol of an energy retiring into union with itself, and which rules over all things according to one intellectual bound. When, therefore, the demiurgus of the universe had established in himself these two principles, the right and the circular line, and had given them dominion, he produced from himself two unities; the one, indeed, energizing according to the circular line, and being effective of intellectual essences; but the other according to the right line, and affording an origin to sensible natures. But because the soul is allotted a middle situation between intellectuals and sensibles, so far, indeed, as she adheres to an intellectual nature, she energizes according to the circle; but so far as she presides over sensibles, she provides for their welfare according to the right line: and thus much concerning the similitude of these forms to the universality of things. But Euclid, indeed, has properly delivered the present definition of a line; by which he shews that a right line alone occupies a space equal to that which is situated between its points: for as much as is the distance of one point from another, so great is the magnitude of the lines terminated by the points. And this is the meaning of being equally situated between its extremes. For if you take two points in a circumference, or in any other certain line, the space of line which is included between these, exceeds their distance from each other; and every line, besides a right one, appears to suffer this property. Hence, according to a common conception, the vulgar also say, that he who walks by a right line, performs only a necessary journey: but that they necessarily wander much, who do not proceed in a right line. But Plato thus defines it; a right line is that whose middle parts darken its

\* In Timæo.

extremes.

extremes. For this passion necessarily attends things which have a direct position ; but it is not necessary that things situated in the circumference of a circle, or in another interval, should be endued with this property. Hence, the astrologers also say, that the sun then suffers an eclipse when that luminary, the moon, and our eye are in one right line ; for it is then darkened through the middle position of the moon between us and its orb. And perhaps, the passion of a right line will evince, that in the things which are, according to processions emanating from causes, the mediums are endued with a power of dividing the distance of the extremes, and their mutual communication with each other. As also, according to regressions, such things as are distant from the extremes, are converted by mediums to their primary causes. But Archimedes defines a right line the least of things having the same bounds. For since, according to Euclid, a right line is equally situated between its points, it is on this account, the least of things having the same bounds : for if a less line could be given, it would not lie equally between its extremes : but all the other definitions of a right line, fall into the same conclusions ; as for instance, that it is constituted in its extremities, and that one part of it is not in its subject plane, but another, in one more sublime : and that all its parts similarly agree to all : and that its extremes abiding, it also abides. Lastly, that it does not perfect figure, with one line similar in species to itself : for all these definitions express the property of a right line, which it possesses from the simplicity of its essence, and from its having one progression the shortest of all from one extremity to another. And thus much concerning the definitions of a right line. But again, Geminus divides a line first into an incomposite and composite ; calling a composite, that which is refracted, and forms an angle ; but all the rest of them, he denominates incomposites. Afterwards, he divides a composite line into that which produces figure, and that which may be infinitely extended. And he calls that which produces figure, a circular line, and the line of a shield \*, and that which is similar to an ivy leaf † ; but that which is not effective of figure, the section of a rectangular and obtuse angular cone, the line similar to a

\* The ellipse.

† The cissoid. For the properties of this curve, see Dr. Wallis's treatise on the cycloid, p. 81.

shell \*, the right line, and all of that kind. And again, after another manner, of the incomposite line, one sort is simple, but the other mixt. And of the simple, one produces figure, as the circular; but the other is indefinite, as the right line. But of the mixt, one subsists in planes, but the other in solids. And of that which is in planes, one coincides in itself, as the figure of the ivy leaf, which is called the *eisloid*; but the other may be produced in infinitum, as the helix. But of that which is in solids, one may be considered in the sections of solids; but the other as consisting about the solids themselves. For the helix, indeed, which is described about a sphere or a cone, consists about solids; but conic, or spirical sections are generated from a particular section of solids. But, with respect to these sections, the conic were invented by Mænechmus, which also Eratosthenes relating, says,

“ Nor in a cone Mænechmian ternaries divide.”

But the spiric by Perseus, who composed an epigram on their invention, to this purpose, “ When Perseus had invented three spiral lines in five sections, he sacrificed to the gods on the occasion.” And the three sections of a cone, are the parabola, hyperbola, and ellipsis: but of spiral sections, one kind is twisted and involved, like the fetlock of a horse; but another is dilated in the middle, and deficient in each extremity: and another which is oblong, has less space in the middle, but is dilated on each side. But the multitude of the other mixt lines is infinite. For there is an innumerable multitude of solid figures, from which there are constituted multiform sections. For a right line, while it is circularly moved †, does not make a certain determinate superficies, nor yet conical, nor conchoidal lines, nor circumferences themselves. Hence, if these solids are multifariously cut, they will exhibit various species of lines. Lastly, of those lines which consist about solids, some are of similar parts, as the helixes

\* The conchoid.

† Thus, a right line, when considered as the side of a parallelogram, moving circularly, generates a cylindrical superficies: when moving circularly, as the side of a triangle, a conical surface; and so in other lines, the produced superficies varying according to the different positions of their generative lines.

about

about a cylinder; but others of dissimilar parts, as all the rest. From these divisions, therefore, we may collect, that there are only three lines of similar parts, the right, the circular, and the cylindric helix. The two simple ones, indeed, existing in a plane, but the one mixt, about a solid. And this Geminus evidently demonstrates, when he shews, that if two right lines are extended from one point, to a line of similar parts, so as to make equal angles upon that line, they shall be equal to each other. And the demonstrations of this may be received by the studious, from his volumes; since in these he delivers the origin of spiral, conchoidal, and cissoidal lines. But we have barely related the names and divisions of these lines, for the purpose of exciting the ingenious to their investigation; as we think, that an accurate enquiry after the method of detecting the properties of each, would be superfluous in the present undertaking: since the geometrician only unfolds to us in this work, simple and primary lines; i. e. the right line, in the present definition; but a circular line, in the tradition of a circle. For he then says, that the line terminating the circle, is the circumference. But he makes no mention of mixt lines, though he was well acquainted with mixt angles, I mean, the hemicircular and cornicular: as also with plane mixt figures, i. e. segments and sectors; and with solids, viz. cones and cylinders. Of each of the rest, therefore, he delivers three species; but of lines only two; i. e. the right and circular: for he thought it requisite in discourses concerning things simple, to assume simple species; and all the rest are more composite than lines. Hence, in imitation of the geometrician, we also shall terminate their explanation with simple lines.

## DEFINITION V.

A SUPERFICIES is that which has only Length and Breadth.

**A**FTER a point and a line, a superficies is placed, which is distant by a twofold interval, length and breadth. But this also remaining destitute of thickness or bulk, possesses a nature more simple than body, which is distant by a triple dimension. On which account the geometrician adds to the two intervals the particle *only*,  
because

because the third interval does not exist in superficies. And this is equivalent to a negation of bulk, as here also he shews the excellency of superficies compared to a solid with respect to simplicity, by negation, or by an addition equivalent to negation : but the diminution which it possesses, if compared with the preceding terms, by the affirmations themselves. But others define a superficies to be the boundary of body, which is almost affirming the same as the definition of Euclid; since that which terminates is exceeded in one dimension, by that which is terminated. And others, a magnitude different by two intervals. Lastly, others declaring the same affection, form its assignation in a somewhat different manner. But they say we have a knowledge of superficies when we measure fields, and distinguish their extremities according to length and breadth ; but that we receive a certain sensation of it, when we behold shadows. For as they are without bulk, because they cannot penetrate into the interior part of the earth, they have only length and breadth. But the Pythagoreans say, that it is assimilated to the triad; because the ternary is by far the first cause to all the figures; which a superficies contains. For a circle, which is the principle of orbicular figures, occultly possesses the ternary, by its centre, interval, and circumference. But a triangle, which ranks as the first among all right-lined figures, on every side evinces that it is enclosed by the triad, and receives its form from its perfect nature.

## DEFINITION VI.

The Extremities of a SUPERFICIES are Lines.

**F**ROM these also, as images, we may understand, that things more simple procure bound and an end to every one of their proximate natures: for soul perfects and determines the operations of nature ; and nature the motion of bodies. And prior to these, intellect measures the convolutions of soul; and unity the life of intellect; for that is the measure of all. Just as in these also, a solid is terminated by a superficies ; but a superficies by a line; and a line by a point ; for that is the boundary of them all. Hence, the line existing

ing uniformly in immaterial forms and imitable reasons, terminates and restrains the various motion of a superficies in its progression, and proximately unites its infinity. But in the images of these, when that which bounds supervenes that which is bounded, it causes, by this means, its limitation and bound. But if it should be enquired how lines are the extremities of every superficies, since they are not the extremes of every finite figure; for the superficies of a sphere is terminated indeed, yet not by lines, but by itself? In answer to this, we must say, that by receiving a superficies so far as it is distant by a two-fold interval, we shall find it terminated by lines according to length and breadth. But if we behold a spherical superficies, we must receive it as that which is endued with figure; which possesses another quality, and conjoins the end with the beginning; and loses its two extremities in the comprehensive embraces of one: and this one extremity subsists in capacity only, and not in energy.

## DEFINITION VII.

A Plane SUPERFICIES is that which is equally situated between its bounding Lines.

**I**T was not agreeable to the ancient philosophers to establish a plane species of superficies; but they considered superficies in general, as the representative of magnitude, which is distant by a two-fold interval. For thus the divine Plato \* says, that geometry is contemplative of planes, opposing it in division to stereometry, as if a plane and a superficies were the same. And this was likewise the opinion of the demoniacal Aristotle †. But Euclid and his followers consider superficies as a genus, but a plane as its species, in the same manner as rectitude of a line. And on this account he defines a plane separate from a superficies, after the similitude of a right line. For he defines this last as equal to the space, placed between its points. And in like manner, he says, that two right lines being given, a plane superficies occupies a place equal to the space situated between those two lines.

\* Inv ii. De Rep.

† In multis locis.

For this is equally situated between its lines ; and others also explaining the same boundary, assert that it is constituted in its extremities. But others define it as that to all the parts of which a right line may be adapted \*. But perhaps others will say, that it is the shortest of superficies, having the same boundaries ; and that its middle parts darken its extremities ; and that all the definitions of a right line may be transferred into a plane superficies, by only changing the genus : since a right, circular, and mixt line, commencing from lines, arrive even at solids, as we have asserted above ; for they are proportionally, both in superficies and solids. Hence also, Parmenides says, that every figure is either right, or circular, or mixt. But if you wish to consider the right in superficies, take a plane, to which a right line agrees in various ways ; but if a circular receive a spherical superficies ; and if a mixt, a conic or cylindric, or some one of that genus. But it is requisite (says Geminus) since a line, and also a superficies is called mixt, to know the measure of mixture, because it is various. For mixture in lines, is neither by composition, nor by temperament only : since, indeed, a helix is mixed, yet one part of it is not straight, and another part circular, like those things which are mixed by composition : nor if a helix is cut after any manner, does it exhibit an image of things simple, such as those which are mixed through temperament ; but in these the extremes are, at the same time, corrupted and confused. Hence, Theodorus the mathematician, does not rightly perceive, in thinking that this mixture is in lines. But mixture in superficies, is neither by composition, nor by confusion ; but subsists rather by a certain temperament. For conceiving a circle in a subject plane, and a point on high, and producing a right-line from the point to the circumference of the circle, the revolution of this line will produce a conical superficies which is mixt. And we again resolve it into its simple elements, by a parallel section : for by drawing

\* This definition is the same with that which Mr. Simson has adopted instead of Euclid's, expressed in different words : for he says, "a plane superficies is that in which any two points being taken, the straight line between them lies wholly in that superficies." But he does not mention to whom he was indebted for the definition ; and this, doubtless, because he considered it was not worth while to relate the trifles of Proclus at full length : for these are his own words, in his note to proposition 7, book i. Nor has he informed us in what respect Euclid's definition is insufficient.

a section between the vertex and the base, which shall cut the plane of the generative right line, we effect a circular line. But the idea of lines, shews that the mode of mixture is not by temperament; for neither does it send us back to the simple nature of elements: on the contrary, when superficies are cut, they immediately exhibit to us their producing lines. The mode of mixture, therefore, is not the same in lines and superficies. But as among lines there were some simple; that is, the right and circular, of which the vulgar also posses an anticipated knowledge without any previous instruction; but the species of mixt lines require a more artificial apprehension: so among superficies, we posses an innate notion of those which are especially elementary, the plane and spherical; but science and its reason investigates the variety of those which are composed through mixture. But this is an admirable property of superficies, that their mixture in generation is oftentimes produced from a circular line; and this also happens to a spiral superficies. For this is understood by the revolution of a circle remaining erect, and turning itself about the same point which is not its centre. And on this account, a spiral also is threefold; for its centre is either in a circumference, or within, or external to a circumference. If the centre is in the circumference, a continued spiral is produced: if within the circumference, an intangled one; if without, a divided one. And there are three spiral sections corresponding to these three differences. But every spiral line is mixt, although the motion from which it is produced is one and circular. And mixt superficies are produced as well from simple lines, (as we have said,) while they are moved with a motion of this kind, as from mixt lines. Since, therefore, there are three conic lines, they produce four mixt superficies, which they call conoids. For a rectangular conoid, is produced from the revolution of the parabola about its axis: but that which is formed by the ellipsis, is called a spheroid; and if the revolution is made about the greater axis, it is an oblong; but if about the lesser a broad spheroid. Lastly, an obtuse-angled conoid is generated from the revolution of the hyperbola. But it is requisite to know, that sometimes we arrive at the knowledge of superficies from lines, and sometimes the contrary; for from conical and spiral superficies, we apprehend conical and spiral lines. Besides,

sides, this also must be previously received concerning the difference of lines and superficies, that there are three lines of similar parts (as we have already observed), but only two superficies, the plane and the spherical. For this is not true of the cylindric, since all parts of the cylindric superficies cannot agree to all. And thus much concerning the differences of superficies, one of which the geometrician having chosen (I mean the plane), this also he has defined; and in this, as a subject, he contemplates figures, and their attendant passions: for his discourse is more copious in this than in other superficies: since, indeed, we may understand right lines, and circles, and helixes in a plane; also the sections of circles and right lines, contacts, and applications, and the constructions of angles of every kind. But in other superficies, all these cannot be beheld. For how in one that is spherical, can we apprehend a right line, or a right-lined angle? How, lastly, in a conic or cylindric superficies, can we behold sections of circles or right lines? Not undeservedly, therefore, does he both define this superficies, and discuss his geometrical concerns, by exhibiting every thing in this as in a subject; for from hence he calls the present treatise plane. And, after this manner, it is requisite to understand that which is plane, as projected and constituted before the eyes: but cogitation as describing all things in this, the phantasy corresponding to a plane mirror, and the reasons resident in cogitation as dropping their images.\* into its shadowy receptacle.

## DEFINITION VIII.

† A PLANE ANGLE, is the inclination of two Lines to each other in a Plane, which meet together, but are not in the same direction.

SOME of the ancient philosophers, placing an angle in the predicament of relation, have said, that it is the mutual inclination of lines or planes to each other. But others, including this in quality,  
as

\* In the Greek *εικός*, but it should doubtless be read *εικών*, *images*, as in the translation of Barocius.

† Mr. Simson, in his note on this definition, supposes it to be the addition of some less skillful editor; on which account, and because it is quite useless (in his opinion) he distinguishes it

as well as rectitude and obliquity, say, that it is a certain passion of a superficies or a solid. And others, referring it to quantity, confess that it is a superficies or a solid. For the angle which subsists in superficies is divided by a line ; but that which is in solids, by a superficies. But (say they) that which is divided by these, is no other than magnitude, and this is not linear, since a line is divided by a point ; and therefore it follows that it must be either a superficies or a solid. But if it is magnitude, and all finite magnitudes of the same kind have a mutual proportion ; all angles of the same kind, i. e. which subsist in superficies, will have a mutual proportion. And hence, the cornicular will be proportionable to a right-lined angle. But things which have a mutual proportion, may, by multiplication, exceed each other ; and therefore it may be possible for the cornicular to exceed a right-lined angle, which, it is well known, is impossible, since it is shewn to be less than every right-lined angle. But if it is quality alone, like heat and cold, how is it divisible into equal parts ? For equality, inequality, and divisibility, are not less resident in angles than in magnitudes ; but they are, in like manner, essential. But if the things in which these are essentially inherent, are quantities, and not qualities, it is manifest that angles also are not qualities. Since the more and the less are the proper passions of quality \*, but not equal and unequal. On this hypothesis, therefore, angles ought not to be called unequal, and this greater, but the other less ; but they ought to be denominated dissimilars, and one more an angle, but the other less. But that these appellations are foreign from the essence of mathematical concerns, is obvious to every one : for every angle receives the same definition, nor is this more an angle, but that less. Thirdly, if an angle is inclination, and belongs to the category of

it from the rest by inverted double commas. But it is surely strange that the definition of angle in general should be accounted useless, and the work of an unskilful geometrician. Such an assertion may, indeed, be very suitable to a professor of experimental philosophy, who considers the useful as inseparable from practice ; but is by no means becoming a restorer of the liberal geometry of the ancients. Besides, Mr. Simson seems continually to forget that Euclid was of the Platonic sect ; and consequently was a philosopher as well as a mathematician. I only add, that the commentary on the present definition is, in my opinion, remarkably subtle and accurate, and well deserves the profound attention of the greatest geometers.

\* For a philosophical discussion of the nature of quality and quantity, consult the Commentaries of Ammonius, and Simplicius on Aristotle's Categories, Plotinus on the genera of beings, and Mr. Harris's Philosophical Arrangements.

relation,

relation, it must follow, that from the existence of one inclination, there will also be one angle, and not more than one. For if it is nothing else than the relation of lines or planes, how is it possible there can be one relation of lines or planes, but many angles? If, therefore, we conceive a cone cut by a triangle from the vertex to the base, we shall behold one inclination of the triangular lines in the semicone to the vertex; but two distinct angles: one of which is plane, I mean that of the triangle; but the other subsists in the mixt superficies of the cone, and both are comprehended by the two triangular lines. The relation, therefore, of these, do not make the angle. Again, it is necessary to call an angle either quality or quantity, or relation; for figures, indeed, are qualities, but their mutual proportions belong to relation. It is necessary, therefore, that an angle should be reduced under one of these three genera. Such doubts, then, arising concerning an angle, and Euclid calling it inclination, but Apollonius the collection of a superficies, or a solid in one point, under a refracted line or superficies (for he seems to define every angle universally), we shall affirm, agreeable to the sentiments of our preceptor Syrianus, that an angle is of itself none of the aforesaid; but is constituted from the concurrence of them all. And that, on this account, a doubt arises among those who regard one category alone. But this is not peculiar to an angle, but is likewise the property of a triangle. For this, too, participates of quantity, and is called equal and unequal; because it has to quantity the proportion of matter. But quality also, is present with this, in consequence of its figure (since triangles are called as well similar as equal); but it possesses this from one category, and that from another. Hence, an angle is perfectly indigent of quantity, the subject of magnitude. But it is also indigent of quality, by which it possesses, as it were, its proper form and figure. Lastly, it is indigent of the relation of lines terminating, or of superficies comprehending its form. So that an angle consists from all these, yet is not any one of them in particular. And it is indeed divisible, and capable of receiving equality and inequality, according to the quantity which it contains. But it is not compelled to admit the proportion of magnitudes of the same kind, since it has also a peculiar quantity, by which angles are also incapable of a comparison with each other. Nor

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can one inclination perfect one angle: since the quantity also, which is placed between the inclined lines, completes its essence. If then we regard these distinctions, we shall dissolve all absurdities, and discover that the property of an angle is not the collection of a superficies or solid, according to Apollonius (since these also complete its essence,) but that it is nothing else than a superficies itself, collected into one point, and comprehended by inclined lines, or by one line inclined to itself: and that a solid angle is the collection of superficies mutually inclined to each other. Hence, we shall find that a formed quantum, constituted in a certain relation, supplies its perfect definition. And thus much we have thought requisite to assert concerning the substance of angles, previously contemplating the common essence of every triangle, before we divide it into species. But since there are three opinions of an angle, Eudemus the Peripatetic, who composed a book concerning an angle, affirms that it is quality. For, considering the origin of an angle, he says that it is nothing else than the fraction of lines: because, if rectitude is quality, fraction also will be quality. And hence, since its generation is in quality, an angle will be entirely quality. But Euclid, and those who call it inclination, place it in the category of relation. But they call it quantity, who say that it is the first interval under a point, that is immediately subsisting after a point. In the number of which is Plutarch, who constrains Apollonius also into the same opinion. For it is requisite (says he) there should be some first interval, under the inclination of containing lines or superficies. But since the interval, which is under a point, is continuous, it is not possible that a first interval can be assumed; since every interval is divisible in infinitum. Besides, if we any how distinguish a first interval, and through it draw a right line, a triangle is produced, and not one angle. But Carpus Antiochenus says, that an angle is quantity, and is the distance of its comprehending lines, or superficies; and that this is distant by one interval, and yet an angle is not on that account a line: since it is not true that every thing which is distant by only one interval, is a line. But this surely is the most absurd of all, that there should be any magnitude except a line, which is distant only by one interval. And thus much concerning the nature of an angle. But with respect to the division of

angles,

angles, some consist in superficies, but others in solids. And of those which are in superficies, some are in simple ones, but others in such as are mixt. For an angle may be produced in a cylindric, conic, spherical, and plane superficies. But of those which consist in simple superficies, some are constituted in the spherical; but others in the plane. For the zodiac itself forms angles, dividing the equinoctial in two parts, at the vertex of the cutting superficies. And angles of this kind subsist in a spherical superficies. But of those which are in planes, some are comprehended by simple lines, others by mixt ones; and others, again, by both. For in the shield-like figure \*, an angle is comprehended by the axis, and the line of the shield: but one of these lines is mixt, and the other simple. But if a circle cuts the shield, the angle will be comprehended by the circumference, and the ellipsis. And when cissoids, or lines similar to an ivy leaf, closing in one point like the leaves of ivy (from whence they derive their appellation) make an angle, such an angle is comprehended by mixt lines. Also, when the hippopeda, or line similar to the foot of a mare, which is one of the spirals, inclining to another line, forms an angle, it is comprehended by mixt lines. Lastly, the angles contained by a circumference and a right line, are comprehended by simple lines. But of these again, some are contained by such as are similar in species, but others by such as are dissimilar. For two circumferences, mutually cutting, or touching each other, produce angles: and these triple, for they are either on both sides convex, when the convexities of the circumferences are external: or on both sides concave, when both the concavities are external; which they call fistroides; or mixt from convex and concave lines, as the lines called lunulas. But besides this, angles are contained in a twofold manner, by a right line and a circumference: for they are either contained by a right line, and a concave circumference, as the semicircular angle; or by a right line and a convex circumference, as the cornicular angle. But all those which are comprehended by two right lines, are called rectilinear angles, which have likewise a triple difference †. The geometrician, therefore, in the present hypothesis, defines all those angles which are constituted in plane superficies, and gives them the common name of

\* That is, the ellipsis.]

† That is, they are either right, acute, or obtuse.

a plane

a plane angle. And the genus of these he denominates inclination: but the place, the plane itself, for angles have position: but their origin such, that it is requisite there should be two lines at least, and not three as in a solid. And that these should touch each other, and by touching, must not lie in a right line, as an angle is the inclination and comprehension of lines: but is not distance only, according to one interval. But if we examine this definition, in the first place it appears that it does not admit, an angle can be perfected by one line; though a cissoid, which is but one, perfects an angle.. And, in like manner, the hippopeda. For we call the whole a cissoid, and not its portions (lest any one should say, that the conjunction of these forms an angle) and the whole a spiral, but not its parts. Each, therefore, since it is one, forms an angle to itself, and not to another. But after this, he is faulty, in defining an angle to be inclination. For how, on this hypothesis, will there be two angles, from one inclination? How can we call angles equal and unequal? And whatever else is usually objected against this opinion. Thirdly, and lastly, that part of the definition, which says, *and not placed in a right line*, is superfluous in certain angles, as in those which are formed from orbicular lines. For without the assistance of this part, the definition is perfect; since the inclination of one of the lines to the other, forms the angle. And it is not possible that orbicular angles should be placed in a right line. And thus much we have thought proper to say concerning the definition of Euclid; partly, indeed, interpreting, and partly doubting its truth.

## DEFINITION IX.

But when the Lines containing the Angle, are right, the Angle is called RECTILINEAR.

**A**N angle is the symbol and image of the connection and compression, which subsists in the divine genera, and of that order which collects divisibles into one, partibles into an imparible nature, and the many into conciliating community. For it is the bond of a multitude of lines and superficies, the collector of magnitude into the

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imparibilty

impartibility of points, and the comprehender of every figure which is composed by its confining nature. On which account, the oracles\* call the angular junctions of figures, knots, so far as they bring with them an image of connecting union, and divine conjunctions, by which discrete natures mutually cohere with each other. The angles, therefore, subsisting in superficies, express the more immaterial, simple, and perfect unions which superficies contain : but those which are in solids, represent the unions, which proceed even to inferiors, and supply a community to things disjunct, and a construction of the same nature, to things which on every side receive a perfect partition. But of the angles in superficies, some shadow forth primary and unmixed unions; but others, such as comprehend in themselves, an infinity of progressions. And some, indeed, are the sources of union to intellectual forms; but others, to sensible reasons; and others, again, are copulative of those forms which obtain between these, a middle situation. Hence, the angles which are made from circumferences, imitate those causes which envelop intellectual variety in coercive union; for circumferences, hastening to coalesce with each other, are images of intellect, and intellectual forms. On the contrary, rectilineal angles, are the symbols of those unions which preside over sensibles, and afford a conjunction of the reasons subsisting in these: but mixt angles represent the preservers of the communion, as well of sensible, as of intellectual forms, according to one immovable union. It is requisite, therefore, by regarding these paradigms, or exemplars, to render the causes of each. For among the Pythagoreans we shall find various angles dedicated to various gods. Thus, Philolaus, consecrates to some a triangular, but to others a quadrangular angle; and to others, again, different angles. Likewise, he permits the same to many gods, and many to the same god, according to the different powers which they contain. And with a view to this, and to the demiurgic triangle, which is the primary cause of all the ornament of the elements, it appears to me, that Theodorus Asinæus the philosopher, constitutes some of the gods, according to sides; but others, according to angles. The first, indeed, supplying progression and power; but the second, the conjunction of the universe, and the collection of progressive na-

\* This oracle is not mentioned by any of the collectors of the Zoroastrian oracles.

tures

tarea again into one. But these, indeed, direct us to the knowledge of the things which are. And we must not wonder that lines are here said to contain an angle. For the one and impenetrable nature which is found in these, is adventitious: but in the gods themselves, and in true beings, the whole, and impenetrable good, precedes things many, and divided.

## DEFINITION X.

When a Right Line standing on a Right Line, makes the successive Angles on each side equal to one another, each of the equal Angles, is a Right Angle; and the insisting Right Line, is called a PERPENDICULAR to that upon which it stands.

## DEFINITION XI.

An OBTUSE ANGLE is that which is greater than a RIGHT ANGLE.

## DEFINITION XII.

But an ACUTE ANGLE, is that which is less than a RIGHT ANGLE.

THESE are the triple species of angles, which Socrates speaks of in the Republic, and which are received by geometers from hypothesis; a right-line constituting these angles, according to a division into species; I mean, the right, the obtuse, and the acute. The first of these being defined by equality, identity and similitude; but the others being composed through the nature of the greater and the lesser; and lastly, through inequality and diversity, and through the more and the less, indeterminately assumed. But many geometers, are unable to render a reason of this division, and use the assertion, that there are three angles, as an hypothesis\*. So that,

\* This, indeed, must always be the case with those geometers, who are not at the same time, philosophers; a conjunction no less valuable than rare. Hence, from their ignorance of principles and intellectual concerns, when any contemplative enquiry is proposed, they immediately ask, in what its utility consists; considering every thing as superfluous, which does not contribute to the solution of some practical problem.

when we interrogate them concerning its cause, they answer, this is not to be required of them as geometers. However, the Pythagoreans, referring the solution of this triple distribution to principles, are not wanting in rendering the causes of this difference of right-lined angles. For, since one of the principles subsists according to bound, and is the cause of limitation, identity, and equality, and lastly, of the whole of a better co-ordination: but the other is of an infinite nature, and confers on its progeny, a progression to infinity, increase, and decrease, inequality, and diversity of every kind, and entirely presides over the worse series; hence, with great propriety, since the principles of a right-lined angle are constituted by these, the reason proceeding from bound, produces a right angle, one, with respect to the equality of every right angle, endued with similitude, always finite and determinate, ever abiding the same, and neither receiving increment nor decrease. But the reason proceeding from infinity, since it is the second in order, and of a dyadic nature, produces two-fold angles about the right angle, distinguished by inequality, according to the nature of the greater and the less, and possessing an infinite motion, according to the more and the less, since the one becomes more or less obtuse; but the other more or less acute. Hence, in consequence of this reason, they ascribe right angles to the pure and immaculate gods of the divine ornaments, and divine powers which proceed into the universe, as the authors of the invariable providence of inferiors; for rectitude, and an inflexibility and immutability to subordinate natures accords with these gods: but they affirm, that the obtuse and acute angles should be ascribed to the gods, who afford progression, and motion, and a variety of powers. Since obtuseness is the image of an expanded progression of forms; but acuteness possesses a similitude to the cause dividing and moving the universe. But likewise, among the things which are, rectitude is, indeed, similar to essence, preserving the same bound of its being; but the obtuse and acute, shadow forth the nature of accidents. For these receive the more and the less, and are indefinitely changed without ceasing. Hence, with great propriety, they exhort the soul to make her descent into generation, according to this invariable species of the right angle, by not verging to this part more than to that; and by not affording

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some things more, and others less. For the distribution of a certain convenience and sympathy of nature, draws it down into material error, and indefinite variety\*. A perpendicular line is, therefore, the symbol of inflexibility, purity, immaculate, and invariable power, and every thing of this kind. But it is likewise the symbol of divine and intellectual measure: since we measure the altitudes of figures by a perpendicular, and define other rectilineal angles by their relation to a right angle, as by themselves they are indefinite and indeterminate. For they are beheld subsisting in excess and defect, each of which is, by itself, indefinite. Hence they say, that virtue also stands according to rectitude; but that vice subsists according to the infinity of the obtuse and acute, that it produces excesses and defects, and that the more and the less exhibit its immoderation, and inordinate nature. Of rectilineal angles, therefore, we must establish the right angle, as the image of perfection, and invariable energy, of limitation, intellectual bound, and the like; but the obtuse and acute, as shadowing forth infinite motion, unceasing progression, division, partition and infinity. And thus much for the theological speculation of angles. But here we must take notice, that the genus is to be added to the definitions of an obtuse and acute angle; for each is right-lined, and the one is greater, but the other less than a right-angle. But it is not absolutely true, that every angle which is less than a right one, is acute. For the cornicular is less than every right-angle, because less than an acute one, yet is not on this account an acute angle. Also, a semi-circular is less than any right-angle, yet is not acute. And the cause of this property is because they are mixt, and not rectilineal angles. Besides, many curve-lined angles appear greater than right-lined angles, yet are not on this account obtuse; because it is requisite that an obtuse should be a right lined angle. Secondly, as it was the intention of Euclid, to define a right-angle, he considers a right-line standing upon another right-line, and making the angles on each side equal. But he defines an obtuse and acute angle, not from the inclination of a right line to either part, but from their relation to a right-

\* Concerning the soul's descent into body, see lib. ix. Ennead iv. of Plotinus; and for the method by which she may again return to her pristine felicity, study the first book of Porphyry's Treatise on Abstinence.

angle.

angle. For this is the measure of angles deviating from the right, in the same manner as equality of things unequal. But lines inclined to either part, are innumerable, and not one alone, like a perpendicular. But after this, when he says, (*the angles equal to one another*) he exhibits to us a specimen of the greatest geometrical diligence; since it is possible that angles may be equal to others, without being right. But when they are equal to one another, it is necessary they should be right. Besides, the word *successive* appears to me not to be added superfluously, as some have improperly considered it; since it exhibits the reason of rectitude. For it is on this account that each of the angles is right; because, when they are *successive*, they are equal. And, indeed, the insisting right-line, on account of its inflexibility to either part, is the cause of equality to both, and of rectitude to each. The cause, therefore, of the rectitude of angles, is not absolutely mutual equality, but position in a consequent order, together with equality. But, besides all this, I think it here necessary to call to mind, the purpose of our author; I mean, that he discourses in this place, concerning the angles consisting in one plane. And hence, this definition is not of every perpendicular; but of that which is in one and the same plane. For it is not his present design to define a solid angle. As, therefore, he defines, in this place, a plane angle, so likewise a perpendicular of this kind. Because a solid perpendicular ought not to make right angles to one right-line only; but to all which touch it, and are contained in its subject plane: for this is its necessary peculiarity.

## DEFINITION XIII.

A BOUND is that which is the Extremity of any thing \*.

**A** Bound, in this place, is not to be referred to all magnitudes, for there is a bound and extremity of a line; but to the spaces which are contained in superficies, and to solid bodies. For he now calls a bound, the ambit which terminates and distinguishes every

\* This Definition too, is marked by Mr. Simson with inverted commas, as a symbol of its being interpolated. But for what reason I know not, unless because it is useless, that is, because it is *philosophical!*

space.

Space. And a bound of this kind, he defines to be an extremity: but not after the manner in which a point is called the extremity of a line, but according to its property of including and excluding from circumjacent figures. But this name is proper to geometry in its infant state, by which they measured fields, and preserved their boundaries distinct and without confusion, and from which they arrived at the knowledge of the present science. Since, therefore, Euclid calls the external ambit, a bound, it is not without propriety that he, by this means, defines the extremity of spaces. For by this, every thing comprehended is circumscribed. I say, for example, in a circle, its bound and extremity is the circumference; but itself, a certain plane space: and so of the rest.

## DEFINITION XIV.

A FIGURE is that which is comprehended by one or more Boundaries.

**B**ECAUSE figure is predicated in various ways, and is divided into different species, it is requisite, in the first place, to behold its differences; and afterwards to discourse concerning that figure which is proposed in this Definition. There is, then, a certain figure which is constituted by mutation, and is produced from passion, while the recipients of the figure are disturbed, divided, or taken away; while they receive additions, or are altered, or suffer other various affections. There is also a figure, which is produced by the potter's, or statuary's art, according to the pre-existent reason, which art itself contains: art, indeed, producing the form, but matter receiving from thence, form, and beauty, and elegance. But there are still more noble and more illustrious figures than these, the skilful operations of nature. Some, indeed, existing in the elements under the moon \*, and having a power of comprehending the reasons those elements contain: but others are situated in the celestial regions, distinguishing their powers, and endless revolutions. For the heavenly bodies, both when considered by themselves, and with relation to each other, exhibit an abundant and admirable variety of figures; and at different times they present to our view different forms, bringing with them a splendid image of intel-

\* That is, the various species of forms, with which the four elements are replete.

tral species; and, by their elegant and harmonious revolutions, describing the incorporeal and immaterial powers of figures. But there are, again, besides all these, most pure and perfect beauties, the figures of souls, which, because they are full of life, and self-motive, have an existence prior to things moved by another; and which, because they subsist immaterially, and without any dimension, excel the forms which are endued with dimension and matter. In the nature of which we are instructed by Timæus, who has explained to us the demiurgic, and essential figure of souls \*. But again, the figures of intellects are by far more divine than the figures of souls; for these, on every side, excel partible essences; are every where resplendent with impartible and intellectual light; are prolific, effective, and perfective of the universe; are equally present, and firmly abide in all things; and procure union to the figures of souls; but recall the mutation of sensible figures to the limitation of their proper bound. Lastly, there are, separate from all these, those perfect, uniform, unknown, and ineffable figures of the gods, which are resident, indeed, in the figures of intellects; but jointly terminate all figures, and comprehend all things in their unifying boundaries. The properties of which the theurgic art, also expressing, surrounds various resemblances of the gods, with various figures. And some, indeed, it fashions by characters, in an ineffable manner; for characters of this kind, manifest the unknown powers of the gods: but others it imitates by forms and images; fashioning some of them erect, and others sitting; and some similar to a heart, but others spherical, and others expressed by different figures. And again, some it fabricates of a simple form; but others it composes from a multitude of forms; and some are sacred and venerable; but others are domestic, exhibiting the peculiar gentleness of the gods. And some it constructs of a severe aspect; and lastly, attributes to others, different symbols, according to the similitude and sympathy pertaining to the gods †. Since, therefore, figure derives

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\* That is, the circle.

† An admirer of the moderns, and their pursuits, will doubtless consider all this as the relics of heathen superstition and ignorance; and will think, perhaps, he makes a great concession in admitting the existence of one supreme god, without acknowledging a multitude of deities subordinate to the first. For what the ancients can urge in defence of this obsolete opinion, I must beg leave to refer the reader to the dissertation prefixed to my translation of Orpheus; in addition to which let him attend to the following considerations. Is it possible

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its origin from the gods themselves, it arrives, by a gradual progression, even to inferiors, in these also appearing from primary causes. Since it is requisite to suppose the perfect before the imperfect, and things situated in the stability of their own essence, prior to those which subsist in others, and previous to things full of their own privation, such as preserve their proper nature sincere. Such figures, therefore, as are material, participate of material inelegance, and do not possess a purity convenient to their nature. But the celestial figures are divisible, and subsist in others. And the figures of souls are endued with division, and variety, and involution of every kind; but the figures of intellects, together with immaterial union, possess a progression into multitude. And lastly, the figures of the gods are free, uniform, simple, and generative; they subsist before all things, containing all perfection in themselves, and extending from themselves to all things, the completion of forms. We must not, therefore, listen to, and endure the opinions of many, who affirm, that certain additions, ablutions, and alterations, produce sensible figures, (for motions, since they are imperfect, cannot possess the principle and primary cause of effects; nor could the same figures often be produced from contrary motions; for the same form is sometimes generated from addition and detraction,) but we must consider operations of this kind as subservient to other purposes in generation, and derive the perfection.

that the machinery of the gods in Homer could be so beautiful, if such beings had no existence? Or can any thing be beautiful which is destitute of all reality? Do not things universally please in proportion as they resemble reality? Perhaps it will be answered, that the reverse of this is true, and that fiction more generally pleases than truth, as is evident from the great avidity with which romances are perused. To this I reply, that fiction itself ceases to be pleasing, when it supposes absolute impossibilities: for the existence of genii and fairies cannot be proved impossible; and these compose all the marvellous of romance. This observation is verified in Spencer's Fairy Queen: for his allegories, in which the passions are personified, are tedious and unpleasant, because they are not disguised under the appearance of reality: while the magic of Circe, the bower of Calypso, the rocks of Scylla and Charybdis, and the melody of the Syrens, in the Odyssey of Homer, though nothing but allegories, universally enchant and delight, because they are covered with the semblance of truth. It is on this account that Milton's battles in heaven are barbarous and ridiculous in the extreme; for every one sees the impossibility of supposing gun-powder and cannons in the celestial regions: the machinery is forced and unnatural, contains no elegance of fancy, and is not replete with any mystical information. On the contrary, Homer's machinery is natural and possible, is full of dignity and elegance, and is pregnant with the sublimest truths; it delights and ennobles the mind of the reader, astonishes him with its magnificence and propriety, and animates him with the fury of poetic inspiration. And this, because it is possible and true.

of figure from other primogenial causes. Nor must we subscribe to their opinion, who assert that figures destitute of matter can have no subsistence, but those only which appear in matter. Nor to theirs, who acknowledge, indeed, that they are external to matter, but consider them as subsisting alone, according to thought and abstraction. For where shall we preserve in safety, the certainty, beauty, and order of figures, among things which subsist by abstraction? For, since they are of the same kind with sensibles, they are far distant from indubitable and pure certainty. But from whence do they derive the certainty, order, and perfection which they receive? For they either derive it from sensibles (but they have no subsistence in these), or from intelligibles (but in these they are more perfect), since, to say from that which is not, is the most absurd of all. For nature does not produce imperfect figures, and leave the perfect without any subsistence. Nor is it lawful, that our soul should fabricate more certain, perfect, and orderly figures, than intellect and the gods themselves. There are, therefore, prior to sensible figures, self-moving, intellectual, and divine reasons of figures. And we are excited, indeed, from the obscurity of sensible forms, but we produce internal reasons, which are the lucid images of others. And we possess a knowledge of sensible figures, by their exemplars resident in soul (*παραδειγματικῶς*), but we comprehend by images (*εἰκονικῶς*) such as are intellectual and divine. For the reasons we contain, emerging from the dark night of oblivion, and propagating themselves in sciential variety, exhibit the forms of the gods, and the uniform bounds of the universe, by which they ineffably convert all things into themselves. In the gods, therefore, there is both an egregious knowledge of universal figures, and a power of generating and constituting all inferiors. But in natures, figures are endued with a power generative of apparent forms; but are destitute of cognition and intellectual perception. And, in particular souls, there is, indeed, an immaterial intellection, and a self-energizing knowledge; but there is wanting a prolific, and efficacious cause. As, therefore, nature, by her forming power presides over sensible figures, in the same manner, soul, by her gnostic energy, drops in the phantasy as in a mirror, the reasons of figures. But the phantasy receiving these in her shadowy forms, and possessing images of the inherent

ideas

inherent reasons of the soul, affords by these the means of inward conversion to the soul, and of an energy directed to herself, from the spectres of imagination. Just as if any one beholding his image in a mirror, and admiring the power of nature, and his own beauty, should desire to see himself in perfection, and should receive a power of becoming, at the same time, the perceiver, and the thing perceived. For the soul, after this manner, looking abroad into the bright mirror of the phantasy, and surveying the shadowy figures it contains, and admiring their beauty and order, pursues, in consequence of her admiration, the reasons from which these images proceed; and being wonderfully delighted, dismisses their beauty, as conversant about spectres alone; but afterwards seeks her own purer beauty, and desires to pass into her own profound retreats, and there to perceive the circle and the triangle, and all things subsisting together, in an imparible manner, and to insert herself in the objects, to contract her multitude into one; and lastly, to behold the occult and ineffable figures of the gods, seated in the most sacred and divine recesses of her nature. She is likewise desirous of bringing into light, from its awful concealment, the solitary beauty of the gods, and of perceiving the circle, subsisting in its true perfection, more imparible than any centre, and the triangle without interval; and lastly, by ascending into an union with herself, of surveying every object which is subject to the power of cognition. The figure, therefore, which is self-motive, precedes that which is moved by another; and the imparible that which is self-motive: but that which is the same with *one*, precedes the imparible itself. For all things are bounded, when they return to the unities of their nature; since all things pass through these as a divine entrance into being. And thus much for this long digression, which we have delivered according to the sentiments of the Pythagoreans. But the geometrician, contemplating that figure which is seated in the phantasy, and defining this, in the first place, (since this definition agrees with sensibles, in the second place) says, that figure is that which is comprehended by one or more boundaries. For, since he receives it together with matter, and conceives of it as distant with intervals, he does not improperly call it finite and terminated \*. [Since every thing

\* The sentence within the brackets is omitted in the Greek.

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which contains either intelligible or sensible matter, is allotted an adventitious bound; and is not itself bound, but that which is bounded.] Nor is it the bound of itself; but one of its powers is terminating, and the other terminated. Nor does it subsist in bound itself, but is contained by bound. For figure is joined to quantity, and subsists together with it; and, at the same time, quantity is subjected to figure; but the reason and aspect of that quantity is nothing else than figure and form. Since, indeed, reason terminates quantity, and adds to it a particular character and bound, either simple or composite. For, since this also exhibits the two-fold progression of *bound* and *infinite* in its proper forms, (in the same manner as the reason of an angle,) it invests the objects of its comprehension with one boundary and simple form, according to *bound*, but with many, according to infinity \*. Hence, every thing figured, vindicates to itself either one boundary, or a many. Euclid, therefore, denominating that which is figured and material, and annexed to quantity figure, does not improperly say, that it is contained by one or more terms. But Posidonius defines figure to be concluding bound, separating the reason of figure from quantity; and considering it as the cause of terminating, defining, and comprehending quantity. For that which encloses, is different from that which is enclosed; and bound from that which is bounded. And Posidonius, indeed, seems to regard the external surrounding bound; but Euclid, the whole subject. Hence, the one calls a circle a figure, with relation to its whole plane, and exterior ambit; but the other with relation to its circumference only. And the one defines that which is figured, and which is beheld together with its subject: but the other desires to define the reason of the circle; I mean that which terminates and concludes its quantity. But if any logician, and captious person, should blame the definition of Euclid, because he defines genus from species (for things contained by one or more terms, are the species of figure,) we shall assert, in opposition to such an objection, that genera also pre-occupy in themselves the powers of species. And when men of ancient authority, were willing to manifest genera themselves, from those powers which genera contain, they appeared, indeed, to enter on their design from species, but, in reality, they

\* That is, the circular form proceeds from *bound*, but right-lined figures from *infinity*.

explained

explained genera from themselves, and from the powers which they contain. The reason of figure, therefore, since it is one, comprehends the differences of many figures, according to the bound and infinity residing in its nature. And he who defined this reason, was not void of understanding, whilst he comprehended in a definition, the differences of the powers it contained. But you will ask, From whence does the reason of figure originate, and by what causes is it perfected? I answer, that it first arises from *bound* and *infinite*, and that which is mixed from these. Hence it produces some species from *bound*, others from *infinite*, and others from the *mixt*. And this it accomplishes by bringing the form of bound to circles; but that of infinite, to right-lines: and that of the mixt to figures composed from right and circular lines. But, in the second place, this reason is perfected from that totality which is separated into dissimilar parts. From whence, indeed, it occasions a whole to every form, and each figure is cut into different species. For a circle, and every right-lined figure may be divided, by reason or proportion, into dissimilar figures; which is the business of Euclid in his book of divisions, where he divides one figure into figures similar to such as are given; but another into such as are dissimilar. In the third place, it is invigorated from accumulated multitude, and, on account of this, extends forms of every kind, and produces the multiform reasons of figures. Hence, in propagating itself, it does not cease till it arrives at something last, and has unfolded all the variety of forms. And, as in the intelligible world, *one* is shewn to abide in that which *is*; and, at the same time, that which *is* in *one*, so likewise, reason exhibits circular in right-lined figures; and, on the contrary, rectilinear comprehended in circular figures. And it peculiarly manifests its whole nature in each, and all these in all. Since the whole subsists in all collectively, and in each separate and apart. From that order, therefore, it is endued with this power. In the fourth place, it receives from the first of numbers \*, the measures of the progression of forms. From whence it constitutes all figures according to numbers; some, indeed, according to the more simple, but others according to the more composite. For triangles, quadrangles, quinquangles, and all multangles, proceed in

\* That is, the number three.

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infinitum, together with the mutations of numbers. But the cause of this is, indeed, unknown to the vulgar, though, to those who understand where number and figure subsist, the reason is manifest. Fifthly, it is replete with that division of forms, which divides forms into other similar forms, from another second totality, which is also distributed into similar parts. And by this, a triangular reason is divided into triangles, and a quadrangular reason into quadrangles. And hence, exercising our inward powers, we effect what I have said in images, since it pre-existed by far the first in its principles. But by regarding these distributions, we may render many causes of figures, reducing them to their first principles. And the more common, or geometrical figure, is allotted an order of this kind, and from so many causes, receives the perfection of its nature. But, from hence it advances to the genera of the gods, and is variously attributed according to its various forms, and energizes differently in different gods. To some, indeed, affording more simple figures; but to others, such as are more composite. And to some, again, assigning primary figures, and those which are produced in superficies; but to others (entering the tumor of solid bodies) such figures, as in solids are convenient to themselves. For all figures, indeed, subsist in all, since the forms of the gods are accumulated, and full of universal powers: but, by their peculiarity, they produce one thing according to another. For one possesses all things circularly, another in a triangular manner, but another according to a quadrangular reason. And in a similar manner in solids.

### DEFINITION XV.

A CIRCLE is a Plane Figure, comprehended by one Line, which is called the Circumference, to which all Right Lines falling from a certain Point within the Figure, are equal to each other.

### DEFINITION XVI.

And that Point is called the CENTRE of the CIRCLE.

A Circle is the first, most simple, and most perfect of figures. For it excels all solids, because it exists in a more simple place; but it is superior to the figures subsisting in planes, on account of its similitude

similitude and identity. And it has a corresponding proportion to bound, and unity, and a better co-ordination of being. Hence, in a distribution of mundane and super-mundane figures, you will always find that the circle is of a diviner nature. For if you make a division into the heavens, and the universal regions of generation, you must assign to the heavens a circular form; but to generation, that of a right line. For whatever among generable natures is circular, descends from the heavens; since generation revolves into itself, through their circumvolutions, and reduces its unstable mutation to a regular and orderly continuance. But if you distribute incorporeal natures into soul and intellect, you will say, that the circle belongs to intellect, and the right line to the soul. And on this account, the soul, by its conversion to intellect, is said to be circularly moved; and it possesses the same proportion to intellect, as generation to the heavens. For it is circularly moved, (says Socrates \*,) because it imitates intellect. But the generation and progression of soul is made according to a right-line. For it is the property of the soul to apply herself at different times to different forms. But if you wish to divide into body and soul, you must constitute every thing corporeal, according to the right-line; but you must assign to every animal a participation of the identity and similitude of the circle. For body is a composite, and is endowed with various powers, similar to right-lined figures: but soul is simple and intelligent; self-motive, and self-operative; converted into, and energizing in herself. From whence, indeed, Timæus also, when he had composed the elements of the universe from right-lined figures, assigned to them a circular motion and formation, from that divine soul which is seated in the bosom of the world. And thus, that the circle every where holds the first rank, in respect of other figures, is sufficiently evident from the preceding observations. But it is requisite to survey its whole series, beginning supernally, ending in inferiors, and perfecting all things, according to the aptitude of the natures which receive its alliance. To the gods, therefore, it affords a conversion to their causes, and ineffable union: it occasions their abiding in themselves, prevents their departing from their own beatitude, strengthens their highest unions, as centres desirable to

\* In Timæo.

inferior.

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inferior natures, and stably places about these the multitude of the powers which the gods possess, containing them in the simplicity of their essences. But the circle affords to intellectual natures, a perpetual energy in themselves, is the cause of their being filled with knowledge from themselves, and of possessing in their essences, intelligibles contractedly ; and of perfecting intellects in themselves. For every intellect, proposes to itself that which is intelligible ; and this is as a centre to intellect, about which it continually revolves : for intellect folds itself, and operates about this, and is united within itself on all sides, by universal intellectual energies. But it extends to souls by illumination, a self-vital, and self-motive power, and an ability of turning, and leaping round intellect, and of returning according to proper convolutions, unfolding the imparibility of intellect. Again, the intellectual orders excel souls after the manner of centres, but souls energize circularly about their nature. For every soul, according to its intellectual part, and the supreme *one*, which is the very flower of its essence, receives a centre: but, according to its multitude, it has a circular revolution, desiring, by this means, to embrace the intellect which it participates. But, to the celestial bodies, the circle affords an assimilation to intellect, equality, a comprehension of the universe, in proper limits, revolutions which take place in determinate measures, a perpetual subsistence, a nature without beginning and end, and every thing of this kind. And to the elements under the concave of the moon's orb, it is the cause of a period, conversant with mutations; an assimilation to the heavens; that which is without generation, in generated natures; that which abides in things which are moved; and whatever is bounded in partible essences. For all things are perpetual, through the circle of generation; and equality is every where preserved on account of the reciprocation of corruption. Since, if generation did not return, in a circular revolution, in a short space of time, the order, and all the ornament of the elements would vanish. But again, the circle procures to animals and plants, that similitude which is found in generations; for these are produced from seeds, and seeds from these. Hence, generation here, and a circumvolution, alternately takes place, from the imperfect to the perfect, and the contrary ; so that corruption subsists together with generation.

generation. But, besides this, to unnatural productions it imposes order, and reduces their indeterminate variety to the limitation of bound; and, through this, nature herself is gracefully ornamented in the last vestiges of her powers. Hence, things contrary to nature have a revolution according to determinate numbers, and not only fertility, but also sterility, subsists according to the alternate convolutions of circles (as the discourse of the Muses evinces), and all evils though they are dismissed from the presence of the gods, into the place of mortals, yet these roll round, says Socrates, and to these there is present a circular revolution, and a circular order; so that nothing immoderate and evil is deserted by the gods; but that providence, which is perfective of the universe, reduces also the infinite variety of evils, to bound, and an order convenient to their nature. The circle, therefore, is the cause of ornament to all things, even to the last participations, and leaves nothing destitute of itself, since it supplies beauty, similitude, formation, and perfection to the universe. Hence too, in numbers it contains the middle centres of the whole progression of numbers, which revolves from unity to the decad (or ten). For five and six exhibit a circular power, because, in the progressions from themselves, they return again into themselves, as is evident in the multiplication of these numbers. Multiplication, therefore, is an image of progression, since it is extended into multitude; but an ending in the same species, is an image of regression into themselves. But a circular power affords each of these, exciting, indeed, as from an abiding centre, those causes which are productive of multitude; but converting multitude after the productions to their causes. Two numbers, therefore, having the properties of a circle, possess the middle place between all numbers: of which one, indeed, precedes every convertible genus of males and an odd nature; but the other, recalls every thing feminine and even, and all prolific series, to their proper principles, according to a circular power. And thus much concerning the perfection of the circle. Let us now contemplate the mathematical definition of the circle, which is every way perfect. In the first place, therefore, he defines it a figure, because, indeed, it is finite, and every where comprehended by one limit, and is not of an infinite nature, but associated to bound. Likewise

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plane,

plane, because, since figures are either beheld in superficies, or in solid bodies, a circle is the first of plane figures, excelling solids in simplicity, but possessing the proportion of unity to planes. But comprehended by one line, because it is similar to one, by which it is defined, and because it does not extrinsically receive a variety of surrounding terms. And again, that this line makes all the lines drawn to it from a certain point within equal, because of the figures which are bounded by one line, some have all the lines proceeding from the middle equal; but others not at all. For the ellipsis is comprehended by one line, yet all the lines issuing from the centre, and bounded by its curvature, are not equal, but only two. Also the plane, which is included by the line called a cissoid, has one containing line, yet it does not contain a centre, from which all the lines are equal. But, because the centre in a circle is entirely one point (for there are not many centres of one circle), on this account, the geometrician adds, that lines falling from one point to the bound of the circle, are equal. For there are infinite points within it, but of all these, one only has the power of a centre. And because this one point, from which all the lines drawn to the circumference of the circle are equal, is either within the circle, or without (for every circle has a pole, from which all the lines drawn to its circumference are equal), on this account he adds, *of the points within the figure*, because, here he receives the centre alone, and not the pole. For he wishes to behold all its properties in one plane, but the pole is more elevated than the subject plane. Hence, he necessarily adds, in the end of the definition, that this point, which is placed within the circle, and to which all right lines drawn from it to the circumference, are equal, is the centre of the circle. For there are only two points of this kind, the pole and the centre. But the former is without, and the other within the plane. Thus, for instance, if you conceive a perpendicular standing on the centre of a circle, its superior extremity is the pole: for all lines drawn from it to the circumference of the circle, are demonstrated to be equal. And, in like manner, in a cone, the vertex of the whole cone, is the pole of the circle at the base. And thus far we have determined what a circle is, and its centre, and what the nature is of its circumference, and the whole circular figure. Again, therefore, from these, let us return

to

to the speculation of their exemplars, contemplating in them the centre, according to one imitable and stable excellence. But the distances from the centre, according to the progressions which are made from one, to multitude infinite in capacity. And the circumference of the circle, according to the regression of the progressions to the centre, by means of which the multitude of powers are rolled round their union, and all of them hasten to its comprehension, and desire to energize about its indivisible embrace. And, as in the circle itself, all things subsist together, the centre, intervals, and external circumference; so in these which are its image, one thing has not an essence pre-existent, and another consequent in time; but all things are, indeed, together, permanency, progression, and regression. But these differ from those, because the former subsist indivisibly, and without any dimension; but the latter with dimension, and in a divisible manner; the centre existing in one place, the lines emanating from the centre, in another; and the external circumference terminating the circle, having a still different situation. But there all things abide in one: for if you regard that which performs the office of a centre, you will find it the receptacle of all things. If the progression distant from the centre, in this, likewise, you will find all things contained. And, in a similar manner, if you regard its regression. When, therefore, you are able to perceive all things subsisting together, and have taken away the defect proceeding from dimension, and have removed from your inward vision, the position about which partition subsists, you will find the true circle, advancing to itself, bounding, and energizing in itself, existing both one and many, and abiding, proceeding, and returning; likewise firmly establishing that part of its essence which is most imitable, and especially singular; but advancing from this according to rectitude, and the infinity which it contains; and rolling itself from itself to one, and exciting itself by similitude and identity to the imitable centre of its nature, and to the occult power of the one which it contains. But this one, which the circle contains, and environs in its bosom, it emulates according to the multitude of its own nature. For that which is convolved, imitates that which abides, and the periphery is as a centre which is distant with interval, and

nods to itself, hastening to receive, and to become one with the centre, and to terminate its regress where it received the principle of its progression. For the centre is every where in the place of that which is lovely, and the object of desire, presiding over all things which subsist about its nature, and existing as the beginning and author of all progressions. And this the mathematical centre also expresses, by terminating all the lines falling from itself to the circumference, and by affording to them equality, as an image of proper union. But the oracles likewise define the centre, after this manner: *The centre is that from which and \* to which all the lines to the circumference are equal.* Indicating the beginning of the distance of the lines, by the particle *from which*; but the middle of the circumference by the particle *to which*: for this, in every part, is joined with the centre. But if it be necessary to declare the first cause, through which a circular figure appears and receives its perfection, I affirm, that it is the supreme order of intelligibles. For the centre, indeed, is assimilated to the cause of bound; but the lines emanating from this, and which are infinite, with respect to themselves, both in multitude and magnitude, represent infinity; and the line which terminates their extension, and conjoins the circular figure with the centre, is similar to that occult ornament, consisting from the intelligible orders; which Orpheus also says, is circularly borne, in the following words, *But it is carried with an unwearied energy, according to an infinite circle.* For, since it is moved intelligibly, about that which is intelligible, having it for the centre of its motion, it is, with great propriety, said to energize in a circular manner. Hence, from these also, the triadic god † proceeds, who contains in himself the cause of the progression of right-lined figures. For on this account, wise men, and the most mystic of theologists, have fabricated his name. [‡ Hence too, it is manifest, that a circle is the first of all figures:] but a triangle is the

\* *πρὸς δὲ οὐ, or, to which,* is wanting in the original, and in all the published collections of the Zoroastrian oracle.

† That is Jupiter, who is called triadic, because he proceeds from Saturn and Rhea; and because his government is participated by Neptune and Pluto, for each of these is called Jupiter by Orpheus.

‡ This sentence, within the brackets, is omitted in the printed Greek.

first of such as are right-lined. Figures, therefore, appear first in the regular ornaments of the gods; but they have a latent subsistence, according to pre-existent causes, in intelligible essences.

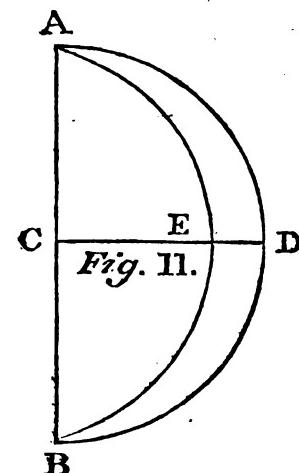
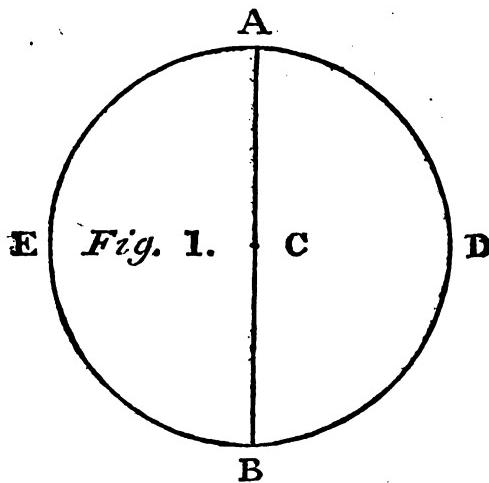
## DEFINITION XVII.

A DIAMETER of a Circle is a certain straight Line, drawn through the Centre, which is terminated both ways by the Circumference of the Circle, and divides the Circle into two equal Parts.

EUCLED here perspicuously shews, that he does not define every diameter, but that which belongs to a circle only. Because there is a diameter of quadrangles and all parallelograms, and likewise of a sphere among solid figures. But in the first of these, it is denominated a diagonal: but in a sphere, the axis; and in circles the diameter only. Indeed, we are accustomed to speak of the axis of an ellipsis, cylinder, and cone; but of a circle, with propriety, the diameter. This, therefore, in its genus, is a right-line; but as there are many right-lines in a circle, as likewise infinite points, one of which is a centre, so this only is called a diameter, which passes through the centre, and neither falls within the circumference, nor transcends its boundary; but is both ways terminated by its comprehensive bound. And these observations exhibit its origin. But that which is added in the end, that it also divides the circle into two eqnal parts, indicates its proper energy in the circle, exclusive of all other lines drawn through the centre, which are not terminated both ways by the circumference. But they report, that Thales first demonstrated, that the circle was bisected by the diameter. And the cause of this bisection, is the indeclineable transit of the right line, through the centre. For, since it is drawn through the middle, and always preserves the same inflexible motion, according to all its parts, it cuts off equal portions on both sides to the circumference of the circle. But if you desire to exhibit the same mathematically, conceive the diameter drawn, and one part of the circle placed on the other.

other \*. Then, if it is not equal, it either falls within, or without; but the consequence either of these ways must be, that a less right-line will be equal to a greater. Since all lines from the centre to the circumference are equal. The line, therefore, which tends to the exterior circumference, will be equal to that which tends to the interior. But this is impossible. These parts of the circle, then, agree, and are on this account equal. But here a doubt arises, if two semicircles are produced by one diameter, and infinite diameters may be drawn through the centre, a double of infinities will take place, according to number. For this is objected † by some against the section of magnitudes to infinity. But this we may solve by affirming, that magnitude may, indeed, be divided infinitely, but not into infinites. For this latter mode produces infinites in energy, but the former in capacity only. And the one affords essence to infinite, but the other is the source of its origin alone. Two semi-circles, therefore, subsist together with one diameter, yet there will never be infinite diameters, although they may be infinitely assumed. Hence, there

\* Thus let a part A E B cut off by the diameter A B (fig. I.) of the circle A E B D be



placed on the other part A D B, as in fig. II. Then, if it is not equal to the other part, either A E B will fall within A D B, or A D B within A E B: but in either case, C E will be equal to C D, which is absurd.

† This objection is urged by Philoponus, in his book against Proclus on the eternity of the world; but not, in my opinion, with any success. See also Simplicius, in his third digression against Philoponus, in his commentary on the 8th book of Aristotle's Physics.

can

can never be doubles of infinites; but the doubles which are continually produced, are the doubles of finites; for the diameters which are always assumed, are finite in number. And what reason can be assigned why every magnitude should not have finite divisions, since number is prior to magnitudes, defines all their sections, pre-occupies infinity, and always determines the parts which rise into energy from dormant capacity?

## DEFINITION XVIII.

A SEMICIRCLE is the Figure contained by the Diameter, and that Part of the Circumference which is cut off by the Diameter.

## DEFINITION XIX.\*

But the CENTRE of the Semi-circle, is the same with that of the Circle.

FROM the definition of a circle Euclid finds out the nature of the centre, differing from all the other points which the circle contains. But from the centre he defines the diameter, and separates it from the other right lines, which are described within the circle. And from the diameter, he teaches the nature of the semi-circle; and informs us, that it is contained by two terms, always differing from each other, viz. a right-line and a circumference: and that this right-line is not any one indifferently, but the diameter of the circle. For both a less and a greater segment of a circle, are contained by a right-line and circumference; yet these are not semi-circles, because the division of the circle is not made through the centre. All these figures, therefore, are biformed, as a circle was monadic, and are composed

\* This definition is no where extant but in the commentaries of Proclus. Instead of it, in almost all the printed editions of Euclid, the following is substituted. *A segment of a circle is the figure contained by a diameter, and the part of the circumference cut off by the diameter.* This Mr. Simson has marked with commas, as a symbol of its being interpolated: but he has taken no notice of the different reading in the commentaries of Proclus. And what is still more remarkable, this variation is not noticed by any editor of Euclid's Elements, either ancient or modern.

from dissimilars. For every figure which is comprehended by two terms, is either contained by two circumferences, as the lunular: or by a right-line and circumference, as the above mentioned figures; or by two mixt lines, as if two ellipses intersect each other (since they enclose a figure, which is intercepted between them), or by a mixt line and circumference, as when a circle cuts an ellipsis; or by a mixt and right-line, as the half of an ellipsis. But a semi-circle is composed from dissimilar lines, yet such as are, at the same time, simple, and touching each other by apposition. Hence, before he defines triadic figures, he, with great propriety, passes from the circle to a biformed figure. For two right-lines can, indeed, never comprehend space. But this may be effected by a right-line and circumference. Likewise by two circumferences, either making angles, as in the lunular figure; or forming a figure without angles, as that which is comprehended by concentric circles. For the middle space intercepted between both, is comprehended by two circumferences; one interior, but the other exterior, and no angle is produced. For they do not mutually intersect, as in the lunular figure, and that which is on both sides convex. But that the centre of the semi-circle is the same with that of the circle, is manifest. For the diameter, containing in itself the centre, completes the semi-circle, and from this all lines drawn to the semi-circumference are equal. For this is a part of the circumference of the circle. But equal right lines proceed from the centre to all parts of the circumference. The centre, therefore, of the circle and semi-circle is one and the same. And it must be observed, that among all figures, this alone contains the centre in its own perimeter, I say, among all plane figures. Hence you may collect, that the centre has three places. For it is either within a figure, as in the circle; or in its perimeter, as in the semi-circle; or without the figure, as in certain conic lines\*. What then is indicated by the semi-circles, having the same centre with the circle, or of what things does it bear an image, unless that all figures which do not entirely depart from such as are first, but participate them after a manner, may be concentric with them, and participate of the same causes? For the semicircle communicates with the circle doubly, as well according to the dia-

\* As in every hyperbola.

ter, as according to the circumference. On this account, they possess a centre also in common. And perhaps, after the most simple principles, the semi-circle is assimilated to the second co-ordinations, which participate those principles; and by their relation to them, although imperfectly, and by halves, they are, nevertheless, reduced to that which is, and to their first original cause.

## DEFINITION XX.

RECTILINEAR FIGURES are those which are comprehended by Straight Lines.

## DEFINITION XXI.

TRILATERAL FIGURES, or TRIANGLES, by three Straight Lines.

## DEFINITION XXII.

QUADRILATERAL, by four Straight Lines.

## DEFINITION XXIII.

MULTILATERAL FIGURES, or POLYGONS, by more than four Straight Lines.

**A**FTER the monadic figure having the relation of a principle to all figures, and the biformed semi-circle, the progression of right-lined figures in infinitum, according to numbers, is delivered. For on this account also, mention was made of the semi-circle, as communicating according to terms or boundaries; partly, indeed, with the circle, but partly with right-lines: just as the duad is the medium between unity and number. For unity, by composition, produces more than by multiplication; but number, on the contrary, is more increased by multiplication than composition: and the duad, whether multiplied into, or compounded with itself, produces an equal quantity. As, therefore, the duad is the middle of unity and number, so likewise, a semi-circle communicates, according to its base, with right-

lines ; but according to its circumference, with the circle. But right-lined figures proceed orderly to infinity, attended by number and its bounding power, which begins from the triad. On this account, Euclid also begins from hence \*. For he says, trilateral and quadrilateral, and the following figures, called by the common name of multilateral : since trilateral figures are also multilaterals ; but they have likewise a proper, besides a common denomination. But, as we are but little able to pursue the rest, on account of the infinite progression of numbers, we must be content with a common denomination. But he only makes mention of trilaterals and quadrilaterals, because the triad and tetrad are the first in the order of numbers ; the former being a pure odd among the odd ; but the latter, an entire even among even numbers. Euclid, therefore, assumes both in the origin of right-lined figures, for the purpose of exhibiting their subsistence, according to all even and odd numbers. Besides, since he is about to teach concerning these in the first book, as especially elementary (I mean triangles and parallelograms) he does not undeservedly, as far as to these, establish a proper enumeration : but he embraces all other right-lined figures by a common name, calling them multilaterals : but of these enough. Again, assuming a more elevated exordium, we must say, that of plane figures, some are contained by simple lines, others by such as are mixt, but others again by both. And of those which are comprehended by simple lines, some are contained by similars in species, as right-lines ; but others by dissimilars in species, as semi-circles, and segments, and apsides, which are less than semi-circles. Likewise of those which are contained by similars in species, some are comprehended by a circular line ; but others by a right-line. And of those comprehended by a circular line, some are contained by one, others by two, but others by more than two. By one, indeed, the circle itself. But by two, some without angles, as the crowns † terminated by concentric circles ; but others angular (*γεγωνικέα*) as the lunula.

\* The Platonic reader must doubtless be pleased to find that Euclid was deeply skilled in the philosophy of Plato, as Proclus every where evinces. Indeed, the great accuracy, and elegant distribution of these Elements, sufficiently prove the truth of this assertion. And it is no inconsiderable testimony in favour of the Platonic philosophy, that its assistance enabled Euclid to produce such an admirable work.

† Concerning these crowns, or annular spaces, consult the great work of that very subtle and elegant mathematician Tacquet, entitled *Cylindrica et Annularia*.

And

And of those comprehended by more than two, there is an infinite procession. For there are certain figures contained by three and four and succeeding circumferences. Thus, if three circles touch each other, they will intercept a certain trilateral space; but if four, one terminated by four circumferences, and in like manner, by a successive progression. But of those contained by right lines, some are comprehended by three, others by four, and others by a multitude of lines. For neither is space comprehended by two right-lines, nor much more by one right-line. Hence, every space comprehended by one boundary, or by two, is either mixt or circular. And it is mixt in a twofold manner, either because the mixt lines comprehend it, as the space intercepted by the cissoidal line; or because it is contained by lines dissimilar in species, as the apsis: since mingling is two-fold, either by apposition or confusion. Every right-lined figure, therefore, is either trilateral, or quadrilateral, or gradually multilateral; but every trilateral, or quadrilateral, or multilateral figure, is not right-lined; since so great a number of sides is also produced from circumferences. And thus much concerning the division of plane figures. But we have already asserted \*, that rectitude of progression is both a symbol of motion and infinity, and that it is peculiar to the generative co-ordinations of the gods, and to the producers of difference, and to the authors of mutation and motion. Right-lined figures, therefore, are peculiar to these gods, who are the principles of the prolific energy of the whole progression of forms. On which account, generation also, was principally adorned by these figures, and is allotted its essence from these, so far as it subsists in continual motion and mutation without end.

\* In the preceding tenth commentary.

## DEFINITION XXIV.

Of three-sided FIGURES: an EQUILATERAL TRIANGLE is that which has three equal Sides.

## DEFINITION XXV.

An ISOSCELES TRIANGLE, is that which has only two Sides equal.

## DEFINITION XXVI.

A SCALENE TRIANGLE, is that which has three unequal Sides.

## DEFINITION XXVII.

A RIGHT-ANGLED TRIANGLE is that which has a Right Angle.

## DEFINITION XXVIII.

An OBTUSE-ANGLED TRIANGLE is that which has an Obtuse Angle.

## DEFINITION XXIX.

An ACUTE-ANGLED TRIANGLE is that which has three Acute Angles.

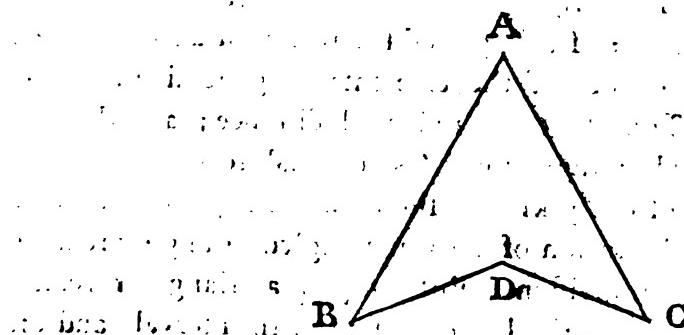
**T**H E division of triangles sometimes commences from angles, but sometimes from sides. And that, indeed, which originates from sides, precedes as known; but that from angles follows as a proper distribution. For these three angles alone belong to right-lined figures, viz. the right, the obtuse, and the acute: but the equality and inequality of sides subsist also in non-rightilinear figures. Euclid says, therefore, that of triangles, some are equilateral, others isosceles,

isosceles, and others scalene : for they have either all their sides equal, or all unequal, or only two equal. And again, that of triangles some are right-angled, others obtuse-angled, and others acute-angled. And he defines a right-angled triangle, that which has one right angle, as likewise an obtuse-angled triangle, that which has one obtuse angle : for it is impossible that a triangle can have more than one right, or obtuse angle \*. But he defines an acute-angled triangle, that which has all its angles acute. For here it is not sufficient that it should have only one acute; since, in this case, all triangles would be acute-angled, as every triangle has necessarily two acute angles †. But, to possess three acute angles, is the property of an acute-angled triangle alone. But Euclid appears to me to have made a separate division into angles and sides, from considering this alone, that every triangle is not also trilateral. For there are quadrilateral triangles, which are called by mathematicians themselves (*ακιδοειδῆς*) that is, similar to the point of a spear ‡: but by Zenodorus (*κυλογύνων*) that is, having an hollow angle. For on one of the sides of a trilateral figure, constitute two right-lines inwardly ; by this means a certain space will be enclosed, which is comprehended by external and internal right-lines, and which has three angles ; one, indeed, contained by the external lines; but two comprehended by these and the internal lines, at the extremities in which these lines are conjoined. A figure of this kind, therefore, is a quadrilateral triangle. And hence, it does not immediately follow, that because a figure has three angles (whether they

\* This is consequence of every triangle possessing angles alone equal to two right.

† This too, follows from the same cause as above.

‡ Thus the following figure A B D C has four sides, and but three angles.



are

are all acute, or one right, or one obtuse), we shall find it trilateral; for it may be, perhaps, quadrilateral. In like manner, you may also find quadrangles having more than four sides. And therefore, we must not rashly determine the number of sides from the multitude of angles. But of this enough. But the Pythagoreans affirm that the triangle is simply the principle of generation, and of the formation of generable natures. On which account, Timæus says, that natural reasons, as well as those of the construction of the elements, are triangular. For they are distant by a triple interval, are on all sides collective of partible, and variously mutable natures, are replete with material infinity, and bear before themselves the conjunctions of material bodies, loosened and free: as, indeed, triangles also are comprehended by three right-lines, but they possess angles which collect the multitude of lines, and afford to them an adventitious angle and conjunction. With great propriety, therefore, Philolaus has consecrated the angle of a triangle to four gods, Saturn, Pluto, Mars, and Bacchus, comprehending in these the whole quadripartite ornament of the elements descending from the heavens, or from the four segments of the zodiac. For Saturn constitutes an essence wholly humid and frigid; but Mars a nature totally fiery; and Pluto contains the whole terrestrial life; but Bacchus governs a humid and hot generation; of which wine also is a symbol, for this is humid and hot. Hence, all these gods differ according to their operations in inferior concerns: but they are mutually united according to their proper natures. And on this account, Philolaus collects their union according to one angle. But if the differences of triangles contribute to generation, we shall very properly confess that a triangle is the principle and author of the constitution of sublunary natures. For a right angle, indeed, affords them essence, and determines the measure of being; and the reason of a right-angled triangle produces the essence of the elements of generable natures; but an obtuse angle assigns to them universal distance; and the reason of an obtuse-angled triangle increases material forms in magnitude, and in mutation of every kind. But an acute angle effects their divisible nature; and the reason of an acute-angled triangle prepares them to receive infinite division. But simply, a triangular reason constitutes the essence of material bodies distant with interval, and on all

all sides divisible. And thus much should we speculate concerning the nature of triangles. But from these divisions you may understand, that all the species of triangles are neither more nor less than seven. For the equilateral triangle is one, since it is acute-angled only; but each of the rest is triple. For the isosceles is either right-angled, or obtuse-angled, or acute-angled; and, in like manner, the scalene triangle possesses this triple difference. If then, these have a triple distinction, but the equilateral has but one mode of existence, all the species of triangles will be seven. But again, you will understand the proportion of triangles to the things which *are*, according to the division of sides; for the equilateral, entirely excelling in equality and simplicity, is allied to divine souls; since it is the measure and equality of things unequal, in the same manner as divinity of all inferior concerns. But the isosceles triangle is allied to the better genera, which govern a material nature, the greater part of which genera is held by the limitation of measure; but their extremes extend to inequality and material immoderation; for the two sides of an isosceles triangle are equal, but the base is unequal. But a scalene triangle symbolizes with partible lives, which are on all sides lame and defective, which prepare themselves for generation, and are replete with matter and material imperfection.

## DEFINITION XXX.

Of Quadrilateral Figures, a QUADRANGLE or SQUARE is that which has all its Sides equal, and all its Angles Right Angles.

## DEFINITION XXXI.

An OBLONG is that which has all its Angles right Angles, but has not all its Sides equal.

## DEFINITION XXXII.

A RHOMBUS, is that which has all its Sides equal, but its Angles are not right Angles.

DEFI-



## DEFINITION XXXIII.

A RHOMBOID is that which has its opposite Sides equal to one another, but all its Sides are not equal, nor its Angles Right Angles.

## DEFINITION XXXIV.

All other Quadrilateral Figures besides these, are called TRAPEZIUMS.

IT is requisite that the first division of quadrilateral figures should take place in two numbers; and that some of them should be called parallelograms, but others non-parallelograms. But of parallelograms some are rectangular and equilateral, as quadrangles; but others neither of these, as rhomboids: others again, are rectangular, but not equilateral, as oblongs: but others, on the contrary, are equilateral, but not rectangular, as the rhombuses. For it is requisite either to possess both, viz. equality of sides and rectitude of angles, or neither; or one of these, and this in a twofold respect. Hence a parallelogram has a quadruple subsistence. But of non-parallelograms, some have only two parallel sides, and not the rest; but others have none of their sides parallel. And those are called Trapeziums, but these Trapezoids. But of Trapeziums, some, indeed, have the sides equal, by which the parallel sides of this kind are conjoined; but others unequal, and the former of these are called isosceles trapeziums; but the latter scalene trapeziums. A quadrilateral figure, therefore, is constituted by us according to a seven-fold distribution. For one is a quadrangle; but the other an oblong; the third a rhombus; the fourth a rhomboides; the fifth an isosceles trapezium; the sixth a scalene trapezium; the seventh a trapezoid. But Posidonius makes a perfect division of right-lined quadrilateral figures into so many members; for he establishes seven species of these; as likewise of triangles. But Euclid could not divide into parallelograms and non parallelograms, because he neither mentions parallels, nor teaches us concerning the parallelogram itself. But trapeziums, and all trapezoids, he calls by a common name,

name, describing trapeziums themselves, according to the difference of those four figures\*, in which the property of parallelograms is verified. And this is to have the opposite sides and angles equal. For a quadrangle and an oblong, and a rhombus, have their opposite sides and angles equal. But in a rhomboides he only adds this, *that its opposite sides are equal*, lest he should define it by negations alone, since he neither calls it equilateral, nor rectangular. For where we want proper appellations, it is necessary to use such as are common. But we should hear Euclid shewing that this is common to all parallelograms. But a rhombus appears to be a quadrangle having its sides moved, and a rhomboides a moved oblong. Hence, according to sides, these do not differ from those; but they vary only according to the obtuseness and acuteness of angles; since the quadrangle and the oblong are rectangular. For if you conceive a quadrangle or an oblong, having its sides drawn in such a manner, that while two of its opposite angles are dilated, the other two are contracted; then the dilated angles will appear obtuse, and the contracted, acute. And the appellation of rhombus † seems to have been imposed from motion. For if you conceive a quadrangle moving after the manner of a rhombus, it will appear to you changed in order, according to its angles: just as if a circle is moved after the manner of a sling, it will immediately exhibit the appearance of an ellipsis. But here you may perhaps enquire concerning the quadrangle, why it has this denomination? and why the appellation of quadrangle may not be applied to other quadrilateral figures, as the name of triangle is common to all those which are neither equiangular nor equilateral, and in like manner of quinquangles or pentagons; for the geometrician, in these, adds only the particle *an equilateral triangle*, or *a quinquangle, which is equilateral and equiangular*, as if these could not be otherwise than such as they are? But when he mentions a quadrangle, he immediately indicates that it must be equilateral and rectangular. But the reason of this is as follows: a quadrangle alone has the best space, both

\* The Greek in this place is very erroneous, which I have restored from the version of Barocius.

† For the Greek word *ῥόμβος* is derived from the verb *ῥίμνω*, which signifies to have a circumvolute motion.

according to its sides and angles. For each of the latter is right, intercepting a measure of angles, which neither receives intention nor remission. As it excels, therefore, in both respects, it deservedly obtains a common appellation. But a triangle, though it may have equal sides, yet will in this case have all its angles acute, and a quadrangle all its angles obtuse. Since, therefore, of all quadrilateral figures, a quadrangle alone is replete with equality of sides, and rectitude of angles, it was not undeservedly allotted this appellation: for, to excellent forms, we often dedicate the name of the whole. But it appeared also to the Pythagoreans, that this property of quadrilateral figures, principally conveyed an image of a divine essence. For they particularly signified by this, a pure and immaculate order. Since rectitude imitates inflexibility, but equality a firm and permanent power: for motion emanates from inequality, but quiet from equality itself. The gods, therefore, who are the authors to all things of stable disposition, of pure and uncontaminated order, and of indeclinable power, are deservedly manifested as from an image, by a quadrangular figure. But, besides these, Philolaus also, according to another apprehension, calls a quadrangular angle, the angle of Rhea, Ceres and Vesta. For, since a quadrangle constitutes the earth, and is its proximate element, as we learn from Timæus, but the earth herself receives from all these divinities, genital seeds, and prolific powers, he does not unjustly consecrate the angle of a quadrangle to these goddesses, the bestowers of life. For some call both the earth and Ceres, Vesta\*, and they say that Rhea totally participates her nature, and that all generative causes are contained in her essence. Philolaus, therefore, says that a quadrangular angle comprehends, by a certain terrestrial power, one union of the divine genera. But some assimilate a quadrangle to universal virtue, so far as every quadrangle from its perfection has four right angles. Just as we say that each of the virtues is perfect, content with itself, the measure and bound of life, and the middle of every thing which, in morals, corresponds to the obtuse and acute. But it is by no means proper to conceal, that Philolaus attributes a triangular angle to four, but a quadrangular angle to three gods, ex-

\* See the Orphic Hymns of Onomacritus to these deities; my translation of which I must recommend to the English reader, because there is no other.

hibiting

Inhibiting their alternate transition, and the community of all things in all, of odd natures in the even, and of even in the odd. Hence, the tetradic ternary, and the triadic quaternary, participating of prolific and efficacious goods, contain the whole ornament of generable natures, and preserve them in their proper state. From which the duodenary, or the number twelve, is excited to a singular unity, viz. the government of Jupiter. For Philolaus says, that the angle of a dodecagon (or twelve-sided figure) belongs to Jove, so far as Jupiter contains and preserves, by his singular union, the whole number of the duodenary. For also, according to Plato, Jupiter presides over the duodenary \*, and governs and moderates the universe with absolute sway. And thus much we have thought proper to discourse concerning quadrilateral figures, as well declaring the sense of our author, as likewise affording an occasion of more profound inspections to such as desire the knowledge of intelligible and occult essences.

## DEFINITION XXXV.

**PARALLEL RIGHT LINES** are such as being in the same Plane, and produced both ways infinitely, will in no part mutually coincide.

**W**HAT the elements of parallels are, and by what accidents in these they may be known, we shall afterwards learn: but what parallel right lines are, he defines in these words: "It is requisite, therefore (says he), that they should be in one plane, and while they are produced both ways have no co-incidence, but be extended in infinitum." For non-parallel lines also, if they are produced to a certain distance, will not coincide. But to be produced infinitely, without coincidence, expresses the property of parallels. Nor yet this absolutely, but to be extended both ways infinitely, and not coincide.

\* These twelve divinities, of which Jupiter is the head, are, *Jupiter, Neptune, Vulcan, Pegasus, Minerva, Mars, Ceres, Juno, Diana, Mercury, Venus, and Apollo*. The first triad of these is demiurgic, the second comprehends guardian deities, the third is vivific, or zoogonic, and the fourth contains elevating gods. But, for a particular theological account of these divinities, study Proclus on Plato's Theology, and you will find their nature unfolded, in page 403, of that admirable work.

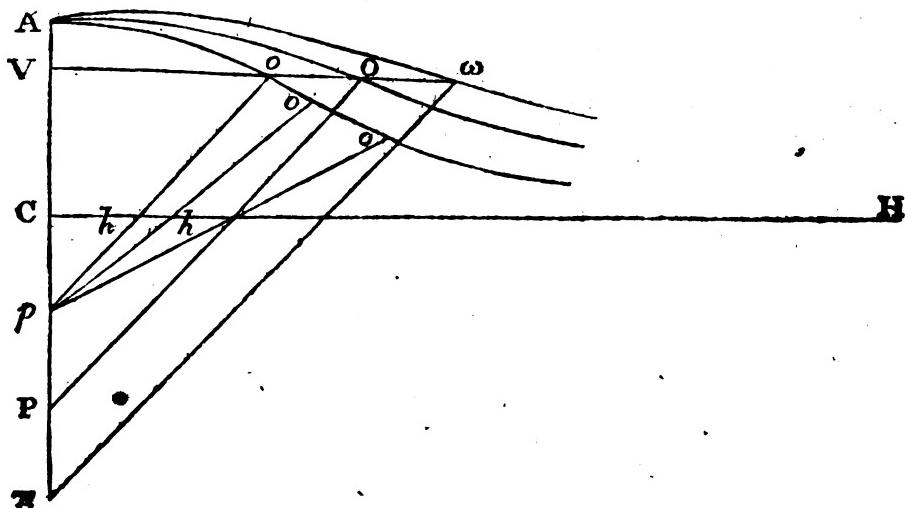
For it is possible that non-parallel lines may also be produced one way infinitely, but not the other; since, verging in this part, they are far distant from mutual coincidence in the other. But the reason of this is, because two right-lines cannot comprehend space; for if they verge to each other both ways, this cannot happen. Besides this, he very properly considers the right-lines as subsisting in the same plane. For if the one should be in a subject plane, but the other in one elevated, they will not mutually coincide according to every position, yet they are not on this account parallel. The plane, therefore, should be one, and they should be produced both ways infinitely, and not coincide in either part. For with these conditions, the right-lines will be parallel. And agreeable to this, Euclid defines parallel right-lines. But Posidonius says, parallel lines are such as neither incline nor diverge in one plane; but have all the perpendiculars equal which are drawn from the points of the one to the other. But such lines as make their perpendiculars always greater and less, will some time or other coincide, because they mutually verge to each other. For a perpendicular is capable of bounding the altitudes of spaces, and the distances of lines. On which account, when the perpendiculars are equal, the distances of the right lines are also equal; but when they are greater and less, the distance also becomes greater and less, and they mutually verge in those parts, in which the lesser perpendiculars are found. But it is requisite to know, that non-coincidence does not entirely form parallel lines. For the circumferences of concentric circles do not coincide: but it is likewise requisite that they should be infinitely produced. But this property is not only inherent in right, but also in other lines: for it is possible to conceive spirals described in order about right lines, which if produced infinitely together with the right lines, will never coincide \*. Geminus, therefore, makes a very proper division in this place, affirming from the beginning, that of lines some are bounded, and contain figure, as the circle and ellipsis, likewise the cissoid, and many others; but others are indeterminate, which may be produced infinitely, as the right-line, and the section of a right-angled, and

\* For it is easy to conceive a cylindric spiral described about a right-line, so as to preserve an equal distance from it in every part; and in this case the spiral and right-line will never coincide though infinitely produced.

obtuse

obtuse angled cone ; likewise the conchoid itself. But again, of those which may be produced in infinitum, some comprehend no figure, as the right-line and the conic sections ; but others, returning into themselves, and forming figure, may afterwards be infinitely produced. And of these some will not hereafter coincide, which resist coincidence, how far soever they may be produced ; but others are coincident, which will some time or other coincide. But of non-coincident lines, some are mutually in one plane ; and others not. And of non-coincidents subsisting in one plane, some are always mutually distant by an equal interval ; but others always diminish the interval, as an hyperbola in its inclination to a right-line, and likewise the conchoid \*. For these, though

\* As the choncoid is a curve but little known, I have subjoined the following account of its generation and principal property. In any given right line A P, call P the pole, A the vertex,



and any intermediate point C the centre of the conchoid; likewise, conceive an infinite right line  $CH$ , which is called a rule, perpendicular to  $AP$ . Then, if the right line  $Ap$  continued at  $p$  as much as is necessary, is conceived to be so turned about the abiding pole  $p$ , that the point C may perpetually remain in the right line  $CH$ , the point A will describe the curve  $Ao$ , which the ancients called a conchoid.

In this curve it is manifest (on account of the right line  $P O$ , cutting the rule in  $H$  that the point  $O$  will never arrive at rule  $C H$ ; but because  $b O$  is perpetually equal to  $C A$ , and the angle of section is continually more acute, the distance of the point  $O$  from  $C H$  will at length be less than any given distance, and consequently the right line  $C H$  will be an asymptote to the curve  $A O$ .

When

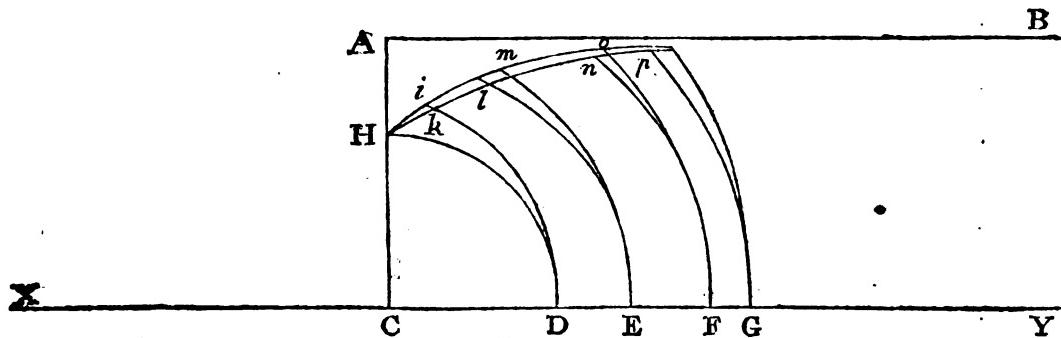
182. COMMENTARIES OF PROCLUS.

though they always diminish the interval, never coincide. And they mutually converge, indeed, but never perfectly nod to each other; which is indeed a theorem in geometry especially admirable, exhibiting certain lines endued with a non-assenting nod. But the right-lines, which are always distant by an equal interval, and which never diminish.

When the pole is at P, so that PC is equal to CA, the conchoid AO described by the revolution of PA, is called a primary conchoid, and those described from the poles  $p$ , and  $\pi$ , or the curves  $A\omega$ ,  $A\pi$ , secondary conchoids; and those are either contracted or protracted, as the excentricity  $P C$ , is greater or less than the generative radius CA, which is called the altitude of the curve.

Now, from the nature of the conchoid, it may be easily inferred, that not only the exterior conchoid  $A\omega$  will never coincide with the right line CH, but this is likewise true of the conchoids  $A\Omega$ ,  $A\pi$ ; and by infinitely extending the right-line  $A\pi$ , an infinite number of conchoids may be described between the exterior conchoid  $A\omega$ , and the line CH, no one of which shall ever coincide with the asymptote CH. And this paradoxical property of the conchoid which has not been observed by any mathematician, is a legitimate consequence of the infinite divisibility of quantity. Not, indeed, that quantity admits of an actual division in infinitum, for this is absurd and impossible; but it is endued with an unwearyed capacity of division, and a power of being diffused into multitude, which can never be exhausted. And this infinite capacity which it possesses arises from its participation of the indefinite duad; the source of boundless diffusion, and innumerable multitude.

But this singular property is not confined to the conchoid, but is found in the following curve. Conceive that the right line AC which is perpendicular to the indefinite line XY, is



equal to the quadrant arch HD, described from the centre C, with the radius CD: then from the same centre C, with the several distances CE, CF, CG, describe the arches IE, EF, FG, each of which must be conceived equal to the first arch HD, and so on infinitely. Now, if the points H, I, L, N, P, be joined, they will form a curve line, approaching continually nearer to the right-line AB (parallel to CY) but never effecting a perfect coincidence. This will be evident from considering that each of the sines of the arches HD, IE, NF, &c. being less than its respective arch, must also be less than the right-line AC, and consequently can never coincide with the right-line AB.

But

nish the space placed between them in one plane, are parallel lines. And thus much we have extracted from the studies of the elegant Geminus, for the purpose of explaining the present definition.

But if other arches  $D_i$ ,  $E_m$ ,  $F_o$ , &c. each of them equal to the right-line  $A C$ , and described from one centre, tangents to the former arches  $H D$ ,  $I E$ ,  $n F$ , &c. be supposed; it is evident that the points  $H$ ,  $i$ ,  $m$ ,  $o$ , &c. being joined, will form a curve line, which shall pass beyond the former curve, and converge still nearer to the line  $A B$ , without a possibility of ever becoming coincident: for since the arches  $D_i$ ,  $E_m$ ,  $F_o$ , &c. have less curvature than the former arches, but are equal to them in length, it is evident that they will be subtended by longer lines, and yet can never touch the right-line  $A B$ . In like manner, if other tangent arches be drawn to the former, and so on infinitely, with the same conditions, an infinite number of curve-lines will be formed, each of them passing between  $H_p$  and  $A B$ , and continually diverging from the latter, without a possibility of ever coinciding with the former. This curve, which I invented some years since, I suspect to be a parabola; but I have not yet had opportunity to determine it with certainty.

END OF THE FIRST VOLUME.

T H E  
PHILOSOPHICAL and MATHEMATICAL  
**COMMENTARIES OF PROCLUS,**  
O N  
THE FIRST BOOK OF EUCLID'S ELEMENTS.

TO WHICH ARE ADDED,  
A History of the Restoration of PLATONIC THEOLOGY,  
BY THE LATTER PLATONISTS:  
And a Translation from the Greek of  
**PROCLUS'S THEOLOGICAL ELEMENTS.**  
IN TWO VOLUMES.



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VOL II.

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LONDON, PRINTED FOR THE AUTHOR:

And Sold by T. PAYNE and SON; B. WHITE and SON; J. ROBSON; T. CADELL;  
LEIGH and Co. G. NICOL; R. FAULDER; and T. and J. EGERTON. 1792.

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## P R E F A C E.

**I** Now present the reader with the remaining part of the Commentaries of Proclus on Euclid: with the addition of his Theological Elements, and a History of the Restoration of the Platonic Theology, by the latter disciples of Plato. Should my design be enquired into, in combining works which the superficial observer will consider as opposite and heterogeneous, I answer that it is no less novel than certain, no less important than apposite and connected. Its novelty will be evident by assuring the reader, that a vindication of philosophical Polytheism, as embraced by the wisdom, and supported by the general voice of antiquity, is the ultimate tendency of its execution. Its connection too with geometry will be manifest to every Tyro in Platonism, and has been so copiously proved in the former volume, that it would be superfluous to repeat the demonstration in the present. I am well aware that nothing has been so much the subject of ridicule and declamation, of ignorant aspersion and impotent contempt, as the theology of the ancients. It has supplied the harangues of the pulpit with an endless variety of popular argument, and an exhaustless source of *priestly* elocution. It has been equally derided by the man of learning and the merchant, by the noble and the vulgar, by the peasant and the priest. But it still lives in the works of the ancients, it is still capable of being supported by sound reasoning, and sublime philosophy; and its intrinsic excellence and truth will extend its existence beyond the wreck of modern systems, and the desolation of ages. Like a strong and capacious ship it sails with majestic security through the ocean of time; and sustains with careless dignity the storms of opposition that roar round its well-compacted sides. The blasts of calumny may indeed impede

## P R E F A C E.

pede its progress, but are unable to shatter its indissoluble fabric ; and the prosperous gales of philosophy will always succeed the tempests of folly, and waft it with rapidity to the *enlightned* regions of mankind. The time perhaps is not far distant, when this fortunate change may commence. Above twelve hundred years have elapsed since the vessel of ancient wisdom visited the civilized parts of the world, and the nations were blest with its invaluable contents : and during this dreadful interval, ignorance and delusion, *jargon* and *reverie*, have held an undisturbed and universal reign. The depravity of the times is the subject of general complaint : genius no longer soars ; learning has evaporated into words ; and philosophy is but a name. Yet, though the restoration of ancient theology is the object of my most ardent desires, I much fear that a period still more barbarous, with respect to philosophy ; that an age still darker and more debased must precede its establishment on the earth. Prodigies and destruction attended, as we shall observe in the ensuing history, its departure from mankind ; and desolation will doubtless be the harbinger of its future appearance. The orb of vicissitude produces renovation and decay in regular succession ; and marks, as it revolves, the dormant events of future periods with *the ruinous* characters of the past. Let us, therefore, patiently wait for, and joyfully expect the happy moment when the breezes of philosophy shall arise with abundance and vigour ; and impel the vessel of theology laden with the riches of wisdom, on our natal coast. The revolution is certain, however remote : and the prospect is of itself sufficient to increase the vigour of exertion, and animate the expectations of hope ; to enable us to brave the storms of ecclesiastical persecution, and vanquish the resistance of folly.

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# C O M M E N T A R I E S

O F

# P R O C L U S.

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## B O O K III.

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### *Concerning Petitions and Axioms.*

\***S**INCE the principles of geometry are triply divided into Hypotheses, Petitions, and Axioms, the difference between these we have explained in the preceding books. But we now intend to discourse more accurately of petition and axiom, as especially necessary to our present design. For hypotheses, which are also called definitions, we have already explained. It is common, therefore, as well to axioms as to petitions, to require no demonstration, and no

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\* In the two preceding books of this work our author has displayed an uncommon degree of philosophic elegance and depth ; and in the present two, he no less manifests the greatest geometrical accuracy and skill. In the former he elevates us from participated truth to truth itself ; and from the glimmering light of universals reflected in the catoptric bosom of the phantasy, to the bright resplendence of ideas. In the latter he combines geometry and philosophy, occasionally cloathes the rigid accuracy of demonstration with the enchanting imagery of divine imaginations, and unites the graces of diction with the precision and sanctity of truth. Yet his genius, though rapid as a torrent, never passes beyond the bounds of propriety ; and though his thoughts are vehement and vast, they are at the same time orderly and majestic. For my own part I confess myself enamoured with the grandeur of his diction, astonished with the magnificence of his conceptions, and enlightened by the irradiations of his powerful genius. And I desire nothing so much as that others may experience similar effects from this admirable work. I only add, that the study of this second part is absolutely necessary to a perfect comprehension of Euclid's method and meaning ; and to the understanding geometry completely and philosophically. It is easy indeed to learn a science in a manner sufficient for mechanical purposes ; for this is accomplished by the *many* : but it is arduous to learn it with a view to the perception of truth ;

geometrical faith : but to be received as manifest, and to become the principles of the rest. But they differ mutually from each other, in the same manner in which we have distinguished theorems from problems. For as in theorems we propose to perceive and know that which follows a subject ; but in problems we are ordered to compare and do something : in the same manner also in axioms, we must receive whatever is manifest of itself, and easily apprehended by our untaught conceptions ; but in petitions we must receive whatever is easy to be done and compared, (since in admitting these, thought is not fatigued) and whatever requires no variety, and no kind of construction. Hence evident and indemonstrable cognition, and unconstrued assumption, distinguish petitions from axioms. Just as demonstrative cognition, and an assumption of things sought, together with construction, separates theorems from problems. For it is every where requisite, that principles in simplicity, indemonstrability, and self-evidence, should excel things posterior to principles. For universally (says Speusippus) of the things which cogitation pursues, some of its energies it produces without a various progression, prepares them for future enquiry, and has a more evident apprehension of these than of visible objects : but others which it is not able immediately to follow, by a transition proceeding from their nature, these it endeavours by consequence to pursue. Thus for example, *to draw a right line from one point to another*, it receives as evident, and easy to be done. For since in this case the line is composed from the indeclinable flux of a point, and at the same time advances in an orderly progression, because it no where more or less declines, it necessarily falls in another point. Again, if one extremity of a right line abiding, the other is moved about it, it will describe a circle without any labour. But if any one wishes to describe a helix of one revolution, it requires a

for this is alone the province of a *few*. It is easy to be knowing in effects, for these are obvious and common ; but it is difficult to investigate causes, for these are occult and rare. In short, a general and confused apprehension of a science may be readily obtained, without much labour and toil ; but a particular and accurate knowledge requires literal application, and patient endurance. For the one is like the distant prospect of a country, in which the larger parts are alone conspicuous to the observer's eye ; but the other resembles a near and distinct view, in which every thing is recognized essential to the perfection of the whole.

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more various operation. For it is generated by various motions. Likewise if any one wishes to construct an equilateral triangle, he will require a certain method for its construction. For the geometrical intellect says, when I understand a right line, which abides according to one of its extremities, but is moved about it according to the other, and at the same time conceive a point, which is moved in the line from the abiding extreme, I have described a helix of one revolution. For when at the same time both the extremity of the right line, which describes the circle, and the point which is moved in the right line, arrive at the same point, and coincide, they produce for me such a helix. And again, when I describe equal circles, and draw right lines from the common section to the centre of the circles, and a right line from one centre to the other, I shall have an equilateral triangle. The production of these, therefore, is very remote from a simple apprehension, and primary notion. For we are content to pursue the progressions of their origin. Hence it happens that these are compared with greater ease or difficulty, and are exhibited with many or fewer mediums, according to the habit of those who enter on this undertaking: but that they require demonstration and construction, on account of the property of the things sought, which wants the evidence of petitions and axioms.

Petition, therefore, and axiom, are simple and easy to be apprehended. But petition, indeed, commands us to fabricate, and provide a certain matter, in order to the assignation of the *symptom*, which possesses an easy and simple apprehension: but axiom pronounces a certain essential accident, of itself known to the hearers. As that *fire is hot*, or any other of those manifest truths, he who doubts of which, we consider as either wanting sense or punishment. Hence, petition and axiom are of the same genus; but they differ in the above-mentioned manner. For each is an indemonstrable principle, but this after one mode, and that after another, as we have already observed. But some think that all these should be called petitions, in the same manner as all problems, *things sought*. For Archimedes beginning his book of *Equiponderants*, *we desire it may be granted* (says he) *that things equally heavy, from equal lengths, will equally ponderate*; though

some would rather chuse to call this an axiom. But others call all these axioms, in the same manner as they denominate every thing a theorem, which requires demonstration. For, according to the same proportion, as it seems they pass from proper names to such as are common. Nevertheless, as a problem differs from a theorem, so petition from axiom: though both these last are indemonstrable, and the former require demonstration. And the one, indeed, is assumed as easy to be done, but the other is granted as easy to be known by the common consent of all men. After this manner, therefore, Geminus distinguishes petitions from axioms.

But others will perhaps say, that petitions are indeed proper to the geometrical matter: but that axioms are common to the universal theory, which is conversant about the *how-much*, and the *how-many*. For the geometrician knows *that* which requires *that all right angles are equal*, and *that every finite right line may be produced straight forwards*: but *that* which says, *things equal to one and the same are equal to each other*, is a common conception, which not only the arithmetician employs, but every one endued with science, accommodating that which is common to his own particular matter. But Aristotle (as we have before observed\*) says, that petition, since it is demonstrable, is not granted by the hearer, yet is received as a principle: but that axiom is of itself indemonstrable, and that this is confessed by all, according to habit, though some, for the sake of disputation, have doubted its evidence. Since then, there are these three differences, according to the first, which by operating, and knowledge only distinguishes petition from axiom, it is manifest that that which says *all right angles are mutually equal*, is not a petition. Nor the fifth, which says, *if a right line falling on two right lines makes the internal angles towards the same parts less than two right, those right lines infinitely produced, shall coincide towards the parts in which the angles less than two right subsist*. For these are neither assumed in construction, nor do they command any thing to be done: but they exhibit a certain symptom, inherent in right angles, and in right lines, departing from

\* See the second section of the Dissertation, Vol. I.

angles less than two right. But, according to the second difference, that will not be an axiom which says, that *two right lines cannot comprehend space*, which some at present consider as an axiom. For this is proper to the geometric matter, as likewise that which affirms that *all right angles are equal*. But according to the third difference, which is Aristotelic, all those which produce their own credibility by a certain demonstration, are petitions; but whatever are indemonstrable, are axioms. Apollonius, therefore, in vain endeavours to deliver the demonstrations of axioms: for Geminus very properly observes, that some have attempted demonstrations of indemonstrables, and have endeavoured from more unknown mediumis, to prove things manifest to all, into which error Apollonius has fallen, who wishes to prove the axiom true, which says, *that things equal to one, and the same, are equal to each other*: but that others assume in the place of indemonstrables, things requiring demonstration. As is the case with Euclid himself, in the fourth and fifth petition. For some say, that this last, as ambiguous, requires demonstration. Indeed, is it not ridiculous, that theorems should be assigned as indemonstrable, the converse of which are demonstrable? For that the internal angles of coincident right lines are less than two right, Euclid himself shews in the theorem, which says, *that two angles of every triangle, however taken, are less than two right*: besides, it may be perspicuously shewn, that not every thing equal to a right angle is a right angle. Hence, says Geminus, the converse of these are not to be granted indemonstrable. It seems therefore, according to the ordination of this man, that there are, indeed, three petitions: but that the other two, and the converse of these, require demonstrating science: and that in the axioms, the one which says, that *two right lines cannot comprehend space*, is superfluously added, since its credibility must be derived from demonstration. And thus much concerning the difference of petitions and axioms. Again, of axioms, some are proper to arithmetic, but others to geometry; and others are common to both: for that which says, *every number is measured by unity*, is an arithmetical axiom. But that which says *equal right lines agree amongst themselves*, as also this which affirms that *every magnitude is divisible in infinitum*, are geometrical axioms:

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axioms : but the one which says that things equal to the same, are mutually equal, and all of this kind are common to both. However, it must be observed, that each science uses such as the last, according to its proper subject ; as geometry in magnitudes, but arithmetic in numbers. In like manner of petitions, some are peculiar to particular sciences, but others are common to all. For you must call the petition which requires to be granted, *that a number may be divided into the least parts*, peculiar to arithmetic : - but this, *that every finite straight line may be produced straight forwards*, peculiar to geometry ; and the one which desires us to grant, *that quantity may be infinitely increased*, common to both ; for this passion is equally found to reside in number and magnitude.

### PETITIONS or POSTULATES.

#### I.

Let it be granted that a straight line may be drawn from any one point to any other point.

#### II.

That a terminated straight line may be produced to any length in a straight line.

#### III.

And that a circle may be described from any centre, at any distance from that centre.

According to the opinion of Geminus, these three are necessarily placed among petitions, as well on account of their facility, as because they command us to do something. For this, *to draw a right line from every point, to every point*, follows the definition, which says, *that a line is the flux of a point*, and a right line *an indeclinable and inflexible flow*. If then we conceive a point to be moved with an un-inclined, and the shortest motion, we shall fall upon another point, and the first petition will be produced, and we shall understand nothing various or difficult. But if when the right line itself is terminated by a point, we conceive its extremity moved with the shortest indeclinable motion,

motion, the second petition will arise from an easy and simple apprehension. But if we again imagine that the terminated right line abides according to its other extreme, but that it moves about that which abides according to the rest, the third petition will be produced ; for the centre is the point which abides; but the interval the right line. Since the distance of the centre, from all parts of the circumference, is always equal to the quantity of this interval. But if any one should doubt how we apply motion in geometrical concerns, which have an immoveable existence ; and how we can move impartibles, (since this is impossible) we request him to call to mind what we have demonstrated in the beginning of these Commentaries. I mean that the reasons of things subsisting in the phantasy, describe there all the images of cogitation, of which cogitation itself possesses the reason : for an intellect of this kind is an unwritten, ultimate, and passive tablet. Hence, it receives forms from another, accompanied with motion ; but we must not understand a corporeal but imaginative motion, and must by no means admit that impartibles are moved with corporeal motions, but that they suffer imaginative progressions. For intellect, though impertible, is moved, yet not according to place, and the phantasy has a proper motion according to the impertible which it contains : but we only regarding corporeal motions, neglect those which are made in things destitute of interval. Impartibles, therefore, are pure from corporeal place, and external motions : but another species of motion, and another place congenial to such motions, is considered in their progressions. For, indeed, we should say, that a point also has position in the phantasy, and should not enquire how an impertible can abide, which is at the same time moved elsewhere, and comprehended by place. Since the place of things, with dimension, possesses itself dimension ; but the place of impartibles is destitute of all dimension. The proper species therefore of geometrical concerns, are different from the things they produce ; and the motion of bodies is different from that of the forms in the phantasy ; and the place of partible is different from that of impertible natures ; and it is requisite, by distinguishing these, neither to confound nor disturb the essences of things.. But it appears that the first of these three petitions declares to us in-  
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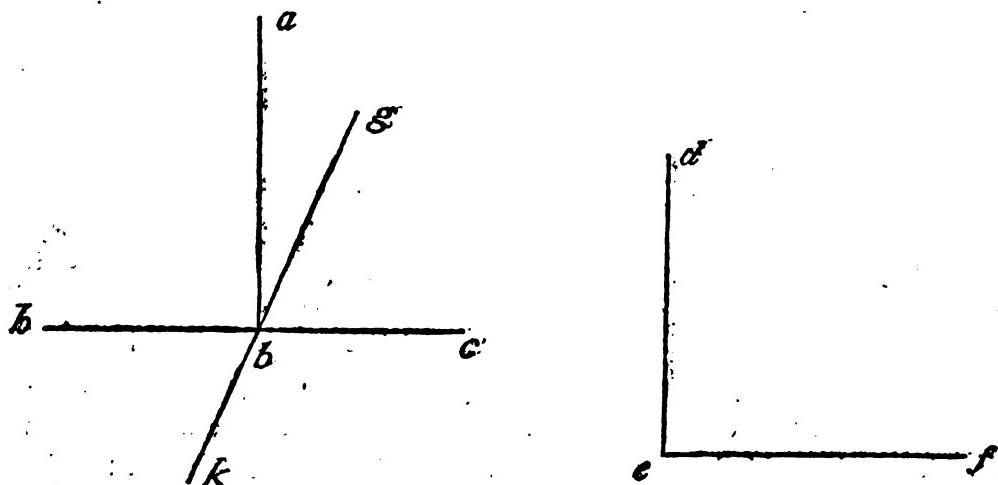
images, how the *things which are*, are contained in their own impenetrable causes, and are terminated by their immaterial bound; and that previous to their constitution, they are on all sides comprehended in their indivisible embrace: for the points existing, a right line is drawn from the one to the other, is terminated by, and received between them. But the second indicates how the *things which are* by possessing proper causes proceed to all things, preserving in them a continuation not derived from the natures into which they proceed; but that through a cause of infinite power, they endeavour to permeate every where, with a never-failing progression. And the third petition shadows forth the manner in which these progressions return again to their proper principles, for the convolution of a point producing a circle, by moving about an abiding point, imitates a circular regression. But it is requisite to know, that every line cannot be infinitely produced, for the circle and cissoid, and all such as describe figure, are incapable of this property; as likewise some which produce no figure. For the helix of one revolution cannot be infinitely produced, since it is constituted between two points; nor any other lines similarly formed. But neither is it possible to extend every line from every point, to every point; for every line cannot subsist between all points: and thus much for the three first petitions; let us now proceed to the rest.

### IV.

All right angles are equal to each other.

If the present petition is considered by us as manifest, and as requiring no demonstration, it is not a petition according to the opinion of Geminus, but an axiom; for it affirms a certain essential accident of right angles, not commanding us to perform any thing according to a simple conception. But neither is it a petition according to the division of Aristotle: for petition, according to his opinion, requires some demonstration. But if we should say it is demonstrable, and enquire after its demonstration, yet according to the opinion of Geminus, it ought not to be placed among petitions. The equality, therefore, of right angles, appears from our common conceptions; for since a right angle has the relation of unity or bound to the infinite increase

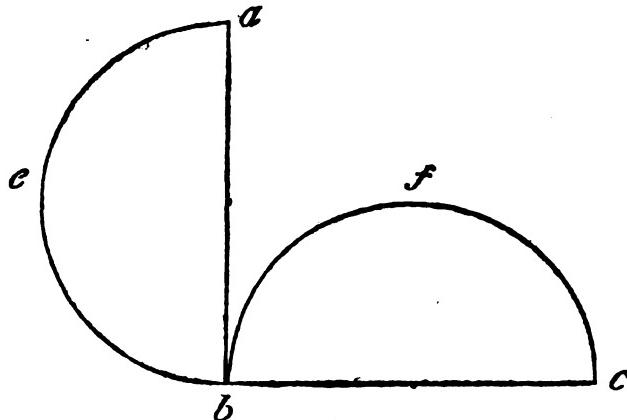
and decrease of the angles on each side, it is equal with respect to every right angle, since we constitute the first right angle after this manner, by a right line making angles on each side of the right line on which it stands equal to each other; but if it be requisite to produce a linear demonstration of this, let there be two right angles, one  $a b c$ . the other  $d e f$ .



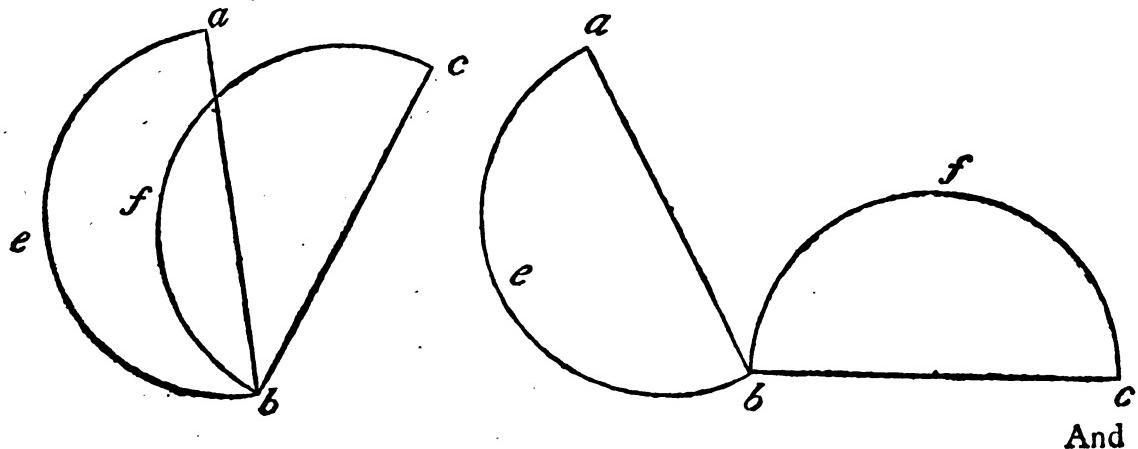
I say that they are equal; for if they are not equal, one of them must be greater, suppose the angle at  $b$ . If then the line  $d e$  be adapted to the line  $a b$ , the line  $e f$  shall fall within. Let it fall as  $b g$ , and let the line  $b c$  be produced to  $b$ ; because, then  $a b c$  is a right angle,  $a b b$  also shall be a right angle, and they shall be mutually equal to each other, from the tenth Definition: the angle  $a b b$  therefore, is greater than the angle  $a b g$ . Let again the line  $g b$  be produced to  $k$ , because, therefore  $a b g$  is a right angle, the successive angle  $a b k$  shall be a right one, and consequently equal to  $a b g$ . Hence, the angle  $a b b$ , shall be less than the angle  $a b g$ ; but it was also greater, which is impossible: but this has been shewn by other expositors, and requires no great consideration: But Pappus very properly admonishes us, that the converse of this Petition is not true; I mean, that every thing equal to a right angle, is a right angle; though if it be rectilinear, it is without doubt a right angle. But a curvilinear angle may also be exhibited equal to one that is right: for let there-

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be conceived two equal right lines,  $a b$ , and  $b c$ , making the angle at the point  $b$ , right;



and on them let the semicircles  $a e b$ ,  $b f c$ , with a proper centre and interval be described; because, therefore, the semicircles are equal, they shall have a mutual congruence, and the angle  $e b a$ , is equal to the angle  $f b c$ , and  $a b f$  is common: the whole right angle, therefore, is equal to the lunular, i. e. to  $e b f$ , and yet the lunular is not a right angle. In the same manner, if the angle  $a b c$  should be obtuse or acute, a lunular angle may be shewn equal to it (for this is that genus of curvilinear angles which agrees with such as are rectilinear), only this is to be observed, that in a right and obtuse angle, it is requisite to add the middle angle, which is contained by the line  $a b$ , and the circumference  $b f$ ; but in an acute angle to take this away: for the right line  $c b$ , in these cases, cuts the circumference  $b e$ . The truth of which, will be evident from the following figures:



And hence, it appears, that all right angles are mutually equal to each other, and that not every thing equal to a right angle, is consequently a right ang'e : for if it be not rectilinear, how can it be called right. But it is also manifest from this Petition, that angular rectitude is allied to equality, in the same manner as acuteness and obtuseness are related to inequality. For rectitude and equality, as also similitude, are of the same co-ordination, (for each exists under bound) : but acuteness and obtuseness, as also dissimilitude, are of the same series with inequality. For they are all produced from *bound* and *infinite*. Hence some, regarding the quantity of angles, say, that a right angle is equal to a right : but others, considering their quality, affirm that one is similar to another. For similitude in qualities is the same as equality in quantities.

## V.

If a right line falling upon two right lines, makes the internal angles towards the same part less than two right, those right lines, if infinitely produced, shall coincide in that part, in which the angles less than two right, are placed.

This ought to be entirely blotted out from the number of Petitions, for it is a theorem including many doubts, which Ptolemy in one of his books proposes to solve ; but it requires in its demonstration both many definitions and theorems ; and Euclid also exhibits its converse as a theorem. But perhaps some, from an erroneous conception, may think that this should be placed among the petitions, as that which produces credibility of itself, respecting the inclination of right lines, on account of the diminution of two right angles. To such as these, Geminus rightly answers, that from the authors of this science, we learn not entirely to give credit to imaginative probabilities, for the purpose of accomplishing geometrical reasons : for it is similar (says Aristotle) to require demonstrations from a rhetorician, and patiently listen to a geometrician, disputing from probability. And Simmias in the Phædo of Plato, says, "I know that those who demonstrate from appearances, are vain." Hence, in the present instance, it is true and necessary that right lines should incline, while right angles are diminished : but this, that the inclining lines, while they are more

more produced, should at length coincide, is probable, but not necessary, unless some reason demonstrates that this is true in right lines: for there are certain lines infinitely inclining, and never coinciding, and though this appears incredible and admirable, yet it is true, and has been observed in other forms of a line. Is it therefore possible that this can be accomplished in right lines which takes place in others? For before we procure conviction of this, from demonstration, the properties exhibited in other lines molest the phantasy by the contrary images they produce. But if the reasons doubting against the coincidence of lines are very strong, ought we not much more to expel this improbable and irrational supposition from our doctrine? And thus it appears that a demonstration is to be sought for of the present theorem, and that it is foreign from the property of Petitions: but how it is to be demonstrated, and by what reasons the objections urged against it are to be removed, we shall shew in our comment on the proposition, where it is used by the institutor of the elements as manifest. For then it will be necessary to exhibit its evidence, since it does not present itself to our view with indemonstrable clearness, but becomes manifest through the medium of demonstration alone.

## AXIOMS.

## I.

Things which are equal to the same, are equal to one another.

## II.

If equals be added to equals, the wholes are equal.

## III.

If equals be taken from equals, the remainders are equal.

## IV.

If equals are added to unequal, the wholes are unequal.

## V.

If equals be taken from unequal, the remainders are unequal.

## VI.

Things which are double of the same, are equal to one another.

## VII.

## VII.

Things which are halves of the same, are equal to one another.

## VIII.

Things which coincide with each other, are mutually equal.

## IX.

The whole is greater than its part.

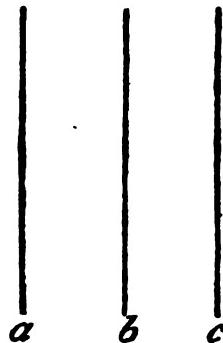
## X.

Two right lines cannot comprehend space.

These are the things which, according to the opinion of all men, are called indemonstrable axioms, so far as their certainty is admitted by all, and no one disputes their evidence. For propositions also are often simply called axioms, of whatever kind they may be, whether they are immediately proper, or require some declaration; and the Stoicks, indeed, are accustomed to call every simple enunciative speech an axiom: and when they write on dialectic arts, they say that they discourse on axioms. But some, distinguishing more accurately axioms from other propositions, give this appellation to a proposition immediate, and producing credibility of itself, on account of its evidence: as also Aristotle and geometricians themselves affirm. For, according to the opinion of these, an axiom is the same as a common conception. By no means, therefore, must we praise Apollonius the geometer, who writ (as it appears) demonstrations of axioms, because he performs the very opposite to Euclid: for he, indeed, enumerates that which is demonstrable among Petitions; but Apollonius endeavours to find out demonstrations of indemonstrables. But these naturally differ from each other, and the genus of the sciences is different: I mean of the things which take place about immediate propositions, which are entirely subject to our knowlege, on account of their evidence; and of things which use demonstrations, which receive principles from them; and which, when received, they orderly employ in their proper conclusions. But that the demonstration of the first axiom, which Apollonius persuades himself he has invented, possesses a medium, not more known, but more dubious than the conclusion

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clusion may be known by any one from a slight inspection. For let (says he)  $a$  be equal to  $b$ , and  $b$  to  $c$ , I say that  $a$  also is equal to  $c$ .



For since  $a$  is equal to  $b$ , it occupies the same place as  $b$ . And because  $b$  is equal to  $c$ , it occupies the same place as  $c$ ; and so  $a$  occupies the same place as  $c$ , they are therefore equal. Now in this demonstration it is requisite that two things must be previously assumed; one, that things occupying the same place, are mutually equal; but the other, that things occupying the same place, with the same thing, mutually occupy the same place: but these are evidently more obscure than the present axiom. For it is proper to enquire how are things, which fill the same place equal, according to the whole, or according to a part; or according to a figure of speech: hence we must by no means admit a transition to place,\* which is more unknown than the

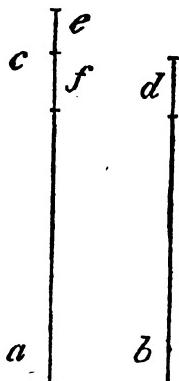
\* The nature of place has been a subject of much curious and deep speculation to the Peripatetic and Platonic philosophers, as may be seen in the very valuable Commentaries of Simplicius on Aristotle's Physics; so that Proclus does not affirm without reason, that place is more obscure than the natures it contains. But as the opinion of our philosopher, concerning place, is so admirably profound, subtle, and remarkable, I persuade myself the following translation from the fourth book of Simplicius on the Physics, containing his sentiments at large on this subject, will not be unacceptable to the liberal English reader. "Proclus (says Simplicius) having proved from the arguments of Aristotle, that place is neither matter nor form, concludes, that it is a certain interval:" after which, he reasons as follows. "This interval then, is either nothing or something; and if nothing, local motion will consist in a transition from nothing to nothing; but all motion subsists according to something. But if it ought to be called something, it is either corporeal or incorporeal; and if incorporeal, an absurdity will ensue; for it is necessary that place should be equal to the thing placed. But how can body, and that which is incorporeal be equal? For equal is found in quantities, and especially in those of a similar kind, as lines are equal to lines, superficies to superficies, and bodies to bodies: place, therefore, is body, if it be a certain interval; but if body, it is either inmoveable or immovable; and if moveable, in whatever manner it may be moved, it must necessarily be moved according to place,

the natures it contains ; for the invention of its essence is difficult and ambiguous. That we may avoid prolixity, therefore, all axioms are to be delivered as things immediate and self-manifest, since they are of themselves known and credible ; for he who brings demonstration to things the most manifest, does not confirm their truth, but diminishes the evidence we possess in the untaught and innate conceptions of the soul : but this is to be received concerning axioms, as a judgment of their peculiarity ; and that all of them are of the common kind of the mathematical sciences ; and that each of them is said to be verified,

so that place again will require another place, which is impossible, as was also evident to Aristotle and Theophrastus ; for Aristotle says, that a vessel is a moveable place, but place an immovable vessel, because place is naturally immovable. But if it be immovable, it is either indivisible, which cannot be divided by the bodies entering its receptacle, since one body cannot penetrate another ; or it is divisible, as air and water are divided by the bodies entering into their yielding natures ; but if it be divisible, the whole being dissected or divided, the divided parts will be moved on each side, and place will be the first mutable, since its parts are moved ; but we have demonstrated that it is immovable. Again, the parts being separated, we ask where that which is divided betakes itself ; for there must be again given or investigated another interval, intervening between the divided parts, which may receive and be placed together with that which is divided ; and this will be the case, in infinitum. Place, therefore, is an indivisible body ; and if an indivisible body, either material, or destitute of matter : but if material it will not be indivisible, for it is requisite that all material bodies, when permeated by other material bodies, should be divided by them, as is the case with our bodies when they fall into water. But immaterials alone resist all division and this from a necessity of nature ; for every body destitute of matter is void of passion ; but every thing which is divided likewise suffers. Since division is a certain affection of bodies, which extirpates and destroys their unity and connection ; for that which is continuous, so far as continuous suffers no other affection or molestation than section, which destroys and takes away its continuity. That we may therefore collect together what we have separately demonstrated, place is an immovable indivisible body, destitute of matter. And if this be admitted, it is evident that it is a body by far less material than the rest, and indeed less than the matter contained in things which are moved. Hence, if light is the most simple of these (for fire is more incorporeal than the other elements, and fire is lucid) it will be manifest, that since light is the purest among the rest, light will be place. Conceive, therefore, two spheres, of which one is composed from many bodies, and the other of light alone, and let both be of equal bulk ; then, by establishing the sphere of light, together with the centre, and giving the composite sphere a revolution in the circumscribing sphere of light, you will perceive the world moved in immovable light, and according to its whole extension, immovable, similar to place, but moved according to its parts, because these are less than place." Now, from this demonstration of Proclus, it follows by a necessary consequence, since contraries are contained under the same genus that darkness, if it be any thing positive, is the most material of all bodies ; and hence, the most material natures will participate the most of darkness, as indeed, is evident in the elements of earth and water. It likewise follows that whatever exists in perfect darkness, exists out of corporeal place, which, however paradoxical, is perhaps, no less true than wonderful to conceive.

not only in magnitudes, but also in numbers, and motions, and times : and this indeed is necessary. For equal and unequal, the whole and part, and the more and the less, are common to discreet and continued quantities. The contemplation, therefore, which is conversant with times and motions, numbers and magnitudes, requires all these, as things evident by their own intrinsic light ; and in all of them both that is true, which says, *things equal to the same, are equal to one another* ; as likewise each of the axioms we have assumed : but as they exist in common, each science uses them according to its proper matter, and one indeed, as in magnitudes ; but another, as in numbers ; and another, as in times ; and after this manner in each science, the conclusions become peculiar and apposite, though the axioms are common. Besides, it is likewise requisite not to contract the number of these to the least, as is done by Heron, who only establishes three axioms ; for this also is an axiom, *the whole is greater than its part*, and the geometrician every where assumes this in his demonstrations ; as also, that *things which mutually coincide, are equal* ; for this is employed with advantage in the solution of the fourth Proposition. Nor is it proper to join some with others, of which some are proper to the geometric matter, as *that two right lines cannot comprehend space*, (since axioms are, as we have said, of a common kind) ; but others are consequent to things established, as that which says, *things double of the same, are equal*. For this is consequent to the axiom, affirming, that *if to equals you add equals, the wholes are equal*, since things equal to the half, because they assume the half, become double to the same, and mutually equal, on account of an equal addition : and according to this reason, not only the doubles, but also the triples, and all multiples of the same quantity will appear equal. But with these axioms, Pappus says, that certain others are to be classed, as if *unequals are added to equals, the excess of the whole, will be equal to the excess of the adjuncts*. And on the contrary, *if equals are added to unequals, the excess of the wholes is equal to the excess or difference of the unequals themselves*. And these also are manifest from themselves, yet they may

may be made manifest as follows. Let  $a$  be equal to  $b$ , and add to



each the unequals  $c$   $d$ , but let  $c$  be greater than  $d$  by  $e$ , and the remainder be  $f$ ; because, therefore,  $a$  is equal to  $b$ , and also  $f$  to  $d$ ;  $af$  will be equal to  $b d$ . For if equals are added to equals, the wholes are equals:  $a c$ , therefore, exceeds  $b d$ , by  $e$  only, by which alone  $c$  exceeds  $d$ . Again,  $c$  and  $d$  are unequals, to which, let the equals  $a$  and  $b$  be added, and let  $e$  be the excess of  $c$ , above  $d$ , and the remainder be  $f$ ; because, therefore,  $a$  is equal to  $b$ , and  $f$  to  $d$   $af$  will be equal to  $b d$ ; the whole, therefore,  $a c$ , will exceed  $b d$ , by  $e$  only, by which  $c$  also exceeds  $d$ . These, therefore, are consequent to the aforesaid axioms, and are, not undeservedly, in many copies, omitted. But whatever others he adds to these, have been previously assumed by definitions, to which they are consequent. As for example: *that all the parts of a plane and a right line mutually agree*; for things placed in their extremities, possess a nature of this kind; and *that a point divides a line, but a line a superficies, and a superficies a solid*. For all things are divided by the natures by which they are proximately bounded; and *that infinite subsists in magnitudes, by addition and diminution, but according to capacity only, in both these respects*: for every thing continuous may be infinitely divided and increased. But, as we have summarily spoken concerning these, it remains that we consider things consequent to principles; for thus far principles extend themselves. But of those who oppose geometry, some very much doubt concerning principles, endeavouring to shew that the terms have no subsistence, whose arguments, indeed, are

known in common, who endeavour to take away all science, and, like hostile foes from a foreign region, demolish the fruits and fecundity of philosophy, as is the case with the Pyrrhonian philosophers; but others only propose to themselves the subversion of geometrical principles, as the Epicureans. Others, again, admitting the principles, affirm, that things consequent to the principles cannot be demonstrated, unless something else is granted, which was not previously assumed in the principles. Zeno exercised this mode of contradiction, who was a Sidonian by birth, but of the Epicurean sect, against whom Posidonius wrote an entire book, exhibiting the whole of his imbecile opinion; and thus much may suffice for the difference of opinions concerning principles. We shall shortly consider the troublesome objection of Zeno: but now, after we have briefly resumed the consideration of theorems and problems, their difference, and the divisions they receive, we shall proceed to an exposition of the things exhibited by the institutor of the elements, gathering the more beautiful observations upon the propositions found in the writings of the antients, and contracting the infinite prolixity of their discourses; but delivering such things as are more artificial, and full of methods producing science, dwelling more on an accurate treatise of things than on the variety of cases and assumptions, to which young men, for the most part, eagerly incline.

### P R O P O S I T I O N I. P R O B L E M.

Upon a given terminated right line to describe an equilateral triangle.

Since all science is two-fold, and one is conversant about immediate propositions, but another about things, which are exhibited and provided from the propositions, and universally about the consequents to principles; this, again, divides itself in geometrical discourses, into the solution of problems, and the invention of theorems. And problems, indeed, geometry denominates things in which it proposes to procure, manifest, and fabricate that, which, in a certain respect, has no existence; but it calls theorems, things in which it appoints to perceive,

perceive, know, and demonstrate that which either exists, or does not exist. For problems command us to undertake the origin, positions, applications, descriptions, inscriptions, circumscriptions, coaptations, and contacts of figures, and every thing of this kind : but theorems endeavour to procure our assent to symptoms, and things essentially inherent in the subjects of geometry, and to convince by demonstrations. For geometry discourses concerning every object of enquiry, which is possible to be effected, referring some things to problems, but others to theorems ; since it enquires concerning the *what*, in a two-fold respect : for it either seeks for the reason and intelligence of the thing ; or for intelligence, and the essence of the subject. I say, for example, as when it requires what a line of similar parts may be : for in an enquiry of this kind, it either desires to find the definition of such a line, as, *that a line of similar parts is that which has all its parts agreeing with all* ; or to receive the species of lines of similar parts, as that it is either *right*, or *circular*, or a *cylindric helix*. Besides, prior to this, it enquires, by itself, concerning the *if*, and this especially in its determinations, agitating, whether the object of its enquiry is possible or impossible, what place it possesses, and in how many ways. It likewise seeks concerning the *what kind* ; for when it considers the essential accidents of a triangle, circle, and parallels, it is manifest, that in such cases it seeks after the *what kind* ; but many have thought that geometry very little contemplated the *cause*, and the *why*. And of this opinion is Amphinomus, led by the decisions of Aristotle : but (says Geminus) an enquiry into these may be found in geometry. For does it not belong to geometry to enquire for *what cause* infinite equilateral multangles may be inscribed in circles, but to describe solid equilateral and equiangular multangles, and constructed from similar planes, in spheres, is impossible ? To whom does an investigation of this kind belong, except to a geometrician ? When, therefore, to geometricians the syllogism is by an impossibility, they alone desire to find the symptom ; but when by a principal demonstration, then again if the demonstrations are in that which is particular or partial, the cause is not yet manifest ; but if in that which is universal, and in all similars, the *why* becomes immediately manifest : and thus much concerning objects of enquiry.

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But every problem and theorem which receives its completion from its own perfect parts, ought to possess in itself all the following parts: *proposition*, *exposition*, *determination*, *construction*, *demonstration*, and *conclusion*. But of these, *proposition* informs us what the object of enquiry is from a given datum; for a perfect proposition is composed from both; but *exposition* receiving the datum essentially, prepares for the question. Again, *determination* separately explains the thing sought for according to the *what*; but *construction* adds to the datum what is wanting to the investigation of the thing sought; and *demonstration* skilfully collects the proposition from the concessions. But the *epilogue*, or conclusion, is again converted to the proposition, by confirming that which is exhibited. And so many, indeed, are all the parts of problems and theorems; but *proposition*, *demonstration*, and *conclusion*, are especially necessary, and exist in all; for it is requisite that the thing sought for should be previously known, and that this should be shewn by proper mediums, and that *what* is exhibited should be concluded; and it is not possible that any one of these three can be wanting; but the rest are, indeed, received in many places; but in many, because they produce no utility, are omitted. For *determination* and *exposition* are not found in the problem, which says, *to construct an isosceles triangle, which will have each of the angles at the base double of the other*; but *construction* has frequently no subsistence in many theorems, the demonstration being sufficient to exhibit the thing proposed from the data, without any addition. When, therefore, shall we say that *exposition* fails, when no datum is given in a proposition? Because, though *proposition*, for the most part, is divided into *datum*, and *the thing sought for*, yet this is not always the case; but sometimes *the thing sought for*, alone affirms that which it is requisite to know or effect, as in the aforesaid problem; for it does not previously say from what datum it is requisite to construct an isosceles triangle, which shall have each of the angles at the base, double of the remaining one; but that it is required to effect this. And here, indeed, the admission of the proposition takes place from things previously known; for we must know the meaning of the terms *isosceles*, *equal* and *double* (since this, as Aristotle observes, is  
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the property of all ratiocinative discipline\*), yet nothing is subjected to us as in other problems, as in that which says, *to bisect a given terminated right line.* For here the right line is given, but we are ordered to divide it into two parts; and the datum is separately determined from the object of enquiry. When, therefore, a proposition has both of these, then also *determination* and *exposition* are found; but when the datum is deficient, these also fail, since *exposition* and *determination* belong to the datum: for this will be the same with the proposition. Indeed, what else do we say, when determining in the aforesaid problem, unless that it is requisite to find an isosceles of this kind? But such was the proposition: if then the proposition has neither this *datum*, nor *thing sought*, *exposition* will, indeed, be silent, because there is no datum; but *determination* will be neglected, lest it should become the same with the *proposition*: but you may find many other problems of this kind, especially in arithmetic, and in the tenth book of these Elements, as, *to find a medium comprehending two right lines commensurable in power, and every thing of this kind.*

But every datum may be given in these four modes, either in *position*, or *proportion*, in *magnitude* or *form*; for a point, indeed, is given in *position* only, but a line and the rest in all the four. Thus, when we say, *to bisect a given rectilineal angle*, we declare the species of the angle given, as that it is right lined, lest we should also seek to bisect a curvilinear angle by the same methods. But when we say, *from the greater of two unequal right lines, to cut off a part equal to the less*, the lines are given in magnitude; for the less and the more, finite and infinite, are the proper predication of magnitude. But when we say, *that if four magnitudes are proportional, they shall be also alternately proportional*, the same proportion is given in the four magnitudes: but when it is requisite, *from a given point to place a right line equal to a given right line*, then the point is given in position. From whence, since position may be various, construction also receives variety; for the point is given either without the right line, or in the right line, and in the extremity, or without the ex-

\* See Section second, of the Dissertation, in Vol. I. of this work.

## 22. COMMENTARIES OF PROCLUS.

tremity of the right line. Since, therefore, a datum has a four-fold acceptation, it is manifest, that exposition also is four-fold; but sometimes it connects two or three modes. Again, we find that demonstration sometimes possesses things proper to demonstration, exhibiting the thing sought for from mediate definitions; for this is the perfection of demonstration, but that sometimes it argues from certain signs. And it ought not to be concealed, that geometrical discourses have every where that which is necessary, on account of the subject matter, but are not every where perfected by demonstrative methods. For when, because *the external angle of a triangle is equal to the two internal and opposite ones*, it is shewn, that *the three internal angles of the triangle are equal to two right*, how is this demonstration from the cause? And is not a sign the medium in this case? For the external angle not yet existing, since the internal angles exist, they are equal to two right, since it is a triangle, though the side is not produced; but when, by a description of circles, the triangle, which is constituted, is shewn to be equilateral, the apprehension takes place from the cause. For we say, that the similitude and equality of the circles is the cause of the triangles equality with respect to its sides.

But geometrical discourses are likewise accustomed to make the conclusion, in a certain respect, two-fold. And this, when they exhibit things agreeable to the data, and reason universally, recurring from a particular conclusion to that which is universal; for when they do not use the property of the subjects, but placing the data before our eyes, describe an angle or right line, they think that which is concluded in this, is to be concluded in every thing similar: they pass on therefore to *universal*, lest we should think that the conclusion is particular. But their transition is effected in the best manner, since they employ, in demonstration, *the things placed*, not considered as such, but considered as similar to others: for it is not because such a particular angle is proposed that they effect a bipartite section, but because it is rectilineal only. But quantity, is indeed, proper to the proposed angle; but rectilineal is common to all right lines: let then the given angle be a right one. If therefore, we receive rectitude in the demonstration, we cannot pass to every species of right lines; but

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if we do not subjoin its rectitude, or being right angled, but alone consider its being rectilineal, the discourse may be adapted to all right lined angles; and all that we have previously observed we may contemplate in this first problem. For that it is a problem, is evident, since it commands us to construct an equilateral triangle: but *proposition* in this, consists from a *datum* and *thing sought*. For a terminated right line is given, but it is *enquired* how an equilateral triangle may be constructed upon it, and the *datum* indeed precedes, but the *thing sought* follows; so that we may say, by conjoining the two, *if there be a terminated right line, it is possible to construct upon it an equilateral triangle*; for a triangle cannot be constructed without the existence of a right line, since it is comprehended by right lines; nor upon an unlimited line, for an angle cannot be constructed unless it is made on one point, but in an infinite line there can be no extremity or bounding point. But after proposition, *exposition* follows, as, *let there be given a terminated right line*. And here we may see that *exposition* alone pronounces the *datum*, but by no means subjoins the *thing sought*; but after this we shall find *determination*: *it is required upon the given terminated right line to construct an equilateral triangle*; and here we may observe that *determination* is in a certain respect, the cause of attention, for it makes us more attentive to the demonstration, by pronouncing the *thing sought*, as *exposition* causes us to be more docile, by placing the *datum* before our eyes. Again, after *determination*, *construction* follows, *from one extremity of the right line, as a centre, but with the remainder as an interval, let a circle be described*. And again, *with the other extremity, as a centre, and with the same interval; let a circle be described; and from the common point of the sections of the circles, to the extremities of the right line, let right lines be continued*: And here we may observe, that Petitions are used in the construction, this for one, *from every point to every point, to draw a right line*; and also this, *with every centre and interval to describe a circle*; for universally Petitions are the sources of utility to constructions, but Axioms to demonstrations; demonstration therefore follows, because, *then each extremity of the given right line is the centre of the circle surrounding it, the right line which reaches to the common*

*common section is equal to the given right line; hence, because the other extremity of the right line is the centre of its containing circle, the right line reaching to the common section of the circles, is also equal to the given line.* And the admonition of these, is derived from the definition of the circle, which says, that *all lines from the centre to the circumference are equal.* Each of these lines, therefore, is equal to the same; but *things equal to the same, are equal among themselves,* by the first axiom. The three right lines, therefore, are mutually equal; hence, *upon this given right line an equilateral triangle is constructed;* and this, indeed, is the first conclusion which follows the exposition. But after this, that universal one, *upon a given right line, therefore an equilateral triangle is constructed:* for whether you make the line double of the one now proposed, or triple; or receive any one greater or less, the same constructions and demonstrations will accord. But to these he adds the particle *which was required to be done,* shewing from hence, that the conclusion is problematical; for in theorems, he adds the particle *which was required to be shewn;* the former announcing the production of something, but this the ostension and invention of a thing required. He therefore subjoins this to the conclusions, for the purpose of shewing that every part of the proposition is accomplished by this means, uniting the end with the beginning, and imitating intellect convolved, and again returning to its principle. But he does not always add the same, but sometimes the particle *which was required to be done,* and sometimes the particle *which was required to be shewn,* on account of the difference between problems and theorems: and thus, in this one problem, we have exercised and made perspicuous all this variety of considerations. But the reader ought to make a similar enquiry in the rest; investigating what propositions receive these leading properties, and in what they are omitted. Likewise in how many ways a *datum* is given, and from what principles we receive either constructions or demonstrations; for a perspicacious contemplation of these affords no small exercise and meditation of geometrical discourses.

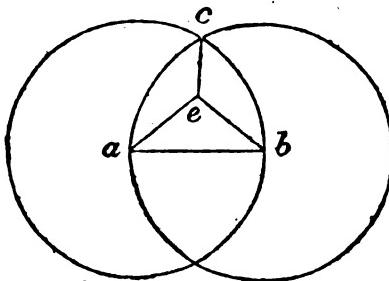
But here it is necessary that we should briefly determine the nature of *assumption, case, corollary, instance, (εργασίς) and induction.* They say

say therefore that *assumption* is often predicated of every proposition assumed in the construction of another proposition, affirming at the same time that the demonstration of such a proposition is composed from so many *assumptions*. But *assumption*, properly considered by those who are conversant in geometry, is a proposition indigent of credibility ; for when either in construction or demonstration we assume any thing which has not been exhibited, but requires a reason for its admission, then that which is assumed, as of itself ambiguous, being considered as worthy of enquiry, we call an *assumption* ; and this differs from Petition and Axiom, because it is demonstrable, but they are assumed without demonstration, for the purpose of giving credibility to others. But the best aid in the invention of *assumptions*, is an aptitude of cogitation ; for we may see many naturally acute in solutions, and discovering them without any method, as was the case with our Cratistus, who was adapted to the investigation of a thing sought from the first and shortest methods possible ; and had a natural promptitude for invention ; but there are nevertheless certain most excellent methods delivered, one which reduces the thing sought, by resolution to its explored principle, which, as they say, Plato delivered to Leodamas, and from which he is reported to have been the inventor of many things in geometry : but the second is that which has a power of division ; because it distributes the proposed genus into articles, but affords an occasion of demonstration, by an ablation of other things from the proposed construction. And this likewise is praised by Plato, as that which affords assistance to all sciences ; but the third is that which by a deduction to an impossibility, does not of itself shew the thing sought, but confutes its opposite, and discovers the truth by accident ; and thus far is the contemplation of *assumption* extended. But *caſe* enunciates different modes of construction, and the mutation of position, points, or lines, superficies, or solids being transposed ; and in fine, all its variety is beheld about description : hence, it is also called *caſe*, because it is the transposition of construction. Again, *Corollary* is affirmed, indeed, of certain problems, as the Corollaries which are ascribed to Euclid ; but Corollary is properly predicated, when, from the things demonstrated, a certain unexpected theorem appears, which on this account they have denominated Co-

rollary, as a certain gain, exceeding the intention of demonstrative science ; but *instance* impedes the whole passage of the discourse, either opposing the construction or the demonstration : and here it is not necessary, that as he who proposes a case, ought to shew the proposition true ; so he who proposes an *instance* : but it is requisite to destroy the *instance*, and convict its employer of falsehood. Lastly, *induction* is a transition from one problem or theorem to another, which being known or compared, the thing proposed is also perspicuous. For example : when the duplication of the cube is investigated, geometers transfer the question into another to which this is consequent, i. e. the invention of two mean proportionals, and afterwards they enquire how between two given right lines two means may be found. But Hippocrates Chius is reported to have been the first inventor of geometrical induction ; who also made a quadrangle equal to a lunula, and invented many other things in geometry, and excelled all in his ingenuity respecting appellations ; and thus much for these.

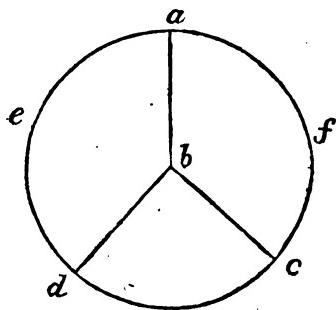
But let us return to the proposed problem : that an equilateral triangle, therefore, is the best among triangles, and is particularly allied to a circle, having all lines from the centre to the circumference equal, and one simple line for its external bound, is manifest to every one ; but the partial comprehension of two circles in this problem, seems to exhibit in images how things which depart from principles, receive from them perfection, identity, and equality. For after this manner, things moving in a right line, roll round in a circle, on account of continual generation ; and souls themselves, since they are induced with transitive intellections, resemble by restitutions and circumvolutions, the stable energy of intellect. The zoogonic or vivific fountain of souls too, is said to be contained by two intellects. If, therefore, a circle is an image of the essence of intellect, but a triangle of the first soul, on account of the equality and similitude of angles and sides ; this is very properly exhibited by circles, since an equilateral triangle is included in their comprehension. But if also every soul proceeds from intellect, and to this finally returns and participates intellect in a two-fold respect ; on this account also it will be proper that a triangle, since it is the symbol of the triple essence of souls,

souls, should receive its origin comprehended by two circles. But speculations of this kind, as from bright images in the mirror of phantasy, recall into our memory the nature of things. And here, because some object to the constitution of an equilateral triangle, thinking by this means to overthrow the whole of geometry, let us briefly answer and confute them. Zeno then, whom we have mentioned before, says, that if any one admits the principles of geometry, yet he will not obtain from common consent, things consequent to the principles, while this is not admitted, that there are not the same segments of two right lines: for unless this is given an equilateral triangle cannot be constructed. For let there be (says he) a right line  $a b$ , upon which an equilateral triangle is to be constructed.



But let circles be described, and from their common section let the right lines  $c e a$ ,  $c e b$ , be extended, having the common segment  $c e$ . It will therefore happen, that the lines extended from the common section, will be equal to the given line  $a b$ , and yet the sides of the triangle will not be also equal, but two will be less than the remainder, that is, than  $a b$ . And so this not being constituted, neither can the rest be constructed. Can then (says Zeno) the rest follow, though the principles are given, unless this also is previously received, that there are no common segments either of circles or of right lines? Against this objection then, we must affirm in the first place, that it was in a certain respect previously understood, that two right lines have no common segment. For the definition of a right line comprehends this property, since *that is a right line which is equally situated between its bounding points*; and the equality of the interval between the points to the right line, causes that which joins the points to be one, and the shortest line; so that if any one adapts it to another line,

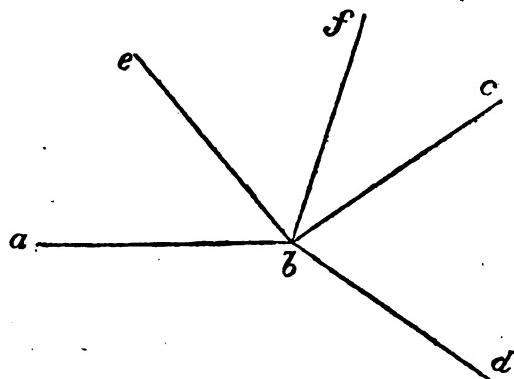
according to one of its parts, it must also agree with the line according to its remaining part; for since it is constituted in its extremities, because it is the shortest line, it is necessary that the whole should fall on the whole. But again, this was manifestly received in the Petitions: for the Petition which says, *that a terminated right line may be produced straight forwards*, perspicuously shews that the produced line ought to be one, and produced by one motion; but if any one is desirous to receive a demonstration of this assumption, let, if possible,  $a b$  be the common segment of  $a c$  and  $a d$ , and with the cen-



tre  $b$ , and interval  $b d$ , let the circle  $a c d$  be described; because therefore the right line  $a b c$ , is drawn through the centre,  $a f c$  is a semicircle; and because the right line  $a b d$  likewise is drawn through the centre,  $a e d$  is a semicircle. The semicircles, therefore,  $a f c$ ,  $a e d$ , are equal to each other, which is impossible. But against this demonstration Zeno will perhaps say, that it is likewise requisite to demonstrate that the diameter bisects the circle, because we previously assume that there is not a common segment of two circumferences. Thus too we take for granted, that one circumference coincides with another, or if it does not coincide, that it either falls externally or internally. But nothing hinders (he will say) that the whole may not coincide with the whole, but according to some part. But to this Posidonius rightly answers, who laughs at the acute Epicurean, as if conscious that though the circumferences do not coincide according to a part, yet the demonstration will succeed; for according to that part in which they do not coincide, the one will fall within, and the other without, and the same absurdities will follow when right lines are extended from the centre to the external circumference; for those

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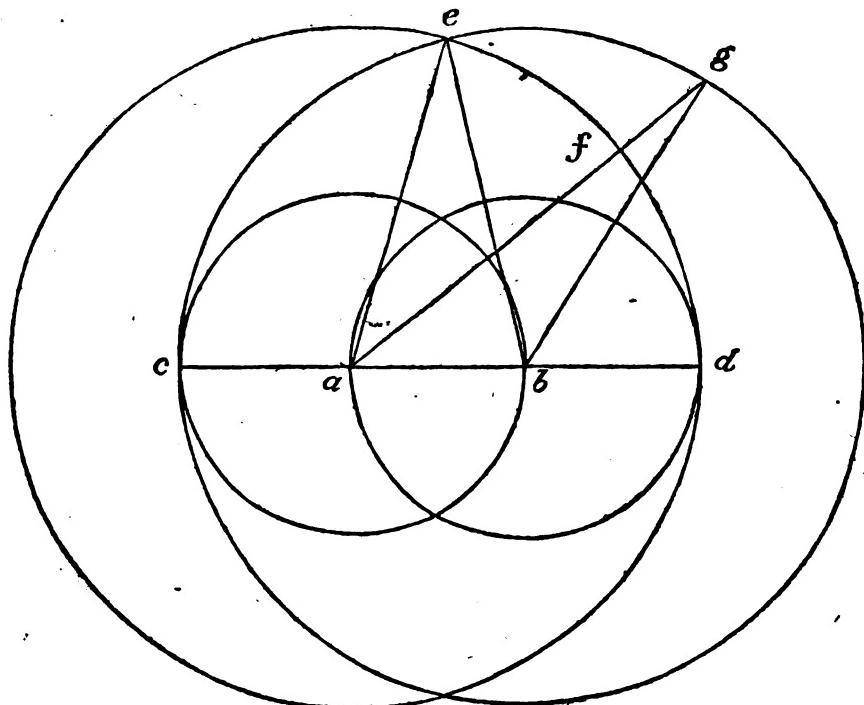
from the centre will be equal, as well the greater which is drawn to the external, as the less which is extended to the internal circle: either therefore the whole will coincide with the whole, and they will be equal; or coinciding according to a part, it will alternately vary according to the remainder, or no part will coincide with no part; and in this case it either falls within or without: but of this, enough. But Zeno also condemns the following demonstration of this particular: Let  $a b$  be the common segment of two right lines  $a c$ ,  $a d$ , and let  $b e$  be erected at right angles to  $a c$ , the angle  $e b c$ , therefore, is a



right one. Hence, if the angle  $e b d$  is also right, they shall be equal, which is impossible; but if not, let  $b f$  be erected at right angles to  $a d$ . The angle  $f b a$ , therefore, is right; but the angle  $e b a$  was also right; and they are therefore mutually equal, which is impossible. This is the demonstration which Zeno opposes, as assuming that which is to be exhibited afterwards; I mean from a given point to raise a right line, at right angles, to a given right line. However, Posidonius observes, that indeed, a demonstration of this kind is never to be introduced into elementary institutions; but that Zeno calumniates Geometricians using their own as a flagitious demonstration; though there is some reason in their conduct. For there are right lines existing at right angles; since any two right lines are capable of forming a right angle; and this is previously assumed in our definition of a right angle. For we alone constitute a right angle from such an inclination; and it may perhaps be this which we have erected. Indeed, Epicurus himself, and all other philosophers admit, that

that not only many things possible may be supposed, but likewise many of an impossible matter, for the purpose of contemplating something consequent ; and thus much concerning an equilateral triangle.

But it is requisite to construct other triangles, and in the first place an isosceles. Let  $a b$ , therefore, be a right line; upon which it is re-



quired to construct an isosceles triangle. Describe circles as in the construction of an equilateral triangle, and produce the line  $a b$  on each side to the points  $c d$ ; the line  $c b$ , therefore, is equal to  $a d$ . Again, with the centre  $b$ , and interval  $c b$ , let the circle  $c e$  be described; and with the centre  $a$ , and the interval  $d a$ , the circle  $d e$ ; and from the point  $e$ , in which the circles intersect each other, to the points  $a$  and  $b$ , let the lines  $e a$ ,  $e b$ , be extended. Because therefore,  $e a$  is equal to  $a d$ ; but  $e b$  to  $b c$ , and  $a d$  is equal to  $b c$ ,  $e a$  will also be equal to  $e b$ ; but they are also greater than  $a b$ . The triangle  $e a b$ , therefore, is isosceles, which it was required to constitute. But let it be ordered to construct a scalene triangle upon the given right line  $a b$ . Describe circles with centres and intervals, as before, and let

let there be taken in the circumference of the circle, whose centre is  $a$ , the point  $f$ , and let the right line  $af$  be extended and produced to the point  $g$ ; and likewise let the right line  $gb$  be extended. Because, therefore,  $a$  is a centre,  $af$  is equal to  $ad$ ; and hence,  $ag$  is greater than  $ad$ , that is, than  $gb$ . But  $b$  also is a centre,  $gb$ , therefore, is equal to  $ba$ ; and hence,  $gb$  is greater than  $ba$ : but  $ga$  is greater than  $gb$ ; the three lines therefore  $gb$ ,  $ba$ ,  $ag$ , are unequal; and hence, the triangle  $abg$  is scalene. Hence too, three triangles are constructed; but these things are commonly known: however, this is beautiful in these triangles, that the equilateral existing on all sides equal, is constructed by one mode alone; but the isosceles, endued with equality in two sides only, has a two-fold construction: for the given right line is either less than both the equal ones (according to our present construction), or it is greater than both; but the scalene being unequal in all its sides, receives a triple construction; for the given right line is either the greatest of the three, or the least; or greater than the one, and less than the other; and indeed, it is proper to be exercised in each supposition, either by enlarging or contracting; but to us, what is already delivered, is sufficient. Let us now contemplate problems universally, some of which are produced simply, but others manifoldly, and others according to infinite modes. But (as Amphinomus observes) those which are simply constructed are *ordinate*: but those which receive a manifold composition, and are constructed according to number, are *middle*; and those which are varied in infinite ways, are *inordinate*. The manner, therefore, in which problems are constructed, simply or manifoldly, becomes manifest in the preceding triangles; for the equilateral is constituted simply; but of the other two, the one receives a two-fold, and the other a triple construction. But problems of the following kind, may take place in infinite modes; I mean to divide a given right line in three proportional parts; for if it be divided in a duple ratio, and the deficient quadrangular form, resulting from the less, be applied to the greater, it will be divided into three equal parts; but if the greater segment be more than double of the less, as for instance, triple, and a deficient quadrangular form, equal to that which results from the less,

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less, be applied to the greater, the line will be divided into three unequal parts. Because, therefore it may be divided into two parts, in infinite ways, the greater of which is either double or triple, (for multiplex proportion proceeds in infinitum), hence, it may be divided into three parts, according to infinite variations.

But it is requisite to know, that problem also is manifoldly predicated ; for whatever is proposed may be called a problem, whether it is proposed for the sake of learning or operating. But in mathematical disciplines, that is properly called a problem, which is proposed for the purpose of contemplative energy. Since that which is performed in these, has contemplation for its end ; and often, indeed, certain things, impossible to be executed, are called problems : but more properly that which is possible to be done, and neither exceeds, nor is deficient, is allotted an appellation of this kind ; and the problem *exceeds*, which says, *to construct an equilateral triangle, having its vertical angle two thirds of one right* ; for this is superfluous, and is added in vain : since it is a property inherent in every equilateral triangle. But of those which exceed, whatever are redundant with incongruous and non-existent symptoms, are called *impossibles* : but whatever are redundant with accidents, are called *greater problems*. But a *defective* problem (which is also called a *less problem*) is that which requires some addition, that it may be reduced from inordination into order and scientific bound, as if any one should say, *to constitute an isosceles triangle* : for this is mutilated and indeterminate, and requires some one who may subjoin, what kind of an isosceles triangle, whether that which has its base greater than either of the equal sides ; or that which has it less. Likewise, whether that which has the vertical angle double of each at the base, as a semiquadrangle ; or that which has each of the angles at the base double of the vertical angle ; or that which possesses these angles according to some other proportion, as triple or quadruple : for it is possible that it may be varied in infinite modes. From hence, therefore, it is manifest, that such things as are properly denominated problems, ought to avoid indetermination, and not to be of the number of things capable of infinite variation ; though such as these are also called problems, through an equivocation of

of the word problem. The first problem, therefore, of these elements, excels the rest in the manner we have explained ; for it neither exceeds, nor is deficient ; it is neither constructed in a variety, nor according to infinite modes ; and such ought to be the conditions of that which is to be the element of the rest.

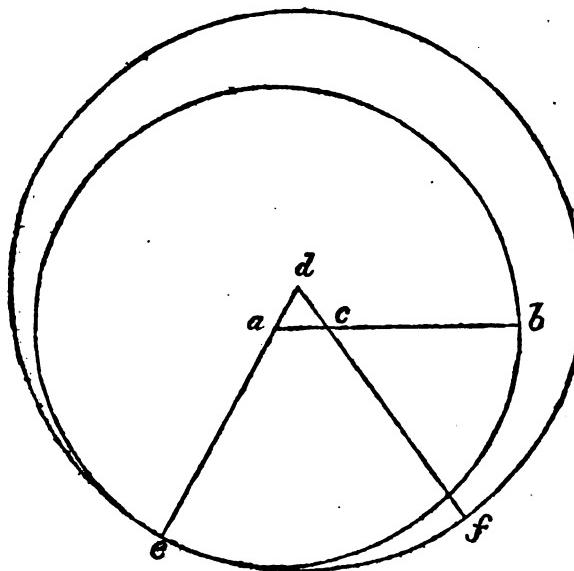
### PROPOSITION II. PROBLEM II.

To a given point to place a right line equal to a given right line.

Of problems, as well as of theorems, some are without case, but others possess a multitude of cases. Whatever, therefore, have the same power acceding to many descriptions, and when their positions are changed, preserve the same mode of demonstration, these are said to have *case* ; but such as proceed according to one position only, and one construction, are without *case* ; for simply, *case*, appears about the construction both of theorems and problems. The second problem, therefore, has many cases, but a point is given in it *in position*, since it can only be given in this manner ; but a right line, both in form and position, (for it is not simply *line*, but of such a kind.) For it is here enquired, *how to a given point to place a right line equal to a given right line*. But it is manifest that the point is entirely in the subject plane, in which the right line exists, and not in one more elevated. For in all problems and theorems respecting planes, we must conceive that one plane is subjected. But if any one should doubt how a line is to be placed equal to a given right line, for what if the given line be infinite ? Since the present datum pertains both to finite and infinite : for every datum signifies that which is proposed and supposed by us for the sake of investigation. But this Euclid himself declares, sometimes, saying, *upon a given terminated right line to construct an equilateral triangle* ; but at other times, *upon a given infinite right line to let fall a perpendicular*. In answer then to this doubt, we must say, that when he orders us to place the line equal to a given

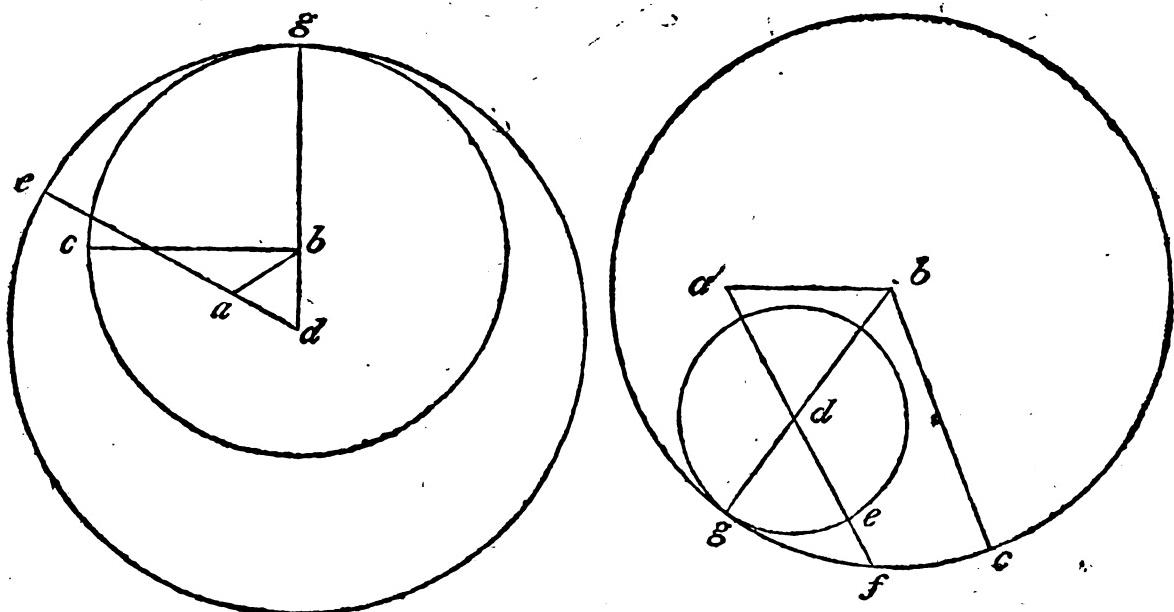
right line, at a given point, he sufficiently evinces that the given line is finite; for every thing placed at a point, is terminated according to that point. Hence, the line equal to that which is given, must have a much prior termination. At the same time, therefore, in which he says *to a given point*, he terminates both the given right line, and its equal which is investigated.

But that the cases of the present problem are formed from the various position of a point, is manifest. For the given point is either placed external to, or in the given right line; and if in it, it will either be one of its extremities, or it will be situated within the extremes; and if external, it will either have a lateral position, so that a line drawn from it to the extremity of the given line will form an angle, or a direct position; so that if the line were produced, it would coincide with the external point. But the geometrician, indeed, considers the point as external, and receives it according to a lateral position; however, for the sake of exercise, all the positions are to be assumed, the more difficult of which we shall exhibit. For let there be given a right line  $a b$ , and a given point  $c$ , which lies between its extremes, and let there be constituted according to the doctrine of



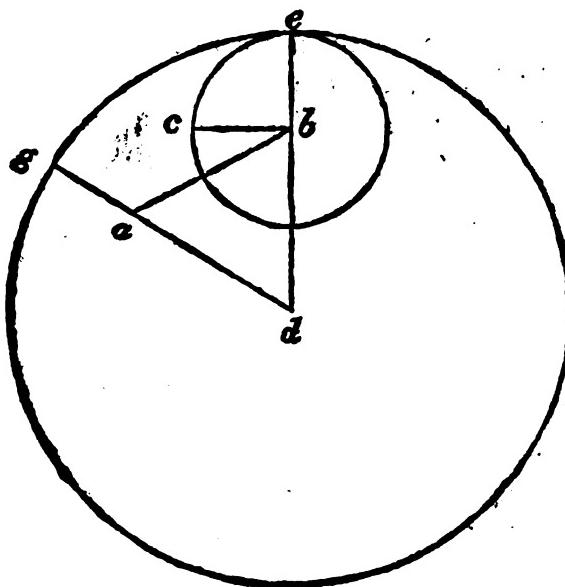
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the elements, an equilateral triangle upon the right line  $a c$ , and let  $d c$ ,  $d a$ , be produced ; then, with the centre  $a$ , and the interval  $a b$ , let the circle  $b e$  be described. And again, with the centre  $d$ , but with the interval  $d e$ , let the circle  $d f$  be designed. Because, therefore,  $a$  is the centre,  $b a$  is equal to  $a e$ ; and hence,  $d e$  is equal to  $d f$ , the parts of which,  $d a$ ,  $d c$ , are equal : for the triangle  $d a c$  was established as equilateral. The remainder, therefore,  $a e$ , is equal to  $c f$ ; but  $a e$ , as it was shewn, is equal to  $a b$ , and hence,  $c f$  is equal to  $a b$ . To a given point, therefore,  $c$ , a right line  $c f$  is placed equal to  $a b$ . With respect to the position of the point then, so many cases arise. But there are many more with respect to the constitution of the equilateral triangle, the extension of its sides, and the description of circles. For let there be assumed, as in this element, a point  $a$ , and a right line  $b c$ , but let  $b a$  be extended. The equilateral triangle,



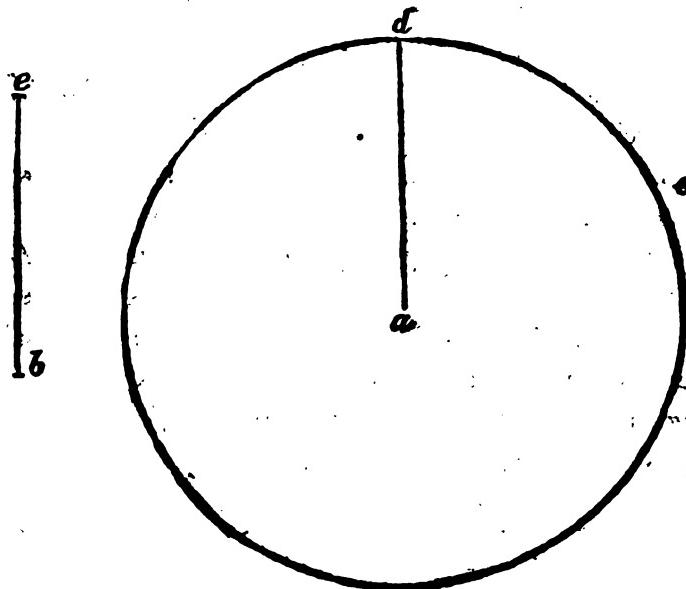
therefore, will not be constituted on  $b a$ , with its vertex above (because there is no place for it), but beneath ; let it, therefore, be  $a d b$ ;  $a d$ , therefore, is either equal to  $b c$ , or greater or less. If then it be equal that which was required is performed. But if less with the

centre  $b$ , and the interval  $bc$ , let a circle be described, and let  $ad$ ,  $db$  be produced to the points  $e$  and  $g$ ; and with the centre  $d$ , but the interval  $dg$ , let a circle  $ge$  be designed. Because, therefore,  $dg$  is equal to  $de$ , for they are drawn from the centre; and likewise because  $ad$  is equal to  $db$ , for the triangle is equilateral, the remainder  $ae$  is equal to the remainder  $bg$ . But  $bg$  is also equal to  $bc$ , for they proceed from the centre; and hence,  $ae$  is equal to  $bc$ , which was required to be done. But if  $ad$  is greater than  $bc$  (for this is the last case), then with the centre  $b$ , and the interval  $bc$ , let a circle



$ec$  be described. The line  $db$ , therefore, shall cut the circle  $ec$ . Again, with the centre  $d$ , and interval  $de$ , let the circle  $eg$  be described. Because therefore,  $d$  is the centre of the circle  $ge$ ,  $gd$  is equal to  $de$ . But  $da$  was also equal to  $db$ ; the remainder, therefore,  $ag$  is equal to the remainder  $be$ . But  $be$  is equal to  $bc$ , for both proceed from the centre. Hence,  $ag$  is equal to  $bc$ ; and it is placed at the point  $a$ , as was required to be done. And though there are many other cases, the description of the above is sufficient for our present purpose. For from these it is possible for the more curious to exercise themselves in the rest. But formerly some destroying the construc-

construction and variety of this problem, reasoned thus. Let  $a$  be a given point, but  $b e$  a given right line, and with the centre  $a$ , but



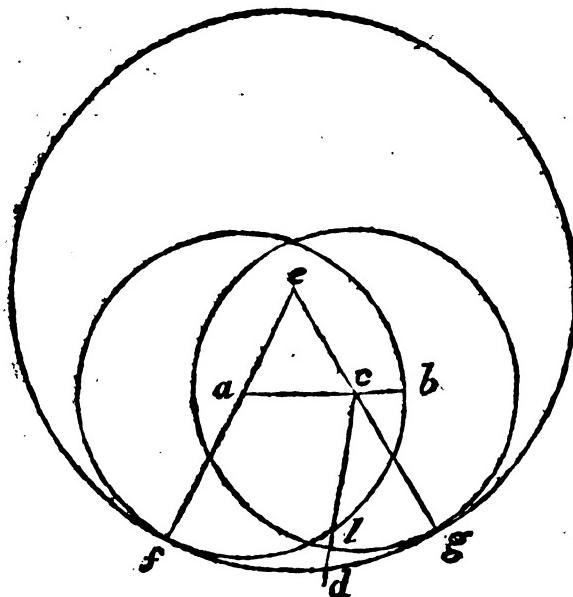
with an interval equal to  $b e$ , let a circle  $d e$  be described. Then let a certain right line  $a d$  be extended from the point  $a$  to the circumference; and this shall be equal to  $b e$ : for the magnitude of the line from the centre, was equal to that of  $b e$ : and so that is done which was required. But he who thus reasons, begs, in the very beginning. For when he says with the centre  $a$ , but interval  $b e$  describe a circle  $e d$ , he receives, in a certain manner, a line equal to  $b e$ , placed at the extremity  $a$ ; and preserving the Petition, he makes one extremity of the interval  $a$  centre, but with the other describes a circle: however, in this case, the centre is in one place, but the interval in another. We by no means, therefore, approve this method of demonstration.

### PROPOSITION III. PROBLEM III.

Two unequal right lines being given, from the greater to cut off a part equal to the less.

This third problem, likewise, has a variety of cases. For the given unequal right lines are either mutually distant from each other, as with the institutor of the elements, or they are united according to one

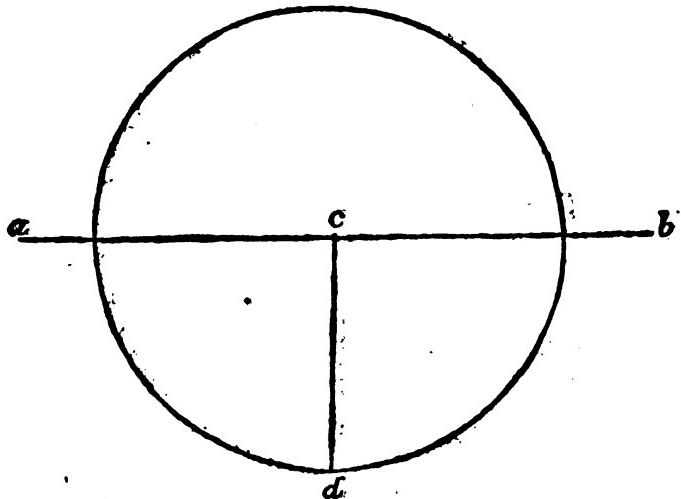
one extreme ; or the one cuts the other according to one of its extremities, and this in a two-fold manner. For either the greater cuts the less, or the less the greater. But if they are united according to one extreme, the demonstration is manifest. For employing the common extremity as a centre, and the lesser of the lines for an interval, you will describe a circle, and cut off from the greater, a part equal to the less ; since as much as the circle intercepts within itself, will be equal to the less. But if the one cuts the other according to its extreme, either the greater will cut the greater, or the contrary. And if they mutually cut each other, they will either be mutually cut into equal parts, or into unequal ; or the one will be cut into equal, and the other into unequal parts, and this in a two-fold respect. For all these present us with an admirable variety of exercise, some of which, out of a many, we shall exhibit. Let there be given the unequal right



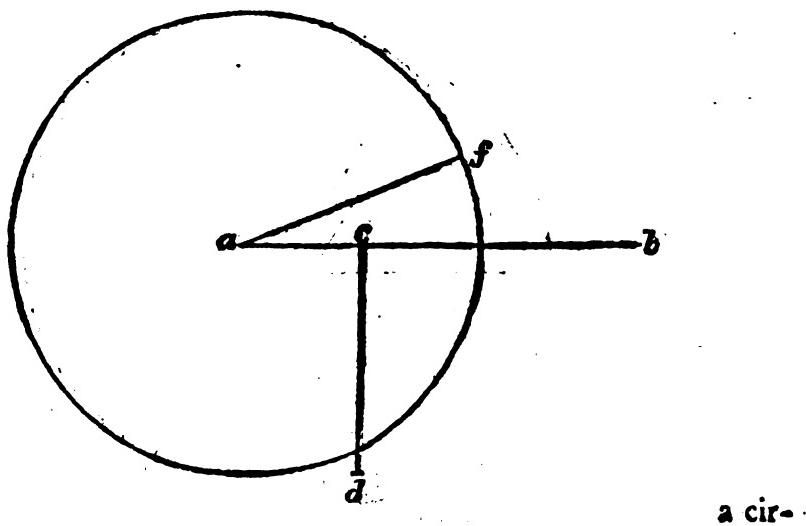
lines  $a b$ ,  $c d$ , the greater of which is  $c d$ , and let it cut  $a b$  in one of its extremities  $c$  ; then with the centre  $a$ , but interval  $a b$ , let a circle  $b f$  be described, and let an equilateral triangle  $a e c$  be constructed upon  $a c$ , and produce  $e a$ ,  $e c$ . Again, with the centre  $e$ , but interval  $e f$ , let the circle  $g f$  be described ; and with the centre  $c$ , and interval  $c g$ , the circle  $g l$ . Because therefore,  $e f$  is equal to  $e g$  (for the centre is  $e$ ) of which  $e a$  is equal to  $e c$ , the remainder  $a f$ , shall be equal

equal to the remainder  $c g$ . But  $a f$  is likewise equal to  $a b$ ; for the centre is  $a$ . Hence,  $c g$  will be equal to  $a b$ , and this is equal to  $c l$ , for the centre is the point  $c$ :  $a b$ , therefore, is equal to  $c l$ , which was required to be done.

But let  $c d$  be less than  $a b$ , and let it cut  $a b$  according to its extremity  $c$ ; either, therefore, it will cut it in the middle, or not in the



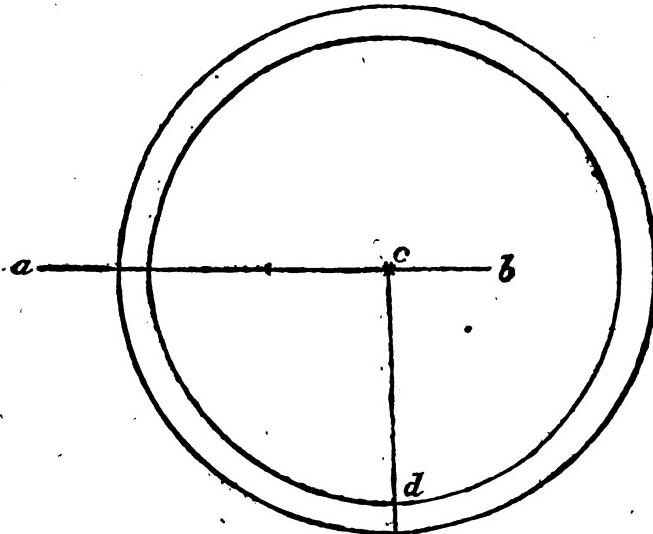
middle. Let it in the first place cut it in the middle,  $c d$ , therefore, is either the half of  $a b$ , and  $a c$  is equal to  $c d$ , or it is less than half. And in this case with the centre  $c$ , and interval  $c d$ , describe a circle, and you will cut off from  $a b$  a part equal to  $e d$ : Or it is greater than half; and then at the point  $a$ , placing  $a f$ , equal to  $c d$ , and describing



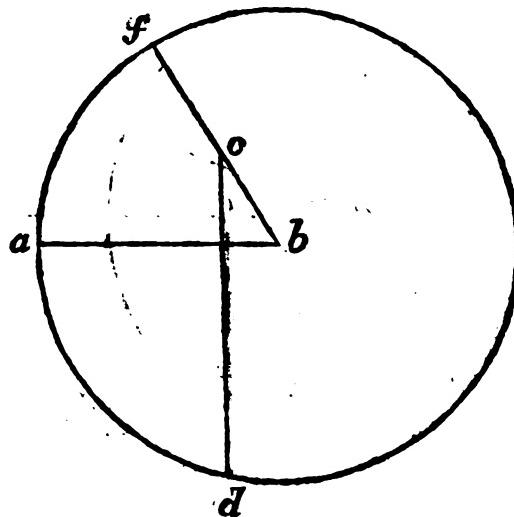
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a circle with the centre  $a$ , and interval  $af$ , you will cut off from  $ab$  a part equal to  $af$ , that is to  $cd$ . But if  $cd$  does not cut  $ab$  in the middle,  $cd$  shall either be its half, or greater than the half, or less. If therefore  $cd$  is the half, or less than the half of  $ab$ , employing  $c$  as a centre, and  $cd$  as an interval, you will cut off from  $ab$ , a part equal



to  $cd$ , as was required to be done. But if  $cd$  is greater than the half, again at the point  $a$ \* placing  $af$  equal to  $cd$ , you will accomplish the same. For with the centre  $a$ , but interval  $af$ , you will describe a circle, cutting off from  $ab$  a line equal to  $af$ , that is, to  $cd$ .



But

\* See the third figure of this problem.

But if they mutually intersect, as  $c d$ ,  $a b$ , then with the centre  $b$ , but interval  $b a$ , describe the circle  $a f$ , and let  $b c$  be extended to the point  $f$ . Because therefore,  $b f$ ,  $c d$ , are the two unequal right lines, and  $c d$  cuts  $b f$ , according to its extremity, it is possible from  $c d$  to make a line equal to  $b f$ ; for this has been shewn in the first case of this problem. It is therefore possible, that a line equal to  $a b$  may be cut off from  $c d$ ; for  $a b$  and  $b f$  are mutually equal. Having, therefore, received these cases from division, we have endeavoured to exhibit their variety. But the demonstration of the elementary institutor is admirable, since it accords with all the preceding constructions. And it is possible, in every position, at the extremity of the greater, to place a line equal to the less, and using the same extreme as a centre, and placing the interval to describe a circle, which shall cut off from the greater, a line equal to the less, whether they mutually intersect, or one cuts the other, or they are constituted in a still different position.

## PROPOSITION IV. THEOREM I.

If two triangles have two sides equal each to each; and have likewise the angles equal; which are comprehended by the equal sides; then they shall have their bases equal; and the two triangles shall be equal; and the remaining angles opposite to the equal sides shall be equal.

This is the first theorem in the institution of the elements, for all those which preceded were problems. The first, indeed, treating concerning the origin of triangles: but the second and third proposing to procure one right line equal to another. And of these the one produced an equal from an unequal line, but the other discovered an equal line by an ablation from one unequal. Since, therefore, equality, which is the first symptom in quantity, is to be constructed by us in a triangle and right line, it is delivered in the following theorem. For how can he who has not previously constructed triangles, and procured their origin, be learned in their essential accidents, and in the equality of angles and sides which they contain? How can he receive sides equal to sides, and right lines to other right lines, who has

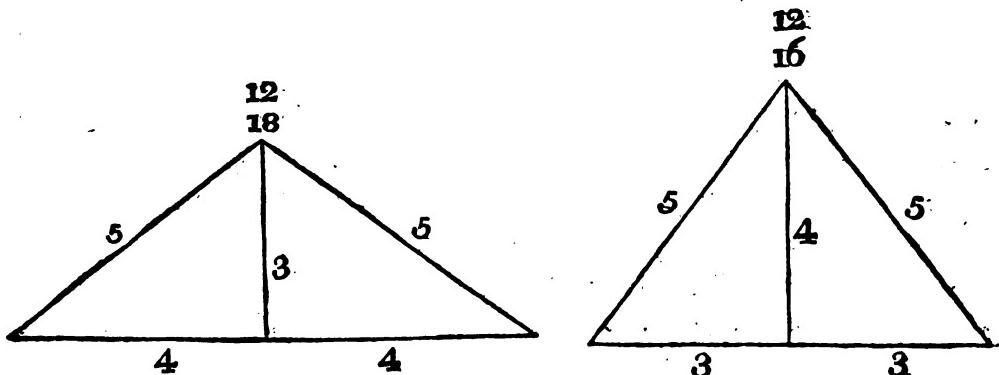
neither problematically investigated these, nor fabricated the invention of equal right lines? For if he should say it may happen before they are fabricated, that if two triangles have *this* for a *symptom*, they shall likewise have *this particular symptom*; would it not, in this case, be easy to object to him, that we by no means know whether a triangle can be constructed? And should it be afterwards inferred, that if there are two triangles, they may have two sides equal to two sides, may we not also doubt this, whether it is possible that right lines may be mutually equal? And this particularly in geometrical forms, in which inequality not entirely existing, equality is likewise inherent. For we must learn that the cornicular is always unequal to an acute angle, and the same is true of the semicircular angle; and the transition from the greater to the less does not entirely take place through that which is equal. The institutor of the elements, therefore, first of all removing these objections, delivers also the construction of triangles (for it is common to three forms) and the origin of equal right lines, in a two-fold order. For he produces the one, not yet existing: but he acquires the other by an ablation from an unequal line. But after these he very properly subjoins the theorem, by which it is shewn how triangles having two sides equal to two, each to each, and the angles comprehended by the equal sides equal, have also the base equal to the base, the area equal to the area, and the remaining angles to the remaining angles. For there are three particulars exhibited in these triangles: but two data. Hence, the equality of the two sides is given, or two equal sides (and it is manifestly given in proportion) and the equality of the angle contained by the equal sides: but three particulars are investigated, the equality of *base to base*, of *triangle to triangle*, and of the *remaining angles*. But because it is possible that triangles may have two sides equal to two, and yet the theorem not be true, because the one is not equal to the other, but both together, on this account he adds in the data, that the sides are equal not simply, but one to the other. For if one of the triangles should have one of its sides of three units, but the other of four; and again, if the sides of the other triangle are respectively two, and five units, the angle comprehended by these being right, the two sides of the one triangle, will, indeed, taken together, be equal to the two sides

sides of the other, or to seven units, yet the two triangles will not be equal. For the area of the one is six units\*, but of the other five. And the reason of this is, because the sides are not equal each to each. Hence, many, not observing this in the division of land, when they have received a greater, have thought just the same as if they had received an equal field; and this because both the sides containing one field, have been together equal to both the sides containing the other field. It is requisite, therefore, to receive the one equal to the other, and to mark wherever the institutor of the elements subjoins this, because he does not add it without occasion. For discoursing on the equality of equal angles, he adds the particle *comprehended by equal sides*, lest by speaking indeterminately we should assume some one of the angles at the bases. Besides, when in triangles no side is previously named, we must conceive the base to be the side opposite to our sight; but when two are previously received, the remaining side is necessarily the base. Hence, here too, the institutor of the elements having previously assumed two sides equal to two, calls the remainder the bases of the triangles. But a triangle is then said to be equal to a triangle, when their areas are equal. For it is possible, that though the ambits are equal, yet the areas may be unequal, on account of the inequality of angles. But I call the area, the space intercepted by the sides of the triangle: as also I denominate the ambit, the line composed from the three triangular sides. Each, therefore, is different, and it is requisite, indeed, that besides the equality of the ambits, according to each side, the angles should also be equal, if also area ought to be equal to area. But it happens in certain triangles, that though the areas are equal, yet the ambits are unequal; and that the ambits being equal, the areas are unequal. For if there be two isosceles triangles, each of whose equal sides contains five units, but the base of the one is eight, and of the other six units; he who is ignorant of geometry, will say that the greater triangle is that whose base contains eight units. For the whole ambit will be eighteen. But

\* This is easily proved from the mensuration of a triangular space, which it is well known is obtained by multiplying the base into half the altitude; and this in the first triangle will be equal to 3 multiplied by 2; and in the second, to 2 multiplied by  $2\frac{1}{2}$ =5.

the geometrician will say, that the area of each triangle contains twelve units, and this he will demonstrate, by drawing in each triangle a perpendicular from the vertex, and multiplying this with either part of the segments of the base †. But it happens (as I have said) that though the ambita are equal, the spaces are unequal. Hence, certain persons formerly fraudulently deceived their partners in the division of fields, on account of the equality according to ambit, receiving a larger field. But one base is said to be equal to another, and one right line to another, when their extremes conjoined make the whole coincide with the whole. For every right line, indeed, agrees with every right line; but equal right lines mutually coincide according to their extremes. Again, one right-lined angle is said to be equal to another, when one of the comprehending sides of one angle being placed upon one of the other, the remaining side also coincides with the remainder: but when one of the remaining sides falls external to the other, the greater angle is that whose side falls externally; and the less whose side falls within. For there, indeed, the one contains, but in this case it is contained. But we must assume the equality of angles according to the convenience of sides in right lines, and in all of the same species, as in lunulars and sytroides \*, and

† The quantity of this perpendicular in each triangle may be easily obtained from the 47th proposition of this book; for in the first triangle it will be three units; and in the second four. Hence, the area of each will be 12 units; but the ambit of the one will be 18, and of the other 16 units, as is evident in the following figures.

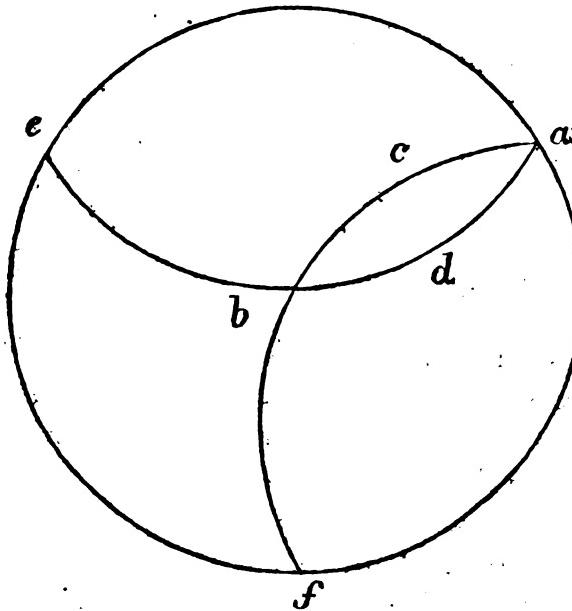


\* That is angles formed from the circumferences of circles cutting or touching each other, when they are on both sides concave.

figures

figures on both sides convex ; because, it is possible that they may be equal, and yet the sides not mutually coincide. For a right angle is equal to a certain lunular angle, and yet it is not possible that right lines can coincide with circumferences. Besides, this also must be previously understood, that the angles are said to subtend the opposite sides. For every triangular angle is contained by two sides of the triangle, but is subtended by the remaining side. Hence, the geometrician, when he says that the angles are equal, adds, *which are opposite to the equal sides*, lest we should conceive it of no consequence whatever angle is received, and should think that he denominated any other two angles of the triangles equal, but we must call those equal which subtend equal sides. For equal sides mutually subtend equal angles. And such are the considerations necessary to the declaration of the present theorem.

But against the objection of our adversary \*, this must be previously assumed, that two right lines cannot comprehend space. For this the geometrician receives as evident. For if (says he) the extremes of the bases mutually coincide, the bases also shall coincide : but if not two right lines, will comprehend space. From whence, therefore, is the impossibility of this derived ? Let there then be two right lines comprehending space  $a c b$ ,  $a d b$ , and let them be infinitely produced.



\* Most probably Zeno, the Epicurean.

Then:

Then with the centre  $b$ , and interval  $a b$ , let a circle  $a e f$  be described. Because, therefore, the line  $a c b f$  is a diameter,  $a c f$  is the half of the circumference. Again, because the line  $a d b e$  is a diameter,  $a e$ , likewise, is one half of the circumference. Hence,  $a e$ , and  $a c f$  are equal to the circumference, which is impossible. Two right lines therefore, cannot comprehend space; which the institutor of the elements knowing laid, in the first Petition, *from every point, to every point, to draw a right line*, because one right line is always capable of uniting two points, but this is impossible for two right lines to effect. Many circumferences, indeed, may conjoin two points, both in the same, and in contrary parts: for by this means the extremities of a diameter conjoin two circumferences, but only one right line. But it is possible that both within and without semicircles, infinite circumferences conjoining given points may be described. And the reason of this is, because a right line is the least of lines, having the same extremes. But there is every where one *minimum*, and this always becomes the measure of the infinity of others. As therefore a right line, since it is one, becomes the measure of the infinity of right-lined angles (for by this we discover their quantity) so likewise a right line procures us the greatest utility in the mensuration of such as are non-rectilineal. And thus much may suffice concerning these.

But that the whole demonstration of the present theorem depends on common conceptions, rising as it were spontaneously, and emerging from the evidence of hypotheses, is manifest to every one. For since two sides are equal to two sides, each to each, they will mutually coincide. But since the angles contained by the equal sides are equal, they also shall mutually coincide. And when angle is placed on angle, and sides on sides, so as to touch, in every part, the extremities of the sides beneath shall also coincide. But if these, then *base*, shall agree with *base*. And if three with three, the whole triangle shall accord with the whole triangle, and all shall be equal to all. Hence, therefore, equality considered in things of the same species, appears to be the cause of the whole demonstration. For here are two axioms endued with a power of containing the whole method of the proposed theorem. One, indeed, affirming, that *things which*

*mutually coincide, are equal*; and this is simply true, requiring no limitation, and is employed by the institutor of the elements both in the base, and in the space, and in the other angles. For these, says he, are equal, because they mutually coincide. But the other affirming that *things which are equal mutually coincide*. This, however, is not true in all, but in those of a similar species. But I call things similar in species, such as a right line when compared with a right line, one circumference with another of the same circle, and the angles comprehended by similar lines endued with a similar position. But of these, I say, that such as are equal, mutually coincide: so that in short, the whole demonstration is of this kind. These equals, therefore, are given, viz. two sides equal to two sides, and the angles which they comprehend, and these accord among themselves. But if these mutually coincide, the base also shall agree with the base, and all coincide with all. And if these accord, they are also equal. If then these are equal, it may at the same time be shewn that all are equal to all. And this appears to be the first mode of knowing triangles on all sides equal. And thus much concerning the whole demonstration.

But Carpus, the mechanist, who, in an astrological treatise, discourses of problems and theorems, says, "that they must not be passed over in silence, since they opportunely present themselves for investigation;" and lastly, entering on their distinction, he observes, "that the problematical genus precedes theorems in order. For in problems (says he) the invention of subjects is investigated prior to symptoms. Likewise a problematical proposition is simple, and requires no artificial intelligence. For this commands us to accomplish something evident, as to construct an equilateral triangle, or from two given unequal right lines, to cut off from the greater a part equal to the less. For what is there in these difficult and obscure? But he affirms, that the proposition of a theorem is difficult, and requires the most accurate power, and a judgment productive of science, that it may appear neither to exceed, nor to be deficient from truth; such, indeed, as the present, which is the first of theorems. Add too, that in problems, there is one common way invented by resolution, by proceeding-

ing:

ing according to which, we can happily accomplish our purpose. For after this manner the more easy kind of problems are investigated. But the treatise of theorems is so very difficult, that even to our time (says he) no one has been able to deliver any common method of their invention. Hence, on account of facility also, the problematical genus is more simple. But these being distinguished, it is on this account (says he) that in the elementary institution problems precede theorems, and from these the institution of the elements begins ; and the first theorem is in order the fourth, not because the fourth is exhibited from the preceding, but because it is necessary they should precede as being problems, and this a theorem, though it should require none of the antecedent propositions for its demonstration. For the present theorem entirely employs common conceptions ; and in a certain respect receives the same triangle in a different position. Since coincidence, and its consequent equality possess a sensible and manifest apprehension. But such being the demonstration of the first theorem, problems with great propriety precede, because they are universally allotted "the primary place." And perhaps, indeed, problems antecede theorems in order ; and particularly among those who ascend to contemplation from the arts, which are conversant with sensible particulars : but theorems excel problems in dignity of nature. And it appears, that all geometry, so far as it conjoins itself with a variety of arts, energizes problematically : but so far as it coheres to the first science, it proceeds theorematically from problems to theorems, from things secondary to such as are first, and from things which more regard the arts, to such as are endued with a greater power of producing science. It is, therefore, vain to accuse Geminus, for affirming that theorems are prior to problems. For Carpus assigns a precedence to problems, according to order : but Geminus to theorems, according to a more perfect dignity. But of this fourth theorem, we have already observed, that in a certain respect it is indigent of the preceding problems, in which we learn the origin of triangles, and the invention of equality. But we now add, that since it is the most simple and principle of theorems (for it is naturally, as I may say, exhibited from primary conceptions alone), but demonstrates a certain symptom appear-

appearing about triangles, having two sides equal to two, each to each, and the two angles equal contained by the equal sides, it is with great propriety placed the first after problems, in which things subject to this symptom, and the data themselves are constructed.

## PROPOSITION V. THEOREM II.

The angles at the base of an isosceles triangle are mutually equal ; and the equal right lines being produced, the angles under the base shall be mutually equal.

Of theorems some are *simple*, but others *composite*. I call those *simple*, which, both according to hypotheses and conclusions, are indivisible, possessing one *datum*, and one object of investigation. Thus for example, if the institutor of the elements had said, *every isosceles triangle has the angles at the base equal*, it would have been a simple theorem. But theorems are composite, which are composed from many particulars, either having composite hypotheses, or conclusions from a simple hypothesis, or both. And of these, some are *complex*, but others *incomplex*. The *incomplex* are such composites as cannot be divided into simple theorems, as the fourth proposition. For in this, both the *datum* is a composite, and its consequent, yet it is impossible that the *datum* can be divided into things simple, and become theorems. For if a triangle has its sides alone equal, or the angle at the vertex, the same consequences will not ensue. But the *complex* are such as may be divided into things simple, as the theorem which says, *triangles and parallelograms of the same altitude, have the same proportion as their bases*. For it is possible to say by division, that *triangles of the same altitude, have the same proportion as their bases*, and in parallelograms after a similar manner. But of all composites, some are composed according to the conclusion, being excited from the same hypothesis : but others have their conclusion according to hypotheses, and infer the same conclusion in all : and others, lastly, are composed both according to the conclusion, and according to hypo-

H theses.

theses. *Composition*, therefore, in the present case, is according to the conclusion, for there are three particulars concluded in this theorem, that the bases are equal, that the triangles are equal, and that the remaining angles, under the base, are equal to the remaining angles. But composition, according to hypothesis, is found in the common theorem of triangles and parallelograms of the same altitude. And according to both, in the theorem that the diameters both of circles and ellipses, bisect as well the spaces as the lines containing the spaces. But of complex theorems, some are universal: but others conclude that which is universal from particulars. For if we should say that a diameter divides a circle, ellipsis, and parallelograms, we receive, indeed, every part of the complex, not universally, but we make that universal which is composed from all. But if we should say, that *in a circle, all lines passing through the centre, mutually bisect each other, and make equal angles of all the segments*, we should affirm a universal. For in an ellipsis all the angles of the segments are not equal, but those only which are formed by the diameter. But these compositions are entirely fabricated, for the sake of geometrical brevity and resolutions. For many things incomposite are not resolved, but composites alone afford convenience to a resolution tending to principles.

In consequence of these previous considerations then, we must call the fifth theorem a composite, and a composite, both with respect to the datum, and the object of investigation; and this the institutor of the elements exhibiting, divides this theorem, being one, and gives a separate position to the *data*, and the *things to be investigated*, for he says that *the angles at the base of an isosceles triangle are equal*; and again, that *the equal sides being produced, the angles under the base are equal*. For we must not think that there are two theorems, but one; and that this is a composite, both according to the *data*, and *thing sought*: and that each of these composites is perfect and true. Hence, conversion also is true in each. For if the angles at the base are equal, the triangle is isosceles: but if those under the base are equal, the equal right lines are produced, and the triangle is isosceles.

isosceles. But the institutor of the elements converts the equality of the angles at the base; but not the equality of those under the base, though this is likewise true; the reason of which we shall shortly explain. But we shall now, in the first place, enquire on what account he demonstrates that the angles under the base are equal. For he never employs this in the construction or demonstration of other problems or theorems. It may be doubted, therefore, why, since it is useless, it was requisite to insert it in the present theorem? To this we must reply, that though it is never employed in the elements, yet it is most useful for the destruction of objections, and the solution of oppositions to theorems\*. But it is artificial, and belongs to science

to

\* Mr. Simpson, in his note on the 7th proposition of this book, positively asserts, that it contains two cases, though there is but one in the Greek text; and ridicules Proclus for asserting that the second part of the present proposition was added, in order to solve objections which might be urged against the seventh. But that Euclid never added any more than one case, is, I think, evident, not only from no such case being found in the Greek copies so early as the age of Proclus; but from his not converting it in the 6th proposition. Besides, it is employed with advantage in the solution of objections against the 9th proposition, as the reader will perceive in its commentary: and the objection there started merits the appellation of a case, as much as the 7th. But Mr. Simpson seems to have been ignorant of Euclid's design in these elements;—the tradition of that only which is accommodated to an elementary institution. Hence, Euclid every where avoids a multiplicity of cases; and anticipates objections where he foresees they may be urged. Mr. Simpson adds in support of his dogmatical assertion, "that the translation from the Arabic has this case explicitly demonstrated." As if an Arabic translation was of greater authority than the Greek text which Proclus consulted! And lastly, he concludes, with observing, that "whoever is curious, may read what Proclus says of this in his commentary on the 5th and 7th propositions; for it is not worth while to relate his trifles at full length." If an accurate knowledge of the nature, beauty, and tendency of a science, or a collection of scientific propositions, is trifling, Proclus, indeed, deserves this accusation; as I doubt not the liberal reader, is, by this time, fully convinced. But Mr. Simpson was no philosopher; and therefore the greatest part of these Commentaries must be considered by him as trifles, from the want of a philosophic genius to comprehend their meaning, and a taste superior to that of a mere mathematician, to discover their beauty and elegance. It is common, indeed, to hear geometers of the present day exclaiming, *What need of a comment on Euclid!* Is he not perspicuous to every one? I will readily admit that such gentlemen know enough of geometry for all mechanical and sensible purposes: but I fear they are totally ignorant of its end; and have never dreamt that when properly studied it is the handmaid of true philosophy, the purifier of the rational soul, and the bridge by which we may pass from the obscurity and delusion of material nature, to the splendor and reality of intellectual vision. I add farther, that I am greatly inclined to doubt, whether such geometers ever considered what kind of substance geometrical forms possess? Whether they have any certainty, or are only imaginary? Where these forms, if real, reside? And a multitude of other questions which are discussed in these Com-

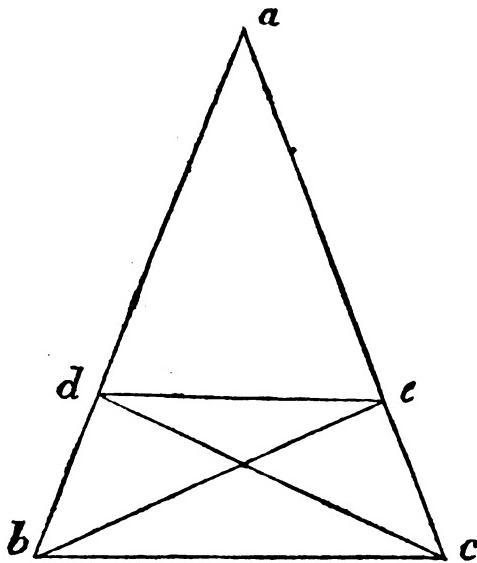
## 52 COMMENTARIES OF PROCLUS.

to prepare solutions of things resisting its propositions, and to provide subsidies of answers; that not only true demonstrations may be fabricated from things previously demonstrated, but that from hence confutations of error may be produced. And from this geometrical order, you will likewise receive a rhetorical emolument. For he who can effect this in the discourses of rhetoric, who can foresee the oppositions to his following heads, and previous to their delivery, can first of all prepare solutions of them to others, he, indeed, will fabricate in a wonderful manner, a most excellent mode of disputation. The instructor of the elements, therefore, teaching us this in reality, previous to the theorems by which we solve opposing objections, employing such as are now exhibited, at the same time demonstrates, that the angles under the base of an isosceles triangle, are equal, and thus prepares a confutation of the falsehood such objections contain. But that from the present theorem we may solve the objections urged in the seventh and ninth propositions, will be perspicuous as we proceed. Hence, it appears, why Euclid does not convert the latter part of this theorem in the sixth, because it does not produce a principal utility, but confers to our advantage, accidentally, with respect to the whole of science.

But if any one should desire us without producing the equal right lines, to prove the angles at the base of an isosceles triangle equal, (for it is not requisite to demonstrate the equality of these, by those under the base) by transposing, in a manner, the construction, and fabricating those constructions within, which are made without the isosceles triangle, we may exhibit the thing proposed. Thus let

mentaries. And lastly, what is most material of all, if geometry be a science, what science itself is? This last question, indeed, they would doubtless consider so trifling and easy of solution, that they would readily and confidently answer with young Theætetus in Plato, "that sciences are such things as may be learned from Mathematicians, geometry, and the like; shoe-making, and other mechanical arts; and that all, and each of them are no other than sciences!" To which admirable definition we may justly reply in the words of Socrates, "Generously and magnificently O my friends, when interrogated concerning *one thing*, have you given instead of something simple, things *many* and *various*."

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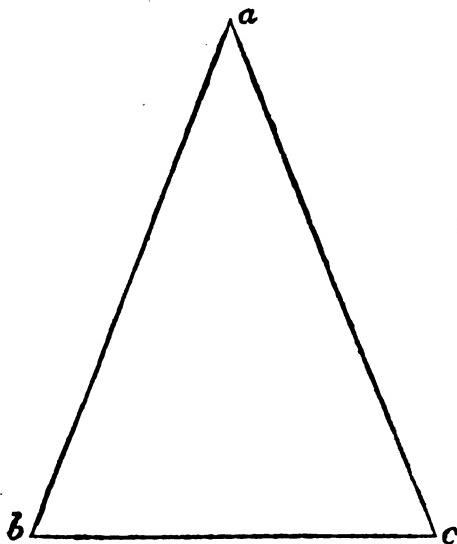


$a b c$  be an isosceles triangle, and in the side  $a b$ , take any point  $d$ , and from  $a c$ , take  $a e$ , equal to  $a d$ , and draw the lines  $b e$ ,  $d c$ ,  $d e$ . Because, therefore,  $a b$  is equal to  $a c$ , and  $a d$  to  $a e$ , and the angle  $a$  is common,  $b e$  also shall be equal to  $c d$ , and the remaining angles to the remaining angles. Hence, the angle  $a b e$ , is equal to the angle  $a c d$ . Again, because  $d b$  is equal to  $e c$ , and  $b e$  to  $d c$ , and the angle  $d b e$  to  $e c d$ ; hence, the base, since it is common to both, is equal to itself, and all are equal to all. The angle,  $e d b$ , therefore, is equal to the angle  $d e c$ : and the angle  $d e b$ , is equal to the angle  $e d c$ . Hence, since the angle  $e d b$ , is equal to the angle  $d e c$ , from which the equal angles  $d e b$ ,  $e d c$ , are taken, the remaining angles  $b d c$ ,  $c e b$  are equal. But the sides also  $b d$ ,  $d c$ , are equal to the sides  $c e$ ,  $e b$ , each to each, and the base  $b c$  is common. All, therefore, are equal to all. Hence, the remaining angles also, subtending equal sides, are equal. The angle, therefore,  $d b c$ , is equal to the angle  $e c b$ . For the angle  $d b c$ , subtends the line  $d c$ : but the angle  $e c b$ , the line  $e b$ . The angles, therefore, at the base of an isosceles triangle, are equal, the equal right lines not being produced.

But Pappus demonstrates this yet shorter, without any addition in the following manner. Let  $a b c$  be an isosceles triangle, having  $a b$ ,

equal

equal to  $a c$ . We must conceive, therefore, this one triangle as if it



was two, and reason thus. Because  $a b$  is equal to  $a c$ , and  $a c$  to  $a b$ , the two sides  $a b$ ,  $a c$ , are equal to the two  $a c$ ,  $a b$ , and the angle  $b a c$ , is equal to the angle  $c a b$ , (for it is the same.) All, therefore, are equal to all. The base  $b c$ , to the base  $c b$ . But the triangle  $a b c$ , to the triangle  $a c b$ ; and the angle  $a b c$ , to the angle  $a c b$ , and the angle  $a c b$ , to the angle  $a b c$ . For they subtend equal sides, i. e.  $a b$ ,  $a c$ . The angles, therefore, at the base of an isosceles triangle, are equal. And it seems that Pappus invented this mode of demonstration, when he considered that the institutor of the elements also, in the fourth theorem, when he had united two triangles, and had made them mutually coincide, thus forming one of two, by this means observed their equality throughout. In like manner it is possible, that we also, by an assumption contemplating two triangles in one, may demonstrate the equality of the angles at the base. Thanks, therefore are to be given to the ancient Thales for the invention of this theorem, as well as a multitude of others. For he, first, is said to have perceived and affirmed, that the angles at the base of every isosceles triangle are equal : and after the manner of the ancients, to have called them similar. But still more deserving of praise are those moderns, who have yet more universally demonstrated (among which number

number is Geminus) that equal right lines falling from one point, on a line of similar parts, form equal angles. For Geminus using this theorem, shews, that there are only three lines, and not more of similar parts, the *right*, the *circular*, and the *cylindric helix*; and this is properly universal, to which this symptom first agrees, just as the possession of two sides greater than the third, is shewn to be essentially inherent in every triangle. It is not, therefore, the property universally of every isosceles, though it belongs to every one, to possess angles at the base equal: but of equal right lines falling on a line of similar parts. For to subtend equal angles, is in these primarily inherent.

### PROPOSITION VI. THEOREM III.

If two angles of a triangle be equal to each other, the sides also which subtend the equal angles, shall be equal to one another.

The present theorem exhibits these two properties of theorems, *conversion*, and *a deduction to an impossibility*. For it is converted, indeed, in the preceding theorem, but its certainty is evinced by a deduction to an impossibility. It is requisite, therefore, to speak of each, whatever belongs to the present treatise. One kind of *conversion* then, among geometers, is denominated principally and properly, when the conclusions and hypotheses alternately receive theorems; so that the conclusion of the former becomes hypothesis in the latter; and hypothesis is inferred as the conclusion. As that the angles at the base of an isosceles triangle are equal. For here the isosceles triangle is the *hypothesis*: but the *conclusion*, the equality of the angles at the base. And that where the angles at the base are equal, the triangles are isosceles, which the present 6th theorem affirms. For here the equality of the angles at the base is the *hypothesis*; but the *conclusion*, the equality of the sides subtending the equal angles. But another kind of conversion, is alone according to a certain mutation of composites. For if the theorem be composite, beginning from many hypotheses, and ending in one conclusion, by receiving the conclusion, and one or more of the hypotheses, we infer some one of the other hypotheses

as

as a conclusion. And after this manner the eighth theorem is the converse of the fourth. For the one says, that *equal bases subtend equal sides and angles*: but the other, that *equal sides being placed on equal bases, contain equal angles*. Of which the predication concerning *equal bases* in the latter proposition, is the *conclusion* of the former: but the predication concerning the position of *equal sides*, is one of the previously assumed hypotheses in the former theorem; and the *comprehension of equal angles* is another hypothesis which this fourth proposition contains. In consequence therefore of these two *conversions*, the one which is called the principle, is uniform and determinate: but the other is various, advancing into a great number of theorems, and not converting in one, but in many, on account of the multitude of hypotheses, in composite theorems. But oftentimes in that which begins from two hypotheses, there is one which is converted, when the hypotheses are not all determinate, but some of them indeterminate.

It is here, however, requisite to observe, that many false and improper conversions take place. As that *every sexangular is a triangular number* \*. For the converse is not also true, that *every triangular number is sexangular*. But the reason of this is, because the one is more common, but the other more particular. And one is alone predicated *totally* † of the other. But things in which, *that which is primary*, is inherent, and according to which it is received, in these, conversion also follows. And these observations, indeed, were not unknown to those mathematicians, the familiars of Menæchmus, and Amphinomus. But of theorems receiving conversion, some are usually called *precedents*, but others *converse*. For when supposing a certain genus, they demonstrate some symptom of its nature, they call this a *precedent theorem*. But when on the contrary, they make the hypothesis a symptom, and the conclusion a genus, they denominate the theorem to which this happens *converse*. As for instance,

\* Triangular numbers, are 1, 3, 6, 10, &c.; and sexangular numbers 1, 6, 15, 28, &c. But concerning their formation, see note to page 95, Vol. I. of this work; by means of which, the truth of this assertion will be evident.

† Concerning the meaning of total predication, see page 45 of the Dissertation, Vol. I. of this work.

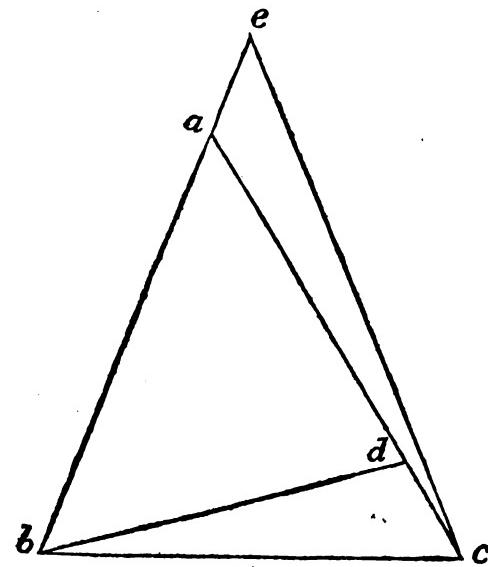
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the theorem which says, *every isosceles triangle has the angles at the base equal*, is a precedent. For that is subjoined which precedes by nature. I mean the genus itself, or the isosceles triangle. But that which says, *every triangle possessing two equal angles, has likewise the sides subtending those equal angles equal, and is isosceles*, is a converse theorem. For it changes the subject, and its passion, supposing the latter, and from this exhibiting the former. And thus much concerning geometrical conversions.

But deductions to an impossibility, entirely end in an evident impossible, the contrary of which is confessed by all. It happens, however, that some of them end in such things as are opposed to Axioms, or Petitions, or Hypotheses; but others in things contradicting prior demonstrations. For the present sixth theorem shews that which happens to be impossible, because it destroys the common conception, affirming that *the whole is greater than its part*. But the eighth theorem falls, indeed, on an impossible, yet not on that endued with a power of destroying a common conception, but that exhibited by the seventh theorem. For what the seventh denies, this affirming exhibits to such as do not admit the object of investigation. But every deduction to an impossibility, which being received, opposes *the thing sought*, and on this hypothesis advances, until it falls upon the explored absurdity, and by this means destroys the hypothesis, corroborates that which was investigated from the first. But it is requisite to know, that all mathematical proofs are either *from principles*, or *to principles*, as Porphyry in a certain place affirms. And the proofs *from principles*, are two-fold. For they either emanate from common conceptions, and things self-evident: or from things previously exhibited. But proofs *to principles* are endued with a power of either *establishing* or *destroying principles*. And those, endued with a power of *establishing principles*, are called *resolutions*; and to these compositions are opposed. For it is possible that we may proceed in an orderly method from those principles to the object of investigation; and this is nothing else than composition. But those possessing a power of *destroying principles*, are called *deductions to an impossibility*. For it

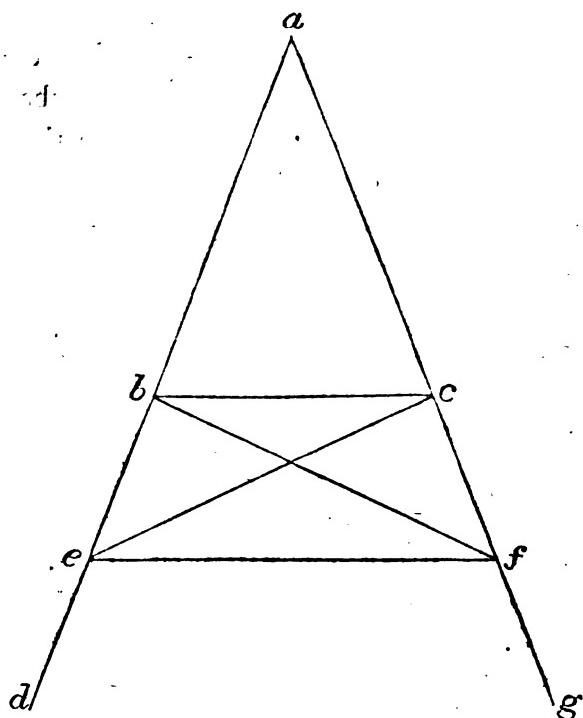
is the business of this mode to destroy some of the concessions, and objects of investigation. And in this, also, there is a certain ratiocination, though not the same as in resolution. For in deductions to an impossibility, *complexion* is according to the second mode of hypothetical reasonings. As if *in triangles possessing equal angles, the sides subtending the equal angles are unequal; and the whole is equal to its part*: but this is impossible. *In triangles, therefore, possessing two equal angles, the sides subtending the equal angles are equal.* And thus much concerning what is called by geometers, deduction to an impossibility.

But the institutor of the elements uses *conversion* in the present proposition, for he receives the conclusion of the fifth as a datum, and adds its hypothesis as an object of enquiry: but he employs *deduction to an impossibility*, in the construction and demonstration. But if any should rise up, and assert that it is not necessary by taking a part from  $a c$  equal to  $a b$ , to make the ablation at the point  $c$ , but at the point  $a$ , upon this hypothesis, we shall fall into the same impossibility. For let  $a b$  be equal to  $a d$ , and having produced  $b a$ , let  $a e$  be placed equal to  $d c$ . The whole  $b e$ , therefore, is equal to the whole  $a c$ .



Let  $e c$  be connected. Because, therefore,  $a c$  is equal to  $b e$ , but  $b c$  is

$b\ c$  is common, the two are equal to the two, and the angle at the point  $b$ , is equal to the angle  $a\ c\ b$ . For so it was established in the hypothesis. All, therefore, are equal to all, by the fourth theorem. Hence, the triangle  $e\ b\ c$ , is equal to the triangle  $a\ b\ c$ , the whole to the part, which is impossible. But because this also is manifest, it remains that we exhibit the rest of the conversion. For the institutor of the Elements converts the whole sixth theorem from a part of the fifth. But it is requisite to adjoin the remaining conversion. This, then, he receives as an hypothesis, *that the angles at the base of a certain triangle are equal*: but he shews that the triangle is *isosceles*. Let  $a\ c\ b$ , therefore, be a triangle, and let  $a\ b$ ,  $a\ c$ , be produced to the



points  $d g$ , and let the angles under the base be equal. I say that the triangle  $a b c$ , is isosceles. For let there be assumed in the line  $a d$ , the point  $e$ , and let  $b e$  be taken equal to  $c f$ ; and connect the lines  $e c$ ,  $b f$ ,  $e f$ . Because, therefore  $b e$  is equal to  $c f$ , but  $b c$  is common, the two will be equal to the two. And the angle  $e b c$ , is

I z equal

equal to the angle  $f c b$ ; for they are under the base. All, therefore, are equal to all, by the fourth theorem. Hence the base  $e c$ , is equal to the base  $f b$ , and the angle  $b e c$ , to the angle  $c f b$ ; and the angle  $c b f$ , to the angle  $b c e$ : for they subtend equal sides. But the whole-angle  $e b c$ , was equal to the whole  $f c b$ , of which the angle  $f b c$ , is equal to the angle  $e c b$ . The remainder, therefore,  $e b f$ , is equal to the remainder  $f c e$ . But  $b e$  is equal to  $c f$ , and  $b f$  to  $c e$ , and they contain equal angles. All, therefore, are equal to all. Hence, also, the angle  $b e f$ , is equal to the angle  $c f e$ . Wherefore, the side  $a e$ , is equal to the side  $a f$  (for it is shewn by the sixth) of which  $b e$ , is equal to  $c f$ . The remainder, therefore,  $a b$ , is equal to the remainder  $a c$ . And hence, the triangle  $a b c$ , is isosceles. It is, therefore, as well isosceles, if it possesses angles at the base equal: as if the sides being produced it has the angles under the base equal. Why then did not the institutor of the Elements convert the remaining part? Shall we say it was because the equality of the angles under the base in the fifth theorem, was exhibited for the sake of solving other doubts. But that proving the triangle to be isosceles, from the equality of the angles under the base, neither confers to a principal demonstration, nor to the solution of things investigated, the truth of which is confirmed in the following theorems, and that from the equality of the angles under the base, he is enabled to demonstrate that the triangle is isosceles? For if every right line, standing upon a right line, and forming two angles, makes them equal to two right; when the angles under the base are equal, those upon the base will be equal. And these being equal, the sides subtending them shall be equal. Euclid, therefore, having used this in the whole elementary institution, was enabled to conclude, that when the angles under the base are equal, the triangle is isosceles. Indeed he requires this also, for the demonstration of certain theorems: For shortly a theorem will appear, evincing, that if a right line standing on a right line, forms angles, it will either make two right, or angles equal to two right. And the theorems, indeed, preceding this, require no such conversion; but those which follow, are indigent of this, and establish their credibility from the present theorem.

PRO-

## PROPOSITION VII. THEOREM IV.

\* Upon the same right line, two right lines cannot be constituted equal to two other right lines each to each, drawn to different points, to the same parts, and having the same extremes with the two right lines first drawn.

The present theorem possesses a rare property, which is not frequently found in propositions producing science. For to be formed by negation, and not by affirmation, is not their sufficiently distinguishing property. Indeed, the propositions, as well of geometrical as of arithmetical theorems, are for the most part affirmations. But the reasons of this is, (as Aristotle says) because, an affirmative universal, especially agrees with sciences, as more proper, and not ~~in-~~digent of negation: but a universal negative requires affirmation, in order to produce evidence; for from negatives alone, there is neither demonstration nor reasoning. Hence, demonstrative sciences exhibit a multitude of affirmations, but rarely employ negative conclusions. However, the proposition of this theorem is full of admirable diligence, and is bound with every addition, by which it is rendered so certain and indubitable, that it cannot be confuted and overturned by the efforts of opposing calumniators. For in the first place, the par-

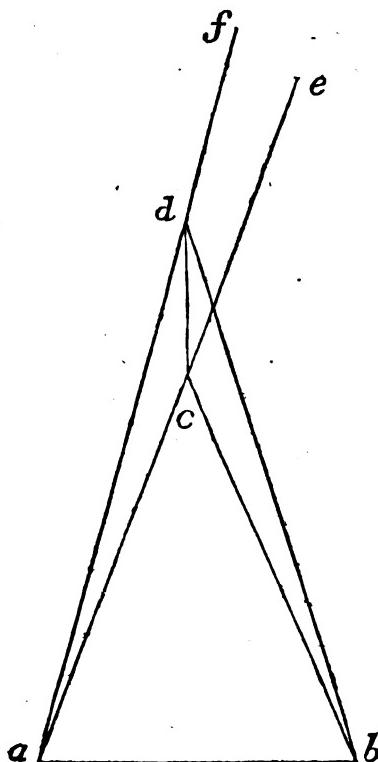
\* Mr. Simson in his note to this proposition observes, that he thought proper to change its enunciation, so as to preserve the same meaning; "because (says he) the literal translation from the Greek, is extremely harsh, and difficult to be understood by beginners." Whatever difficulty learners may find in conceiving this proposition abstractedly, is easily removed by its exposition in the figure; and therefore, I conceive, that Mr. Simson acted very injudiciously in altering its enunciation. Besides, the following comment of Proclus shews, that there is great beauty in Euclid's statement of this proposition; the greatest part of which is lost in Mr. Simson's indiscreet alteration. It would appear strange that such liberties should be taken by one, who professes in his preface, *to remove blemishes, and restore the principal books of the Elements to their original accuracy*, if Mr. Simson had not informed us in his note, that the present Commentary of Proclus unfolding the beauty and accuracy of this proposition, is too trifling to merit a relation! See more concerning this proposition in the note to Prop. 5.

ticle upon the same right line, is assumed, lest we should exhibit upon another, two right lines equal each to each, and employ the proposition for the purpose of circumvention. In the second place, he does not say upon what right line, to constitute two right lines simply equal to two (for this is possible) but *each to each*. For what wonderful thing is it, that he should take both equal to both, who extends one of the constituted lines, and contracts the other? But each to each, (says he) is impossible. In the third place, he adds the particle, *to different points*. For what, if some one, when he has formed two lines equal to the first two, each to each, should connect these with those in the same point, which joins the subject right lines in the vertex; and should constitute these? For the extremes of equal right lines perfectly coincide. In the fourth place, he adds the particle *to the same parts* †. For what if one subject right line being given, we should place two of the right lines on one side, and the other two on the opposite side, so that this common right line should be the basis of the two triangles with opposite vertexes? Lest, therefore, we should form an erroneous figure, and charge our deception on the institutor of the Elements, he adds the particle *to the same parts*. In the fifth place, he subjoins, *having the same extremes with the two right lines first drawn*. For it is possible to constitute upon the same right line, two right lines equal to two, each to each, drawn to different points, and to the same parts, by employing the whole right line, and constructing upon it, these two right lines; but then the lines last drawn, will not have the same extremes with those constituted at first. For if we conceive in a quadrangle two diagonals drawn on one of its sides, two lines shall be equal to two; a side and diameter to its parallel side, and the other diameter. But in this case the equal right lines will not have the same extremes. For neither the parallel sides, nor the diameters, will mutually possess the same extremes; and yet they will be equal. These distinctions, therefore, being preserved, the truth of the proposition, and the certainty of the reasoning, is evinced.

† See the Comment of Clavius on this proposition.

But

But perhaps, some, notwithstanding all these terms producing science, will dare to object, that these hypotheses being admitted, it is possible to effect what the geometrician affirms to be impossible. For



let there be a right line  $a b$ , and upon this two lines  $a d, d b$ , equal to two  $a c, c b$ , and let the former be external to the latter, being drawn to different points  $d c$ , and terminated in the same extremes  $a$  and  $b$ . Let  $a c$  too, be equal to  $a d$ : but  $b c$  to  $b d$ . This objection, then, we shall confute, by connecting the line  $d c$ , and producing the lines  $a c$ , and  $a d$ , to the points  $e f$ . For these being constructed, it is manifest that the triangle  $a c d$  is isosceles,  $a d$ , being equal to  $a c$ , from hypothesis; and the angles under the base  $e c d, f d c$  are equal. The angle  $f d c$ , therefore, is greater than the angle  $b d c$ . Much more then is the angle  $b c d$  greater than the angle  $b d c$ . But again, because the line  $d b$ , is equal to the line  $b c$ , the angles also at the base are equal, i. e. the angle  $b c d$ , to the angle  $b d c$ . The same angle, therefore, is both greater and equal, which is impossible. And this

is what we said in our exposition of the fifth theorem, that though the equality of the angles under the base, was not useful to the demonstrations of the following theorems, yet it procured the greatest utility in the solution of objections. For in the present instance we have confuted the objection, by inferring that, because  $a c$ , and  $a d$ , are equal, the angles  $e c d$ , and  $f d c$ , are also equal. In a similar manner in other theorems, it will appear to be peculiarly useful for the solution of doubts\*.

But if any one should say that there may be constituted upon the right line  $a b$ , right lines  $b d$ ,  $b c$ , equal to the right lines  $a c$ ,  $a d$ , of which  $b c$  may be equal to  $a c$ , but  $b d$  to  $a d$ ; and that in this case they will be drawn to different points  $a$  and  $b$ , to the same parts, and will have the same extremes with  $a c$ , and  $a d$ , viz.  $c$ , and  $d$ , what shall we reply to this assertion? Shall we say that it is requisite to constitute the first lines, upon the right line  $a b$ , and their equals upon the same right line? For this is what the institutor of the Elements affirms in the proposition. But here,  $a c$ , and  $a d$ , are not constituted upon the right line  $a b$ , but only on one of its points. Hence, the lines  $a c$ ,  $c b$ , and  $a d$ ,  $d b$ , which stand on the right line  $a b$ , are different from the right lines, which were placed in the beginning, and to which they ought to be constituted equal. Though at the same time it is necessary that the right lines constituted upon  $a b$ , should be equal to those constituted upon  $a b$ . And thus much may suffice for objections against the present question. But that the present theorem is exhibited by the institutor of the elements, by a deduction to an impossibility, and that this impossible opposes the common conception, affirming that *the whole is greater than its part*; and that *the same thing cannot be both greater and equal*, is sufficiently manifest. But this theorem seems to have been assumed for the sake of the eighth theorem. For it confers to its demonstration, and is neither simply an element, nor elementary: since it does not extend its utility to a multitude. And hence, we find it very rarely employed by the geometrician.

\* And from hence, also appears the emptiness and arrogance of Mr. Simson's note to this proposition, which we have already exploded.

PROPO-

## PROPOSITON VIII. THEOREM V.

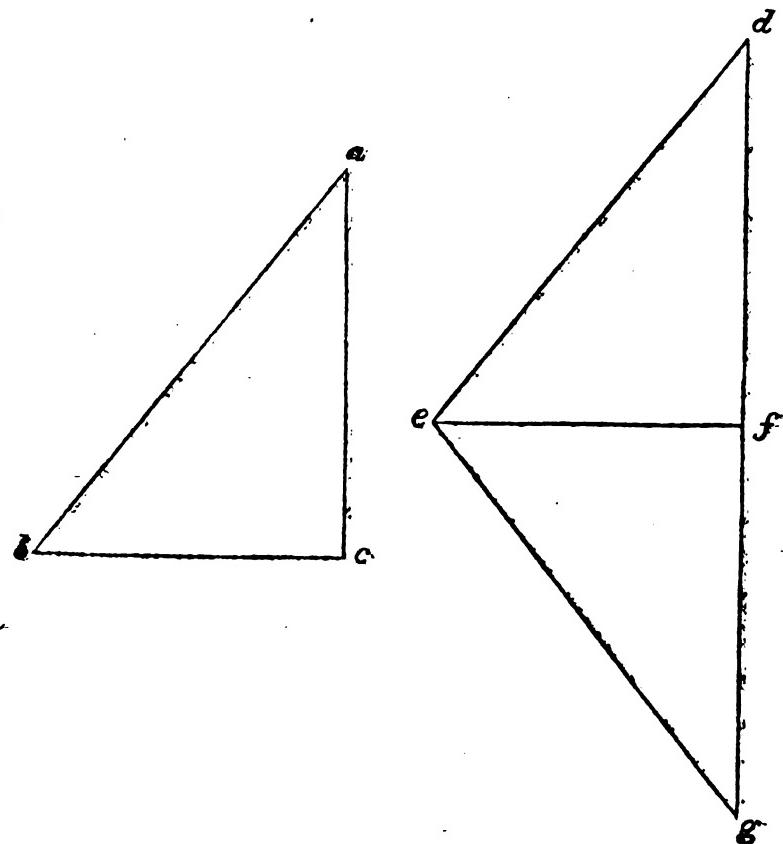
If two triangles have two sides equal to two, each to each, and have the base equal to the base : then the angles contained by the equal right lines, shall be equal to each other.

This eighth theorem is the converse of the fourth : but it is not assumed according to a principal conversion. For it does not make the whole of its hypothesis a conclusion ; and the whole conclusion an hypothesis. But connecting together some part of the hypothesis of the fourth theorem, and some part of the objects of enquiry, it exhibits one of the data which it contains. For the equality of two sides to two, is in each an hypothesis ; but the equality of base to base, is, in the fourth, an object of investigation, but in the present a datum ; and the equality of angle to angle, is, in the former, a datum, but in the latter, an object of enquiry. Hence, a change alone of data, and objects of investigation, produces conversion. But if any one desires to learn the cause why this theorem is placed in the order of the eighth proposition, and not immediately after the fourth, as its converse, in the same manner as the sixth after the fifth, of which it is the converse, since many converted propositions follow their precedents, and are exhibited after them without any intervening medium, to this we must reply, that the eighth, indeed, is indigent of the seventh proposition. For its truth is evinced by a deduction to an impossibility, but the nature of an impossible becomes known from the seventh. And, this again, in its demonstration, is indigent of the fifth. Hence, the seventh and fifth theorems were necessarily assumed, previous to the present. But because the *converse* to the fifth obtained a demonstration easy, and from *things first*, it was very properly placed after the fifth, on account of its alliance with that theorem ; and be-

## 65 COMMENTARIES OF PROCLUS.

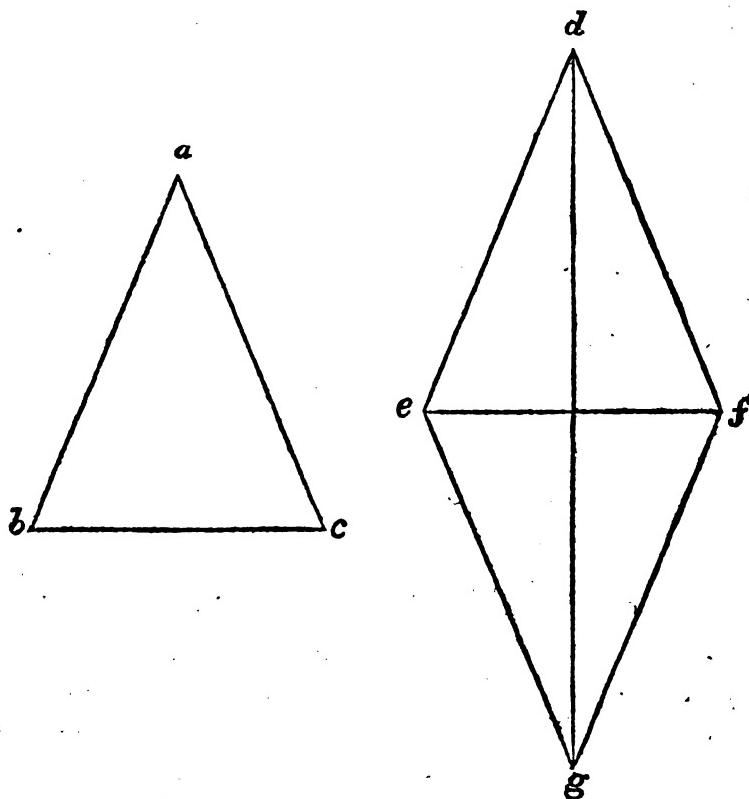
cause, since it is shewn by a deduction to an impossibility, it confutes that which is impossible from common conceptions, and not as the eighth from another theorem. For things opposing common conceptions, are more evident for the purpose of confutation than such as contradict theorems: since these are assumed by demonstration, but the knowledge of axioms is better than demonstration. But the institutor of the elements exhibits what is now proposed from the previously demonstrated seventh theorem.

But the familiars of Philo assert, that they can demonstrate this theorem, without being indigent of any other. For let there be conceived (say they) two triangles,  $a b c, d e f$ , having two sides equal



to two, and the base  $b c$  equal to the base  $e f$ . Likewise let the bases coincide

coincide with each other ; and let the two triangles  $a b c$ ,  $d e f$ , be so placed in the same plane, that their vertices may be opposite, and so that  $e f g$  may be the equal substitute of  $a b c$ . And let  $e g$  be equal to  $d e$ , but  $f g$  to  $d f$ . Hence,  $f g$  will either be placed in a right line with  $d f$ , or not in a right line. And if not in a right line, it will either make with it an angle according to the internal part, or according to the external. Let it first be placed in a right line. Because, therefore,  $d e$  is equal to  $e g$ , and  $d f g$  is one line, the triangle  $d e g$ , is isosceles, and the angle at the point  $d$ , is equal to the angle at the point  $g$ . But if it does not lie in a right line, it will make an angle inward ; and in this case let  $d g$  be connected.

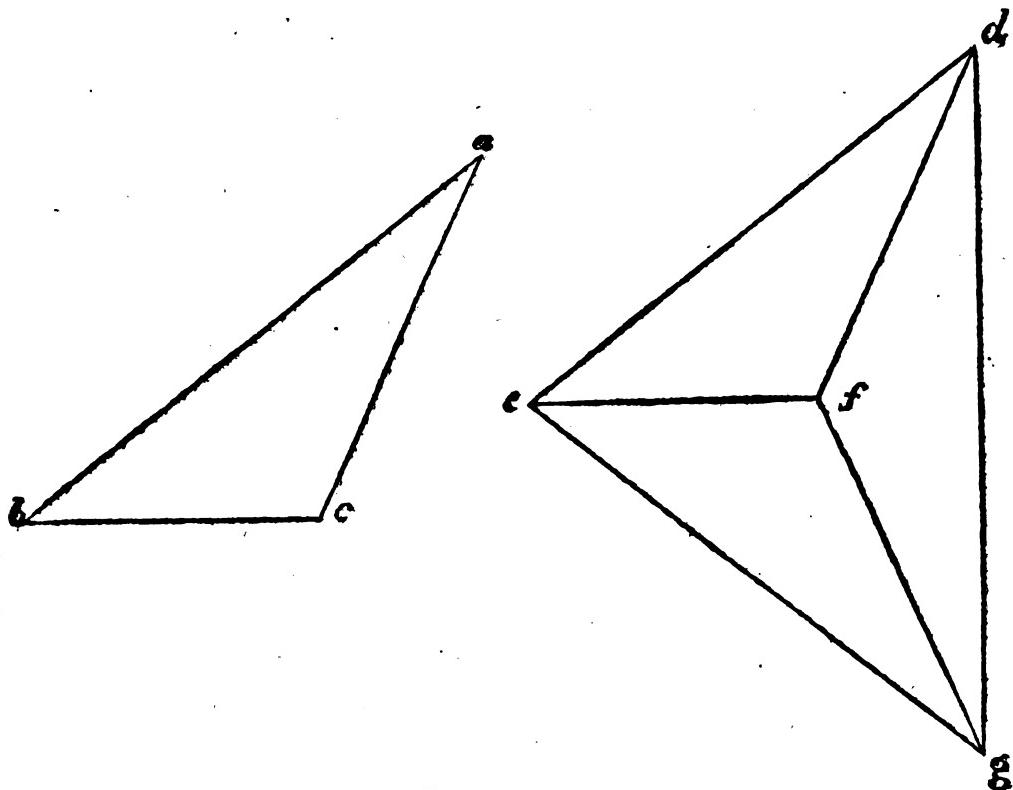


cause, therefore  $e d$ ,  $e g$ , are equal, and the base is  $d g$ , the angle,  $e d g$  also, is equal to the angle  $e g d$ . Again, because  $d f$  is equal to

K 2

 $f g$ ,

$f g$ . and the base is  $d g$ , the angle, also,  $f d g$ , is equal to the angle  $f g d$ . But the angle  $e d g$  was also equal to the angle  $e g d$ . Hence, the whole  $e d f$ , is equal to the whole  $f g e$ , which was required to be demonstrated. But in the third place, let  $f g$  make an angle with  $d f$ , externally, and let the right line  $d g$  be connected. Because, there-



fore  $d c$ ,  $e g$ , are equal, and the base is  $d g$ , the angles  $e d g$ ,  $d g e$ , are equal. Again, because  $d f$ ,  $f g$ , are equal, and the base is  $d g$ , the angle  $f d g$ , is equal to the angle  $f g d$ . But the whole angles  $e d g$ ,  $d g e$ , were mutually equal. Hence, the remaining angles  $e d f$ ,  $f g e$ , will be equal to each other. And thus the thing proposed is invented according to any position of the right line  $f g$ , and we may demonstrate the theorem, without employing the seventh proposition.

Is

Is, then (say they), the seventh proposition introduced in vain by the institutor of the elements? For if we only assume it on account of the eighth, but the eighth may be exhibited without it, does not the seventh appear entirely useless? To these enquiries we must reply in the words of our predecessors, that the seventh theorem, being demonstrated, is of the greatest utility to such as are skilled in astronomical concerns, when they discourse concerning the eclipses of the sun and moon. For, employing this theorem, they shew that three consequent eclipses, distant from each other by an equal space, cannot subsist. I say, in such a manner, that the second may be distant from the first by as great a space of time as the third from the second. For example, if the second is produced after the first, when six months and twenty days are elapsed; the third, will by no means be produced after the second, by the same, but by either a greater or less interval of time. But that this is the case may be demonstrated by the seventh theorem. And the institutor of the elements has not only exhibited the present as conferring to astronomy, but a multitude of other theorems and problems. For to what other end shall we say that the last problem of the fourth book was proposed, by which we are taught how to inscribe the side of a figure of fifteen angles in a circle, than for its relation to astronomy? For those who describe in a circle a quindecangle passing through the poles, will, by this means, obtain the distance of the poles of the equator from the poles of the zodiac. Since they are distant from each other by the side of a quindecangle. The institutor of the elements, therefore, appears by regarding astronomy, to have previously exhibited many things preparative to our advancement in that science. But when, at the same time, he saw that this seventh theorem is exhibited from the fifth, and proves the eighth without any variety, he assigned it the present place. The addition of Philo is, indeed, beautiful, but is not sufficiently adapted by its variety of cases to an elementary institution. And thus much in reply to the present question.

But

But if any one should doubt why he does not add so much in the eighth as in the fourth theorem, I mean, *that the triangles and the remaining angles are equal*; we must say, that because the equality of the vertical angle is demonstrated, it follows, that all are equal to all, by the fourth theorem. It was therefore alone necessary to demonstrate this by itself, but to assume all the rest as consequents. But it seems that the equality of the vertical angles causes the equality of the bases, and of the sides comprehending those angles. For when the bases are unequal, the same angles will not remain, though the containing equal sides are supposed, but while the base becomes less, the angle is at the same time diminished, and while that increases, the angle also receives a correspondent increase. Nor while the same bases remain, but the sides become unequal, will the angle remain; but while they are diminished, it will be increased; and while they are increased, it will be diminished: for angles, and their containing sides, suffer a contrary passion. Thus, if upon the same base, you conceive the sides descending to the lower part, you will diminish the sides, but increase the angle which they comprehend, and enlarge their distance from each other. But if you conceive the sides to be elevated, and to receive an addition as they rise, you will diminish the angle which they contain: for they will coincide the longer, when their vertex is more remote from the base. We may therefore certainly affirm that the identity of the basis and equality of the sides, in a triangle, determine the equality of its angle.

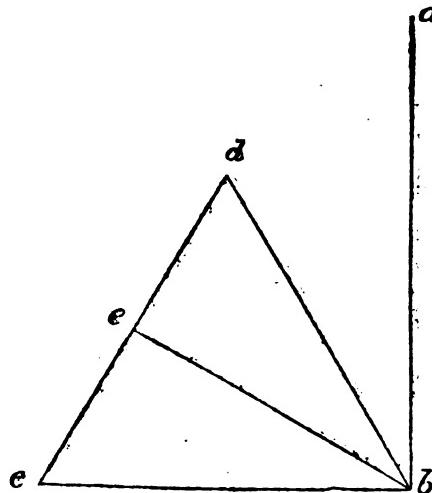
### PROPOSITION IX. PROBLEM IV.

To bisect a given rectilineal angle.

Our authormingles theorems with problems, and connects problems with theorems, and through both completes the whole of his elementary institution, comparing as well subjects as the *symptoms* subsisting about subjects themselves. Since, therefore, he had shewn in the preceding propositions, both in one triangle, from the equality

lity of the sides, the consequent equality of the angles, and the contrary: and in a similar manner in two triangles, with this exception, that the mode of conversion in one and two triangles is different, he now passes to problems, and orders us to bisect a rectilineal angle. And it is manifest, that the angle here is given according to form: for it is called right-lined, and not of any kind whatever. Indeed, we cannot bisect every angle by the elementary institution; since it is doubtful whether every triangle can be bisected. For, perhaps, you may doubt whether it is possible to bisect a cornicular angle. But the ratio of the section is also distinguished in this problem, and this again not in vain. For to divide an angle in any given ratio, transcends the present construction: as, for example, into three, four, or five equal parts. Indeed, to trisect a right angle is possible, by employing a few of the propositions which are afterwards delivered \*:

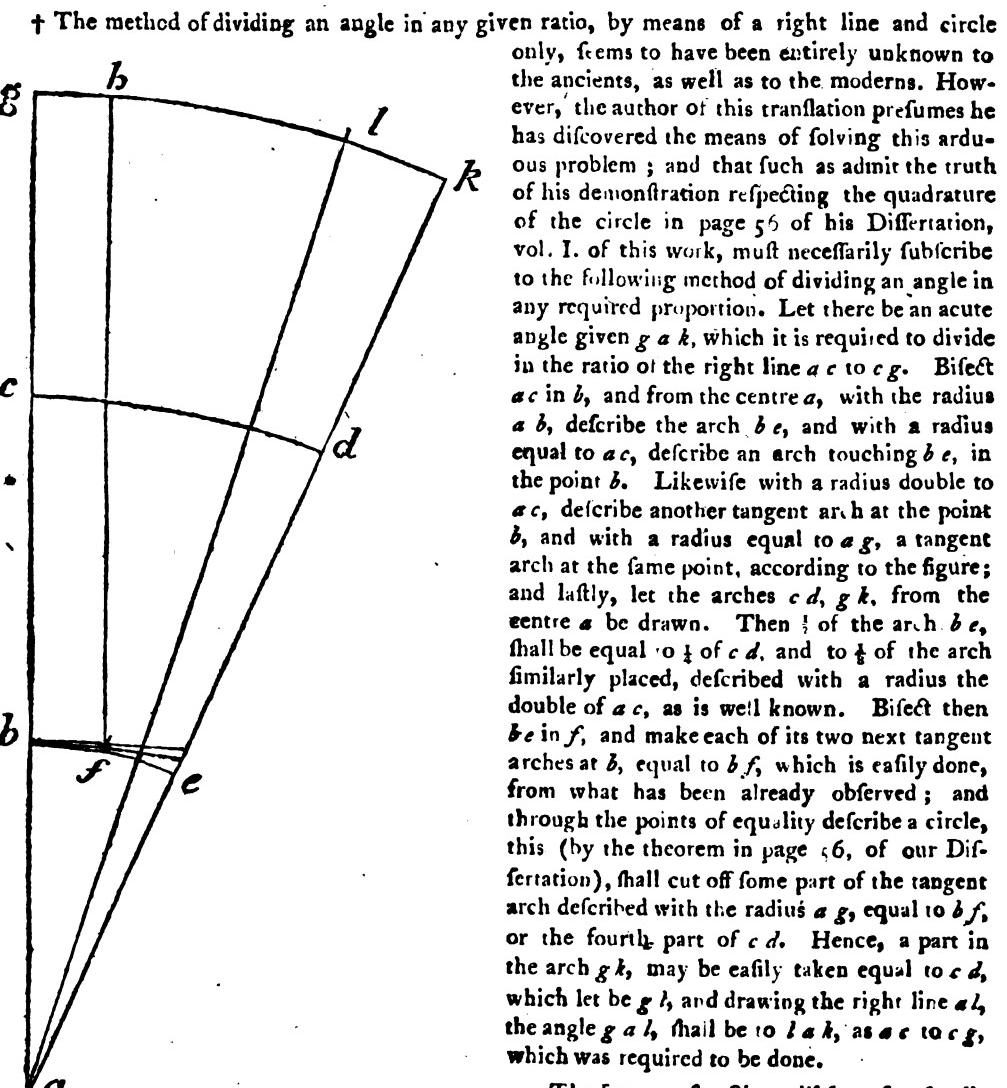
\* This too may be easily effected by means of the first problem, and the present. Thus let  $a b c$  be a right angle, which it is required to trisect; then, upon the side  $a b$ , describe an equilateral.



triangle  $e d b$ , and bisect the angle  $d b c$ , and the angles  $a b d$ ,  $d b e$ ,  $e b c$ , shall be equal. For the angle  $d b c$ , is one third of two right angles, or two thirds of one right, and consequently the angle  $d b a$ , is one third of a right angle; and this is equal to  $d b c$ , the half of  $d b c$ . Therefore they are all equal.

but:

but this cannot be effected in an acute angle, without passing on to other lines of a mixt species. † And this is manifested by the geo-



The same construction will serve for the division of a right angle in any given ratio, as is evident; and if the given angle be obtuse, the problem may be solved by a two-fold operation, that is, by bisecting the obtuse angle, and dividing either of the equal sections in the given ratio; for when this is effected, the whole angle may be easily divided in the same proportion. Hence, too, a right line may be speedily obtained equal to a given arch of a circle.

I

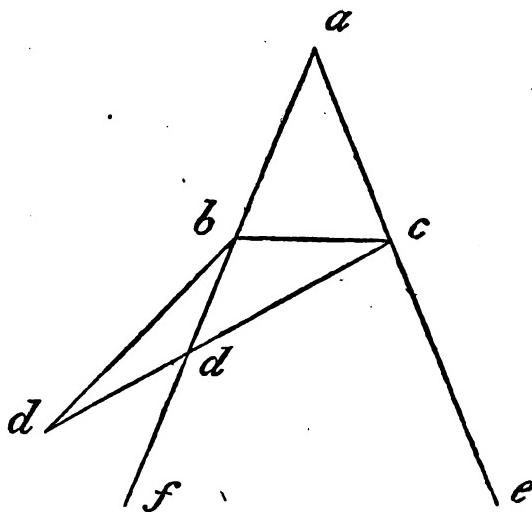
metricians,

metricians who propose to trisect a given rectilineal angle. For *Nichomedes*, indeed, from conchoidal lines, the origin, order, and symptoms of which, he delivers, as he was the inventor of their properties, trisects every right-lined angle. But others effect this from the quadrantal lines of *Hippias* and *Nichomedes*, by employing mixt quadrantal lines. Others, again, being incited from the Helices of *Archimedes*, divide a given rectilineal angle, in a given ratio. But the consideration of these, because difficult to learners, we shall for the present omit ; as it will, perhaps, be more convenient to examine this in the third book \*, where the institutor of the Elements bisects a given circumference. For there the same mode of enquiry presents itself with respect not only to bisection, but also trisection ; and the ancients endeavoured, by employing the same lines, to divide every circumference into three equal parts. With great propriety, therefore, he who only mentions a right line and a circumference, alone bisects a right angle and a circumference. But conceiving that the species composed from these, through mixture, are difficult to explain and enumerate, without a curious examination, he omits all such enquiries as involve mixt lines in their consideration, and proposes to investigate in first and simple forms alone, such things as can either be produced or considered from these. And such, indeed, is the position of the present problem, *to bisect a given right lined angle*. For in the construction of this he uses one petition, and the first and third problem : but in the demonstration he employs the eighth theorem alone. Since problems entirely require demonstration (as we have already observed †) and through this they obtain a power of producing science. But perhaps, some may oppose the geometer, by asserting that an equilateral triangle may be constituted by him, not having its vertex within the two right lines, but either upon, or external to each ; and that this may be manifested by the elements. For let there

\* In the 3rd Prop.

† See Chap. 8. Book 2d.

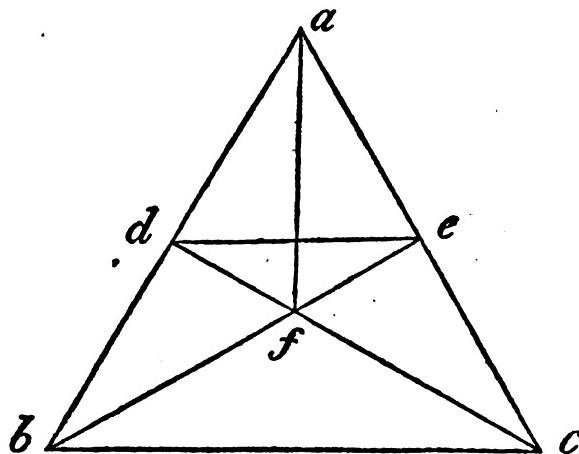
be an angle  $bac$ , which it is required to bisect. Then let  $ba$  be taken equal to  $ac$ , and let  $bc$  be connected, and upon it, let an



equilateral triangle  $bcd$  be constructed. This point  $d$ , therefore, is either within the right lines  $ab$ ,  $ac$ , or upon  $ab$ , or  $ac$ , or external to both. Now the institutor of the Elements assumes them within ; and hence, those who oppose the demonstration, will say the point is either placed on one of the right lines, or external to both. Let the point  $d$  then be placed on the line  $ab$ , so that the triangle  $bcd$  may be equilateral :  $db$ , therefore, is equal to  $dc$ , and the angles at the base  $c b d$ ,  $b c d$ , are equal. Hence, the whole,  $bce$ , is greater than the angle  $c b d$ . Again, because  $ab$ ,  $ca$ , are equal, the triangle  $abc$ , is isosceles, and the angles under the base  $bc$ , will be equal. The angle, therefore,  $bce$ , is equal to the angle  $c b d$ . But it was also greater, which is impossible. Hence, the vertex of the equilateral triangle cannot be in the right line  $abd$ . In like manner we may shew that it cannot be in the right line  $ace$ . Let it therefore, if possible be placed externally. Because, then  $bd$  is equal to  $cd$ , the angles at the base are equal, viz.  $bcd$ , and  $c b d$ . Hence, the angle,

angle  $b c d$ , is greater than the angle  $c b f$ . Much more, therefore, is the angle  $b c e$ , greater than  $c b f$ : but it is also equal, because these angles are under the base  $b c$ , of an isosceles triangle  $a b c$ , and this is impossible. Hence, the point cannot fall in these parts external to the two right lines; and it may be similarly shewn that this is impossible in other parts. Here too you may again observe, that we destroy objections by using the second part of the fifth proposition, *that the angles under the base of an isosceles triangle are equal.* And this is what we have previously observed, that many things opposing science, are shewn to be debile, and easy of confutation, by the assistance of this theorem; and that such is the utility it affords to geometry.

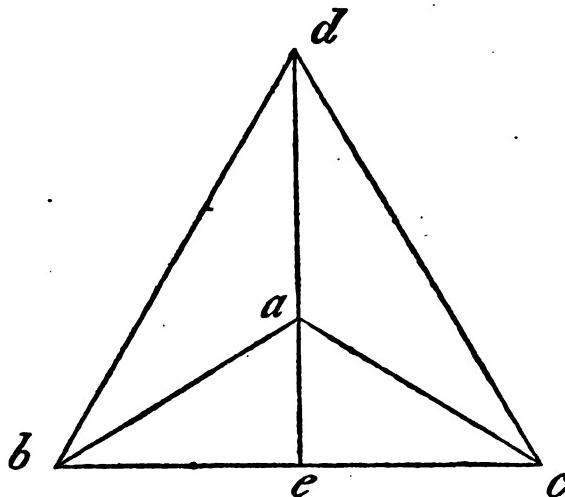
But if any one should say that there is no place under the base, and yet that it is requisite to constitute the equilateral triangle at the same parts, in which the lines  $b a$ ,  $a c$ , are situated; it will be necessary that the lines which are constituted should either coincide with  $b a$ ,  $a c$ , if they also are equal to the base  $c b$ : or that they should fall external to them, if they are less than the base  $b c$ : or within, if  $b a$ ,  $a c$ , are greater than  $b c$ . Let them, in the first place, coincide, and let  $b a c$  be an equilateral triangle, and let there be taken in the side



$a b$ , the point  $d$ , and make  $a e$  in the side  $a c$ , equal to  $a d$ , and connect  
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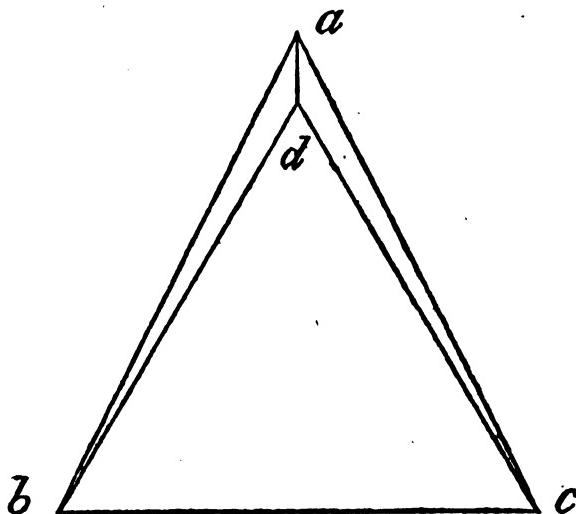
nect the lines  $d e$ ,  $b e$ ,  $c d$ ,  $a f$ . Because, therefore,  $a b$  is equal to  $a c$ , and  $a d$  to  $a e$ , the two  $b a$ ,  $a e$ , are equal to the two  $c a$ ,  $a d$ , and they comprehend the same angle. Hence, they are all equal to all, and the angle  $d b e$ , is equal to the angle  $e c d$ . But  $d b$  is also equal to  $e c$ , and  $b e$  to  $c d$ . All, therefore, are equal to all. Hence, the angle  $d e b$ , is equal to the angle  $e d c$ : for they subtend equal sides. And  $d f$  is equal to  $e f$ , (by the fifth.) Because, therefore,  $a e$  is equal to  $a d$ , and  $a f$  is common, and the base  $d f$ , is equal to the base  $e f$ , the angle  $d a e$  is bisected, which was required to be done.

But if the sides of the equilateral triangle fall external to the right lines  $b a$ ,  $a c$ , let them be  $b d$ ,  $d c$ , and having connected  $d a$ , let it be produced to the point  $e$ . Because, therefore  $b d$ ,  $d c$ , are equal,



but  $d a$  is common, and the bases  $b a$ ,  $a c$ , are equal, the angle, also,  $b d a$ , (by the eighth) is equal to the angle  $c d a$ . Again,  $b d$ ,  $d c$ , are equal, and  $d e$  is common, and they contain equal angles as we have shewn, the base also  $b e$ , is equal (by the fourth) to the base  $e c$ . Because, therefore,  $a b$  is equal to  $a c$ , and  $a e$  is common, the angle, also  $b a e$ , is equal to the angle  $c a e$ , which was to be shewn.

But if the sides of the equilateral triangle fall within the right lines  $a b$ ,  $a c$ , as  $b d$ ,  $d c$ , let again  $a d$  be connected. Because, therefore,  $b a$ , is



is equal to  $a c$ , and  $a d$  is common, but the base  $b d$ , is equal to the base  $c d$ , hence, the angle  $b a d$  (by the eighth) is equal to  $c a d$ . The angle, therefore, at the point  $a$ , is bisected, in whatever manner the equilateral triangle may be constituted. And having thus summarily spoken concerning these, we shall now proceed to the following theorems, only adding, that the given angle may be given in a four-fold respect. *In position*, as when we say *to this right line, and to this point to place an angle*: for after this manner it is given. But *in form*, as when we call the angle right, or acute, obtuse, right-lined, or mixed. And *in proportion*, as when we call it double, or triple, greater, or less. And lastly, *in magnitude*, as when we call it the third part of a right angle. But the present angle is only given in form.

#### P R O P O S I T I O N X. P R O B L E M V.

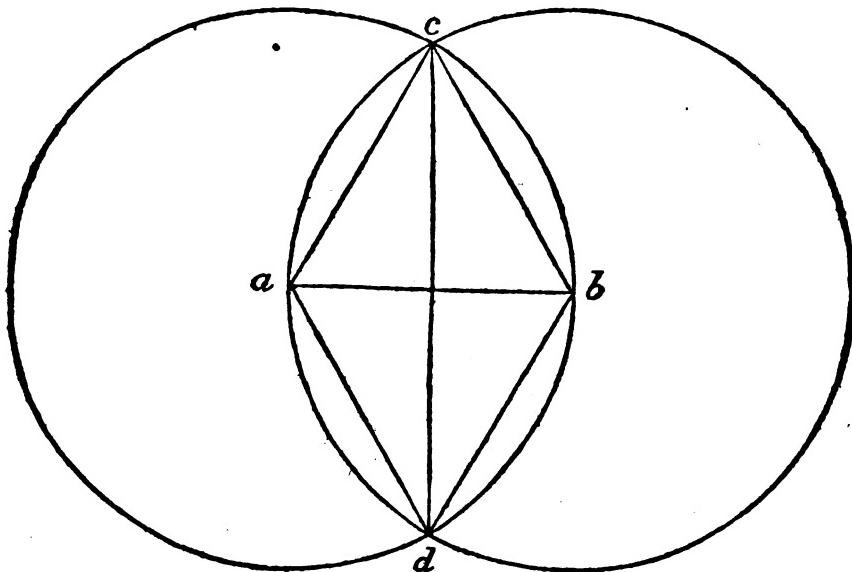
To bisect a given finite right line.

This, also, is a problem which supposes a finite right line, since we cannot terminate a line on both sides infinite. But the section of a line infinite on one side only, wherever the point is assumed, is made in

in unequal parts. For that part of the section which takes place on the infinite side, is necessarily greater than the remainder, because finite. Hence, the line required to be bisected, must be necessarily both ways finite. But perhaps, some excited by this problem, may think, that the doctrine of a line, not being composed from imparables, is only previously received by geometers as an hypothesis. For if it consists from imparables, it either becomes finite, and receives its completion from *odd*, or from *even* parts. But if from such as are *odd*, it will appear that an imparible also may be cut, while a right line is bisected. And if from such as are even, the section will be unequal, because, one part, as composed from more imparables, will be greater than the remainder. It is therefore impossible to bisect a given right line, if magnitude consists from imparables. But if it be not composed from imparables, it may be divided in infinitum. It appears, therefore, (say they) to be received by common consent, and to be a geometrical principle, that magnitude is among the number of things infinitely divisible. Against these we reply in the words of Geminus, that geometers previously receive according to a common conception, that continued quantity is divisible. For we call that continuous, which is composed from conjoined parts, and this it is in every respect possible to divide. But that continued quantity may be infinitely divided, they do not previously assume, but demonstrate from proper principles. For when they shew that incommensurability is found in magnitudes, and that all are not commensurable with each other, what else can we say they evince by this means, except this, that every magnitude may be divided into parts always divisible, and that we can never arrive at an imparible, by the most unwearyed analysis, since this minimum would be the common measure of all magnitudes? This then is demonstrable, but that which says, *every thing continuous is divisible*, is an axiom. Hence, since a finite line also is continuous, it is divisible. And from this conception the institutor of the Elements cuts a finite right line into equal parts,

parts, but not as pre-assuming, that it is divisible in infinitum. For to be merely divisible, and to be infinitely divisible is not the same.

But the discourse of Zenocrates inferring indivisible lines, is confuted by this problem. For if it be a line, it is either right, and may be bisected; or circular, and it is greater than a certain right line; (since every circular has a certain right line less than itself); or it is mixt, and on this account is the more divisible, since composed from simple divisible lines. But this must be deferred to some posterior speculation. However, the geometrician bisects a finite right line, employing in the construction the first and ninth propositions; but using in the demonstration the fourth alone; for by the angles he shews the equality of the bases. But Apollonius Pergæus bisects a given finite right line after the following manner. Let there be (says he) a finite right line  $a b$ , which we are required to bisect, and with



the centre  $a$ , but interval  $a b$ , let a circle be described. And again, with the centre  $b$ , but interval  $b a$ , let another circle be described, and let the right line  $c d$ , connect the common sections of the circles;

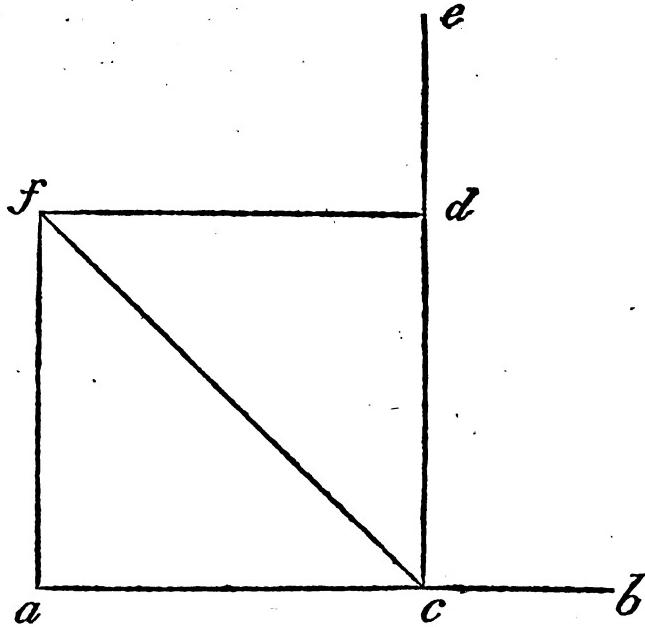
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this shall bisect the right line  $a b$ . For let the equal lines  $d a$ ,  $d b$ ,  $c a$ ,  $c b$ , be connected ; these being equal, because each is equal to  $a b$ . But  $c d$  is common , and  $d a$  is equal to  $d b$  on the same account. Hence the angle  $a c d$ , is equal to the angle  $b c d$ ; and so (by the fourth)  $a b$  is bisected. Such then, according to Apollonius, is the demonstration of this problem, assumed, also, from an equilateral triangle ; but instead of exhibiting the bisection of the line, from the bisection of the angle at the point  $c$ , it shews this from the equality of the bases. The demonstration, therefore, of the institutor of the Elements, is much better, since it is both more simple, and emanates from principles.

### PROPOSITION XI. PROBLEM VI.

To raise a right line at right angles, to a given right line,  
from a given point in that line.

Whether we receive a right line on both sides finite, or on both sides infinite, or on one side infinite but on the other finite, and a point in it, the construction of the present problem will conveniently succeed to the geometrician. For though the given point should be on the extremity of the right line, by producing it we can accomplish our purpose. But it is manifest that the point in the present problem is given in *position*, since it can only be placed in position in a right line. But the right line is given according to *form*; since its magnitude is not distinguished either by proportion or position. Hence, the institutor of the Elements, employing the first and third problem, together with the eighth proposition, and the tenth definition, exhibits the thing proposed. But if any placing the point on the extremity of the right line, should ask us without producing the line, to erect upon this a right line at right angles, we can likewise shew that this is possible to be effected. For let there be a right line  $a b$ , and a given



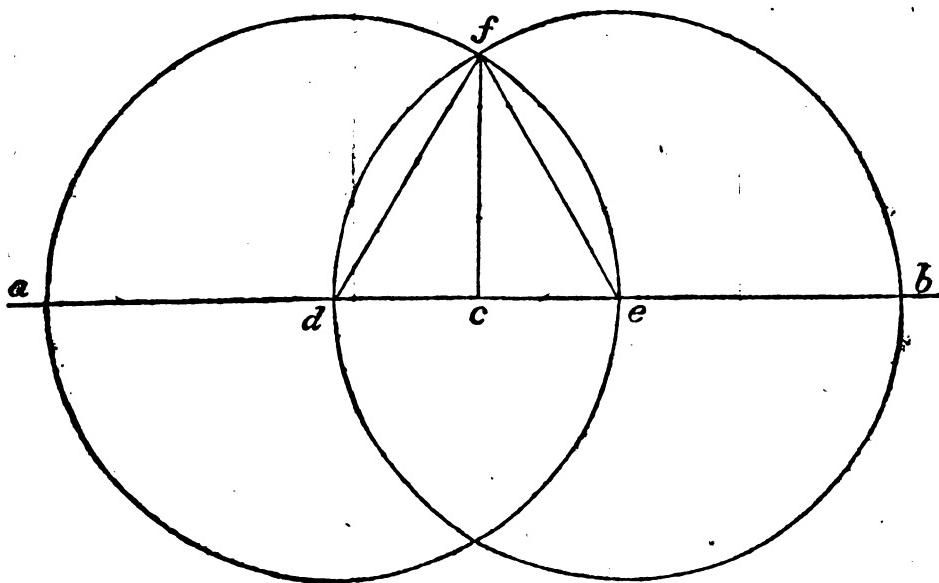
a given point in it  $a$ , and let there be assumed in the line  $ab$ , any point  $c$ , and from this (as the present element teaches us) let a right line  $ce$  be erected at right angles to  $ab$ . Then from  $ce$ , let  $cd$  be taken equal to  $ca$ , and let the angle at the point  $c$  be bisected by the line  $cf$ , and at the point  $d$  let a right line be erected at right angles, coinciding with  $fc$  in  $f$ ; and lastly from the point  $f$ , to the point  $a$ , let  $fa$  be connected. I say that the angle at the point  $a$  is right. For since  $dc$  is equal to  $ca$ , but  $cf$  is common, and contains equal angles, (for the angle at the point  $c$  was bisected) hence,  $df$  is equal to  $fa$ , and all in like manner (by the fourth) are equal to all. The angle, therefore, at the point  $a$ , is equal to the angle at  $d$ . But the angle at the point  $d$  is right; and so consequently is the angle at  $a$ . And thus the thing required is effected. But the institutor of the Elements was not indigent of any such artifice: for he commands us to raise a line at right angles, but not at one right. It is requisite, therefore, not to receive the point in the extremity of the right line, because the perpendicular line forms angles with its subject right line, but not one angle alone.

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But Apollonius raises a perpendicular as follows. Let the given right line, says he, be  $ab$ , and a given point in it  $c$ , but let there be



assumed in  $ac$  any point  $d$ , and from  $cb$ , take away  $ce$ , equal to  $cd$ . Then with the centre  $d$ , but interval  $de$ , let a circle be described; and again with the centre  $e$ , but interval  $ed$ , let another circle be described, and let a right line be drawn from  $f$  to  $c$ . I say that  $fc$  is a perpendicular. For if  $fd$ ,  $fe$ , are connected, they shall be equal. But  $dc$ ,  $ce$ , are equal, and  $fc$  is common. Hence, also, the angles at the point  $c$  (by the eighth) are equal. They are therefore right.. And here, is it not again obvious, that this demonstration is more various than that of Euclid, and requires the description of circles, that by this means an equilateral triangle may be described upon  $de$ , and the problem exhibited? For all the rest are common to the demonstrations. But the demonstration by a semicircle is not worthy to be remembered, since it supposes many things which are afterwards exhibited, and entirely falls from the order of an elementary institution.

PROPO-

## PROPOSITION XII. PROBLEM VII.

Upon a given infinite \* right line, and from a given point which is not in that line, to let fall a perpendicular.

Oenopides first investigated this problem, believing it useful for astrological purposes. But he calls a perpendicular, after the manner of the ancients, a gnomon, because a gnomon, also, is at right angles to the horizon, but the same line is at right angles with a perpendicular, from which it differs only in habitude, since, as he observes a gnomon has the same subject with a perpendicular. But again, a perpendicular is two-fold, that is, it is either plane or solid. Hence, when the point from which the perpendicular right line is drawn, is in the same plane, the perpendicular is called plane; but when the point is on high, and external to the subject plane, it is called solid. And the plane perpendicular, indeed, is drawn to a right line: but the solid to a plane. Hence, it is necessary, that this last should not only form right angles, with one right line, but with all right lines in the same plane. For the perpendicular is let fall on a plane. In the present problem, therefore, the institutor of the Elements proposes to let fall a plane perpendicular. For the deduction is proposed to a right line, and the discourse proceeds, so far as all are supposed to be in the same plane. Hence, in the line at right angles we do not require infinity, because the point is supposed to be in that right line. But in the present problem, respecting a perpendicular, he supposes the given right line infinite, because the point from which the perpendicular is to be drawn is placed external to the right line. For if

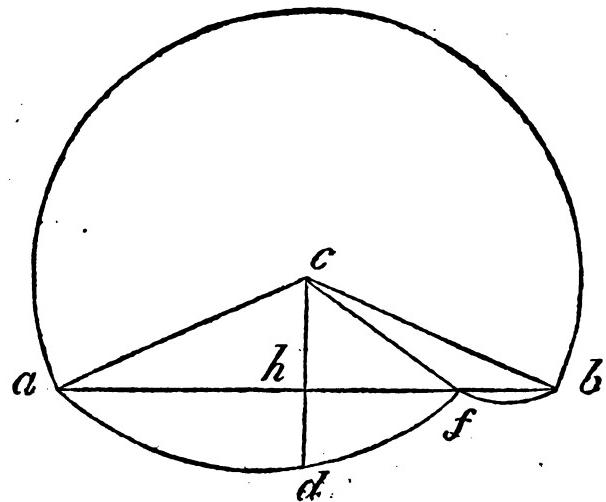
\* Mr. Simson having a great objection to the word infinite, though it is adopted by Euclid, substitutes in its place the word *unlimited*; but not in my opinion with any success. For if by *unlimited*, he means infinite, the alteration is ridiculous: but if he means only *indefinite*, or a line which has boundaries, though they are not ascertained, the problem will not succeed, as the ensuing commentary most beautifully evinces. I only add, that the reader, if he be a man of taste, and possesses any spark of the philosophic genius, must be greatly delighted with the digression of Proclus in this comment, concerning the nature of infinite, as it is perfectly philosophical and truly sublime.

it was not infinite, the point might be received externally, and yet in a direct position, so that the protracted right line would fall upon it, and the problem not succeed. Hence, he places the right line infinite, so that the point may be received at either of its parts; and that no place may be left, in which it can be in the same direction with the given right line, unless it is in the line, and has not an external position. And on this account the right line to which the perpendicular is to be drawn is considered as infinite.

But in what manner infinite can subsist, is a matter well worthy our contemplation. For it is manifest that a right line existing infinite, a plane also will be infinite, and this in energy, if the thing proposed by Euclid be true. That among sensible particulars, therefore, there can be no magnitude infinite, according to any distance, both the dæmoniacal Aristotle, and those who received their philosophy from him, have abundantly shewn. For neither that which is moved circularly, nor any other simple body can be infinite; since the place of each is limited. But neither in separate and imparible reasons is an infinite of this kind possible. For if they neither contain dimension, nor magnitude, much less can they contain infinite magnitude. It remains, therefore, that infinite can alone subsist in the phantasy, which at the same time the phantasy does not comprehend. For as soon as it understands, it induces form and bound to that which is understood, stops the transit of the phantasm by its intellection, pursues its progress, and infolds it in its shadowy embrace. The phantasy, therefore, is not infinite by intellection, but rather by advancing infinitely about that which is understood; and calling whatever it leaves innumerable, and incomprehensible by intelligence, infinite. For as the sight by not seeing understands darkness; so the phantasy by not understanding perceives infinite. Hence it pursues the progress of the infinite, because it is endued with an imparible power, capable of perpetually advancing: but it understands as if stopping in its progression, because infinite surpasses its comprehension. For it calls that infinite, which it leaves as unable to pass over in its pursuit. On this account when we place a given infinite line in the phantasy, in  
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the same manner as we establish all other geometrical species, viz. triangles, circles, angles, lines, and all of this kind, we must not wonder how a line is infinite in energy, and how advancing infinitely, it applies itself to finite intellec[t]tions. But cogitation, in which reasons and demonstrations reside, does not use infinite for the purpose of science, since infinite is by no means perceptible by science, but receiving it from hypothesis, it employs finite alone in its demonstrations, and assumes infinite not for the sake of infinite, but of that which is bounded and finite. For if we should grant to cogitation, that the given point, neither lies in a right line with the given finite right line, nor yet is so distant from it, that no part of the right line is subjected to the point, we shall no longer require an infinite line. That cogitation, therefore, when employing a right line, may use it without controversy and reproof, she supposes it to be infinite; and employs the infinity of the phantasy, as the foundation of infinite generation. And thus much may suffice for the present concerning the nature of infinite.

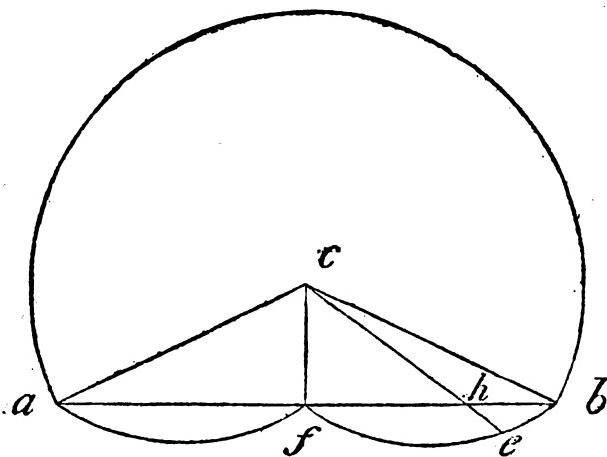
But it is now requisite that we should consider the objections which are urged against the construction of this problem. Let there be re-



ceived, say they, an infinite right line  $a b$ , and let the given point be  $c$ , from which it is required to let fall a perpendicular; and let  $d$  be a point on the other side, according to the geometrician. But the circle

cle which cuts the right line  $ab$ , in the points  $a$  and  $b$ , will cut it also in  $f$ , and will have a situation according to the figure. In answer to this, we must say, that it affirms an impossible case. For let the right line  $ab$  be bisected in  $b$ , and let  $cb$  be connected, and produced to the circumference, to the point  $d$ , and let  $ca$ ,  $cb$ ,  $cf$ , be connected. Because, therefore, these lines are from the centre, and  $ab$  is equal to  $bb$ , but  $cb$  is common, all are equal to all. Hence  $cb$  forms right angles at the point  $b$ . Again, because  $ca$ ,  $cb$ , are equal, they form equal angles at the points  $a$  and  $b$ . But  $ca$  also, is equal to  $cf$ , on which account the angle  $caf$ , is equal to the angle  $cfa$ . In like manner the angle  $cbf$  is equal to the angle  $cfd$ . Because, therefore, the angles at the points  $a$  and  $b$ , are equal, the angle, also,  $cfa$ , is equal to the angle  $cfd$ , and they are successive, and consequently right. But each of the angles at the point  $b$  is right. Hence,  $cb$  is equal to  $cf$ . But  $cf$  is also equal to  $cd$ , since they are from the centre. Therefore  $cb$  is equal to  $cd$ , which is impossible. Hence, the circle does not cut the right line in any other points than  $a$  and  $b$ .

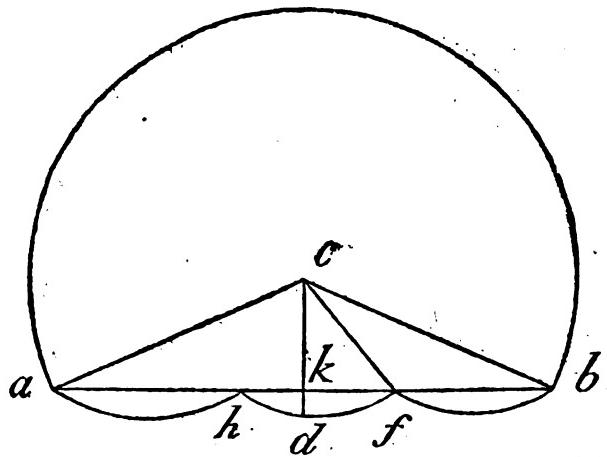
But if any one should say, that he who describes a circle will bisect  $ab$  in  $f$ , we can again shew that this is impossible. For let all be described as before, and let the right line  $fb$ , be bisected in the point  $h$ .



Because, therefore,  $af$ ,  $fb$ , are equal, but  $cf$  common, and the base  $ca$ , is equal to the base  $cb$ , all are equal to all. Hence, the angles at the

the point  $f$  are right. Again, because  $fb$  is equal to  $bb$ , and  $cb$  being connected, is common, and the base  $cf$  is equal to the base  $cb$ , for they are from the centre, the angles at the point  $b$ , are right; for they are equal and successive. Because, therefore, each of the angles  $cfb$ ,  $cbf$ , is right,  $c f$  is equal to  $c b$ . But  $c f$  is equal to  $c e$ , for they are from the centre, and hence  $c b$  is not unequal to  $c e$ , which is impossible.

It now remains that we run over the third objection. For the circle which is described (say they) will cut the right line in the points  $a$ ,  $b$ , and in the points  $f$ ,  $b$ . We therefore bisecting the right line  $ab$ .

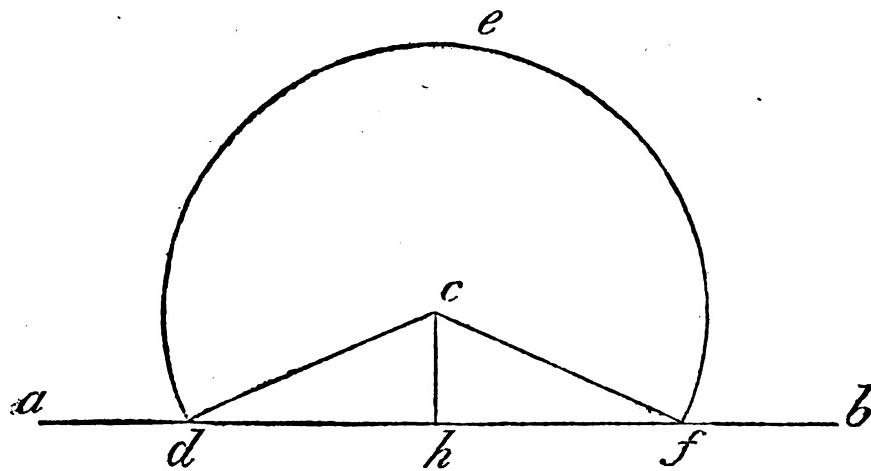


in the point  $k$ , and connecting the lines  $ca$ ,  $cf$ ,  $ck$ ,  $cb$ , can shew that this is impossible. For since  $ka$ ,  $kb$ , are equal, and  $ck$  is common, and the bases  $ca$ ,  $cb$ , are equal, hence the angles at the points  $a$  and  $b$  are equal, and those at the point  $k$  right. But each of the lines is equal to  $cf$ ; and hence, the angles at the point  $f$ , are right; for they are equal, because successive. Therefore,  $cf$  is equal to  $ck$ : for they subtend right ang'es. But  $cf$  is equal to  $cd$ , since they are from the centre; and hence,  $cd$  is equal to  $ck$ , which is impossible. Hence then, it is impossible that the circle which is described shou'd cut the line  $ab$  in one, two, or in more points than  $ab$ . And such are the objections against the present problem.

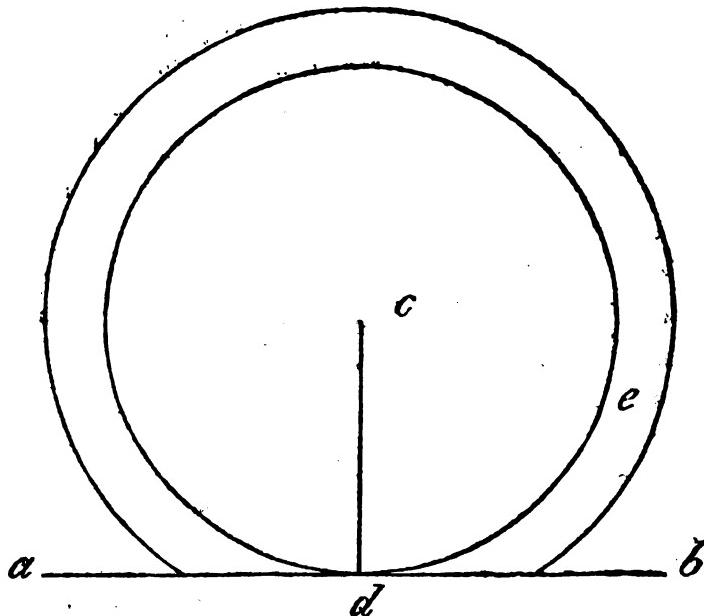
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But there are also cases of the construction of this problem, which are to be distinguished from the objections. For case is not the same with objection; since the former shews the same thing differently, but the latter leads the objection to an inconvenience. But other expositors, not distinguishing these from one another, bring all into the same, so that it is uncertain, whether they enunciate to us in their writings, cases, or objections. We therefore distinguishing these, having enumerated the objections, shall now describe the cases of the problem. Let there be then an infinite right line  $a b$ , and a given



point  $c$ . Now it may be said that there is no farther place in the other part of the perpendicular right line, but in that only where the point  $c$  lies. Taking, therefore, in the right line  $a b$  a point  $d$ , with the centre  $c$ , and interval  $c d$ , let us describe the circumference of a circle  $d e f$ , and bisecting  $df$  in  $b$ , let us connect the lines  $c d$ ,  $c b$ ,  $c f$ . Because, therefore,  $db$  is equal to  $bf$ , but  $ch$  is common, and  $cd$  is equal to  $cf$ , (for they are from the centre,) hence, the successive angles at the point  $b$ , are equal. They are, therefore, right. And hence,  $ch$  is a perpendicular to  $df$ . But if any one should also say that the described circle does not cut the right line  $a b$ , but touch it as the circle  $d e$ , by taking the point  $e$  externally, and using the centre  $c$ , and interval



interval  $c\ e$ , as in the preceding, we shall obtain the object of our enquiry. And thus much we have said concerning the cases of the problem, for the sake of exercising the attention of the reader.

But if we are desirous of adding contemplation likewise to these two problems, a right line erected at right angles, seems to imitate a life tending on high from inferior concerns, ascending purely, and without contamination, and abiding inflexibly with regard to natures subordinate to its own. But a perpendicular is the image of a life perpendicularly descending, and the least of all replete with generative infinity. For a right angle is the symbol of an energy inflexible, and restrained in the comprehension of equality, bound, and finite. From whence, indeed, Timæus also calls the *other circle* in the divine soul, possessing the reasons of sensible natures, *right*; for in our souls it is bent with flexions of every kind, and suffers various contortions and perturbations from the unceasing whirls of generation: but among *wholes* it resides immaculate, uncontaminated, firm, and indeclinable, prior to sensible forms. But if likewise an infinite right line is the symbol of the whole of generation, which is moved infinitely and indeterminately, and besides this, of matter itself, which is deprived

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of bound and form : and if a point placed externally bears an image of an essence impartible, and separate from material natures, doubtless the deduced perpendicular will imitate that life which proceeds into generation with an undefiled progress from unity, and an impartible essence. But if a perpendicular cannot be shewn without circles, this also will be the symbol of an inflexibility inherent in life, through the medium of intellect. For life, indeed, since it subsists by itself as motion, is indeterminate : but it becomes terminated, and is filled with a pure and immaculate power, by participating and adhering to the circulations of intellect.

PROPOSITION XIII. THEOREM VI.

When a right line standing upon a right line forms angles, it either forms two right, or angles equal to two right.

Euclid again passes on to theorems, consequent to things exhibited by problems. For after a perpendicular had been drawn to a right line, and a right line erected at right angles, it remained, to enquire if it should not be a perpendicular, what angles it would form, and how it would be affected to the line upon which it stands. This then he proves universally, that every right line standing upon a certain line, and forming angles, either forms two right, if its state be indeclinable, firm, and never verging : or angles equal to two right, if it declines in one part, but is more distant from its subject line, in the other part. For as much as it takes away from a right angle by its declination in one part, so much it adds by its distance in the other. But it is requisite to take notice, that in this proposition also, the Geometrician employs diligent care. For he does not simply say that every right line, standing upon a right line, forms either two right angles, or angles equal to two right, but he adds, if it forms angles. For what if standing on the extremity of a right line, it should form one angle, will it happen that this may be equal to two right? This certainly is impossible. Since every rectilineal angle is less than two right, as also every solid angle is less than four right. Hence, though you

you shou'd receive that which appears to be the greatest of all obtuse angles, this also must will increase, as that which does not yet receive the measure of two right angles. It is requisite, therefore, that the right line should stand in such a manner, that it may form angles. And these observations regard the productive diligence of science.

But what does he mean by adding the particle, *either two right, or equal to two right?* For when he has constituted two right, he forms angles equal to two right; since right angles are equal to each other. Shall we say that one of the equal angles is also common, but that the other of the equals is only proper? But we are accustomed when both *proper* and *common* is verified, to express every particular from that which is proper, but when we cannot effect this, we are content with that which is common for the explication of the subject concerns. This then, the equality of the successive angles, is common to right angles, but is not predicated of these alone: but this, that they are right, is peculiar to their equality. Hence, the assertion, *equal to two right*, alone signifies the inequality of the angles. For in these it is alone verified, but by no means in such as are equal. And this also the institutor of the Elements divides in opposition to two right. For since it is predicated by itself, it has a power of signifying that the angles on each side are unequal. But through these observations we may also perceive, that equality is the measure and bound of inequality. For though the increase and decrease of an obtuse and acute angle is indeterminate and infinite, yet it is said to receive limitation, and bound from a right angle; and each of them, indeed, separately, recedes from a similitude to the right; but both, according to one harmonizing union, are reduced to its bound. But as they can by no means perfectly equal the simplicity of a right angle, they receive an equality to it when doubled, the duad being the exemplar of their infinity, as of itself endued with an infinite nature. And this seems to procure a manifest image of the progression of primary causes; and of their abiding according to one boundary, in a manner perpetually the same, about the infinity of generation. For how could otherwise generation, which participates of the more and the less, and is carried

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in indefinite whirls, agree with intelligibles, and be equalled with them in a certain respect, unless by participating their natures, whilst they advance with prolific powers, and only multiply themselves in their progressions? For things which abide in their own simplicity and imparibility, are entirely separated from generable natures. And thus much is assumed from the present theorem, and applied to the knowledge of universals.

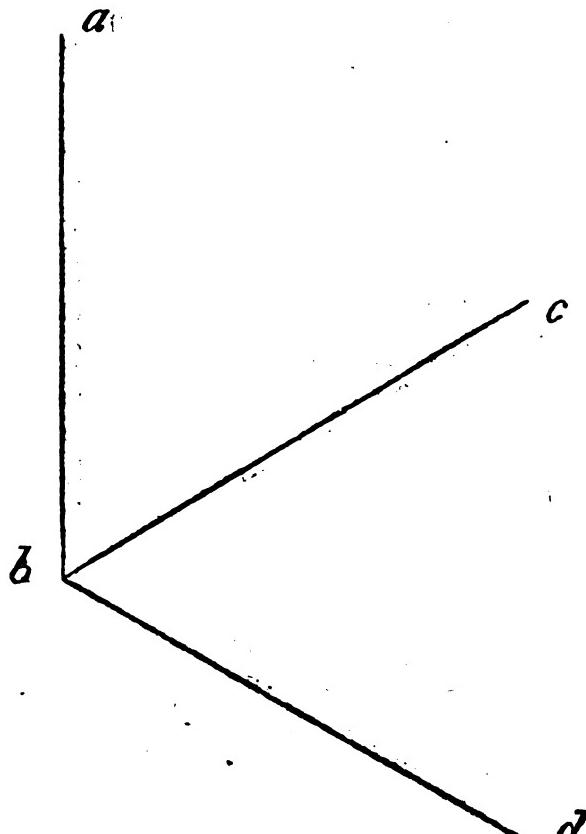
## PROPOSITION XIV. THEOREM VI.

If to any right line, and at a point in it, two right lines being placed in a consequent order, and not towards the same parts, make the successive angles equal to two right, those right lines shall be in a direct position to each other.

The present theorem is the converse of the foregoing: for such as are converse are always consequent to preceding theorems. Since, therefore, the former had constituted a right line upon a right line, and had shewn that it made the successive angles either two right, or equal to two right; in the present theorem he receives the equality of the angles to two right, which are formed at some right line, but he shews that it is one right line which produces their equality. Hence, that which was a *datum* in the former, is in the present theorem an object of enquiry; and is shewn by a deduction to an impossibility. For after this manner the converse of theorems ought to be exhibited; but in problems they should receive principal demonstrations. But in this theorem we may also perceive the greatest and most admirable diligence of this proposition producing science. For in the first place, after he had said, *if to any right line*, he adds, *and at a point in it*; for what if the two extremes of the right line existing, one of the right lines should be drawn from the one extreme, but the other from the remaining one, and should form angles at the right line, equal to two right, would they on this account have a direct position? And how

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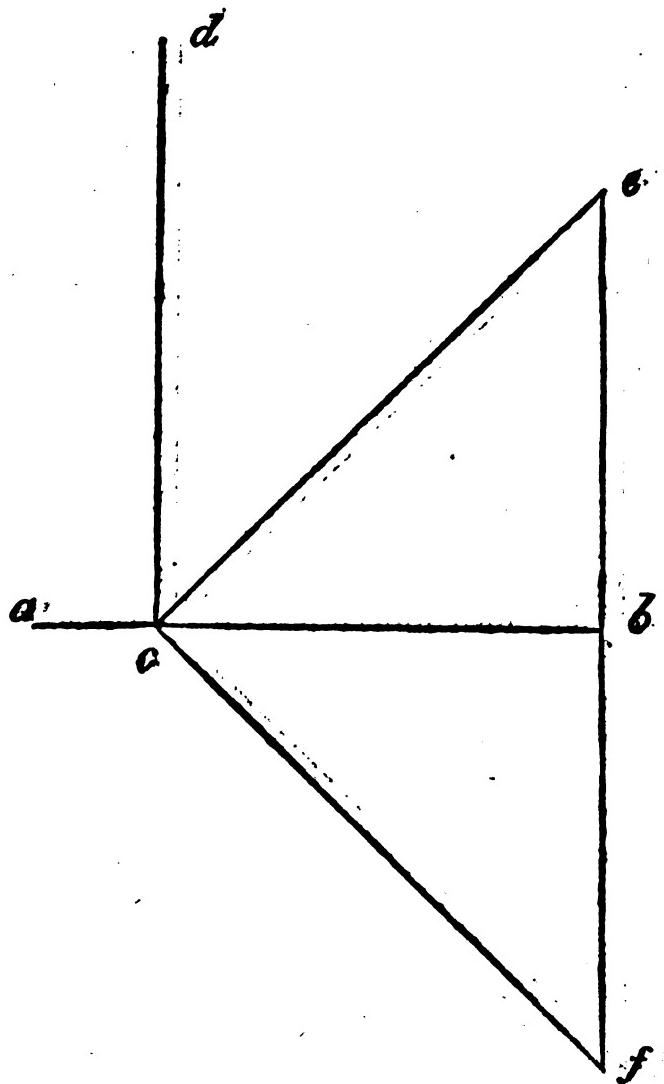
can this take place in lines drawn from different points of the right line? It is on this account also, that he adds, *and at a point in it*, since he is willing that both should be in the same point. But in the second place, because it is possible that the right lines which are drawn, may be at the same point, and not consequent (since we may receive infinite right lines placed at the same point) he adds the particle, *in a consequent order*. And in the third place, because the word *consequent* may be considered as well at the same parts as on both sides: but because it is impossible that lines which are consequent at the same parts should be mutually in a direct position, this indeed he explains; but affords us an opportunity of considering that consequent right lines are to be received in position on both sides; since these also can be shewn to be in a right line. Let there be placed at the right line  $a\ b$ , and at a point in it  $b$ , towards the same parts, two right lines  $b\ c$ ,  $b\ d$ ,



these,

these, therefore, shall be consequent to each other. For no other right line is situated between them. But those things are successive, between which there is nothing similar. Thus we call the columns *consequent*, between which there is no other column : for though the air intervenes, yet nothing of the same kind is situated in the middle. Because, therefore, they lie towards the same parts, they are by no means in a direct position, although they form two angles equal to two right ; I mean the angles at the point  $b$ . For nothing hinders but that the angle  $a b d$ , may contain in itself, one right, and a third part of a right angle : and that the angle  $a b c$ , may be two thirds of a right angle. And thus much concerning the proposition.

But one petition is employed in the construction, viz. the second, which begs to produce a right line straight forwards, as in the demonstration he uses the preceding theorem, and two axioms ; i. e. the one which says, *things equal to the same, are equal to one another* ; and also the one which affirms, *that if from equal things equals are taken away, the remainders shall be equal*. But at the collection of the impossibility, he employs the axiom, which says, *the whole is greater than its part*. For it is equal one common angle being taken away, which is impossible. But that it is possible to the same right line, and at a point in it, two right lines in a consequent position, and yet, towards the same parts, may form angles belonging to that one right line, equal to two right, we may shew with Porphyry, as follows. Let there be a certain right line  $a b$ , and any point in it  $c$ , and let  $c d$  be raised at right angles to  $a b$ , and let the angle  $d c b$  be bisected, by the line  $c e$ . Then from the point  $e$ , to the line  $a b$ , let there be drawn the perpendicular  $e b$ , and let  $e b$  be produced, and place  $f b$  equal to  $e b$ , and connect  $c f$ . Because, therefore,  $e b$  is equal to  $b f$ , but  $b c$  is common, and they contain equal angles (for they are right), hence, the base  $e c$ , is equal to the base  $c f$ . All, therefore, are equal to all. Hence, the angle  $e c b$ , is equal to the angle  $f c b$ . But the angle  $e c b$  is the half of a right angle : because the right angle  $d c b$  was bisected by the line  $e c$ . Hence, also the angle  $f c b$ , is the half



half of one right. The angle, therefore,  $d c f$ , is equal to one right, and the half of a right angle. But the angle  $d c e$ , also, is the half of a right angle. Hence to the right line  $c d$ , and to a point in it  $c$ , two right lines are consequently placed towards the same parts, viz.  $c e$  and  $c f$ , forming angles equal to two right,  $c e$  causing the half of a right angle, and  $c f$  one and a half. Let, therefore, we should enquire after things impossible to be effected, viz. how the right lines  $c e$ ,  $c f$ , forming angles at the right line  $d c$ , equal to two right, can be.

be in a direct position to one another, the Geometrician adds the particle *not towards the same parts*. It is requisite, therefore, that the right lines which form angles equal to two right, should be placed on both sides of the right line, being raised, indeed, from one point, but drawn to different parts of the right line.

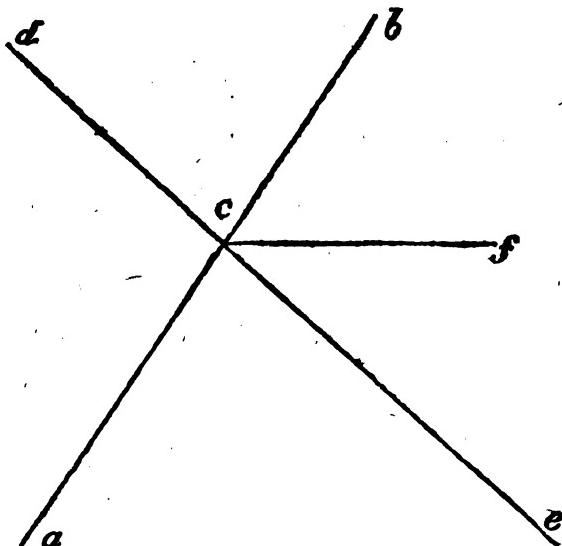
### PROPOSITION XV. THEOREM VIII.

If two right lines cut one another, they will form the angles at the vertex equal.

We must call *successive* angles different from such as are *vertical*. For these last originate from the section of two right lines: but the former from the mere dissection of the one by the other. Thus, if a right line remaining itself without section, but cutting another in its extremity, forms two angles, we denominate these *successive* angles. But if the two right lines mutually cut each other, they form *vertical* angles. And they are so called, because they have their vertices conjoined in the same point. But their vertices are the points, at which the planes, while they are contracted, form angles. This, therefore, is what the present theorem evinces, that when two right lines mutually cut each other, the vertical angles are equal. And it was first invented (according to Eudemus) by Thales: but was thought worthy of a demonstration producing science by the institutor of the Elements. But it is not exhibited from all the particulars requisite to a perfect proposition. For construction is wanting in the present theorem: but demonstration, which must be necessarily inherent, depends on the thirteenth theorem. But he uses two axioms, one of which is, *that things equal to the same, are equal among themselves*: and the other, *if from equal things equals are taken away, the remainders will be equal*. The theorem, indeed, of Euclid, is manifest, but another such is converted to the present theorem. *If to any right line, and at a point in it, two right lines, not assuned towards the same parts, make the verti-*

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cal angles equal, those right lines shall be in a direct position to each other. For let there be a certain right line  $a b$ , and any point in it  $c$ ,



and at the point  $c$ , let two right lines  $c d$ ,  $c e$ , not towards the same parts be assumed, forming equal angles  $a c d$ ,  $b c e$ . I say that  $c d$ ,  $c e$ , are in a right line. For since the right line  $c d$ , infests upon the right line  $a b$ , it forms angles equal to two right, i. e.  $d c a$ ,  $d c b$ . But the angle  $d c a$ , is equal to the angle  $b c e$ . Therefore, the angles  $d c b$ ,  $b c e$ , are equal to two right. Because, therefore, to a certain right line  $b c$ , and at a point in it  $c$ , two consequent right lines  $c d$ ,  $c e$ , not placed towards the same parts, form the successive angles equal to two right, those right lines  $c d$ ,  $c e$ , are in a direct position to each other. The converse, therefore, to the present theorem, is exhibited. But the Geometrician seems to have neglected this, because it is easy to evince its truth, by the same method of deduction to an impossibility as we employed in exhibiting the fourteenth proposition. For the same things being supposed, I say that the right line  $c d$ , is in a direct position to  $c e$ . For if it be not, let  $c f$  be taken in a right line with  $c d$ . Because, therefore, two right lines  $a b$ ,  $d f$ , intersect each other, they will form the angles at the vertex equal. Hence, the angles  $a c d$ ,  $b c f$ , are equal. But  $a c d$ ,  $b c e$ , were also equal.

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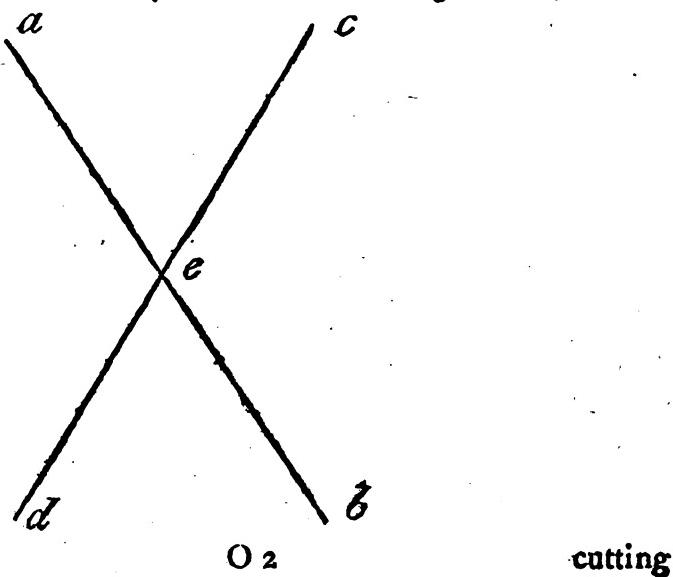
The angle, therefore,  $bce$ , is equal to the angle  $bcf$ , the greater to the less, which is impossible. Hence, no other right line, besides  $cd$ , is in a direct position to  $ce$ . The right lines, therefore,  $cd$ ,  $ce$ , are in a direct position to each other, the angles at the vertex being supposed equal.\* Since then, there is the same demonstration which was preassumed in the fourteenth theorem, would it not have been superfluous to have produced this conversion? But for the sake of exercise, we have proved it as well by a deduction to an impossible, as by an ostensive method. However, this fifteenth theorem seems to rest upon the similitude of the parts of right lines, and their situation in their extremities. Because lines with these conditions, and mutually cutting each other, must necessarily possess similar inclinations on both sides to each other. Since circumferences, and universally non-right lines cutting one another, do not necessarily form the vertical angles equal, but sometimes equal, and sometimes unequal. For if two equal circles cut each other through the centres, or even not through the centres, they will form the lunular angles at the vertex equal: but not likewise the remaining angles, viz. those on both sides concave, and on both sides convex, but the one will be greater than the other. But in right lines, the situation in the extremities, causes the distance of one segment, to be equal to the distance of another.

### C O R O L L A R Y.

From hence it is manifest that if two right lines cut each other, they will make four angles equal to four right.

Corollary is one of the geometrical appellations, but it has a two-fold signification. For they denominate corollaries, whatever theorems are proved together with the demonstrations of others, becoming as it were the unexpected gain and emolument of the investigator: and likewise, whatever is the object of enquiry, but is indigent of invention, and is neither investigated for the sake of generation alone, nor of simple contemplation. For that the angles at the bases of isos-

celes triangles are equal, it is requisite to contemplate, and the knowledge of things in existence is of this kind. But to bisect an angle, or constitute a triangle, to cut off, or place an equal right line, all these demand that something may be performed. And again, to find the centre of a given circle, or two commensurable magnitudes being given to find their greatest common measure, with every thing of this kind, are, after a manner, situated between problems and theorems. For neither is the origin of *objects of enquiry* inherent in these, nor contemplation alone, but invention. Since it is requisite to place the object of enquiry conspicuously and before our eyes. Such then are whatever corollaries Euclid wrote, for he constructed a book of corollaries. But we must now omit to speak of corollaries of this kind. However, such as occur in the elementary institution, appear at the same time with the demonstrations of other things, but they themselves are not principally investigated, as is evident in that which is proposed at present. For the design of the proposition is to enquire whether if two right lines mutually cutting each other, the angles at the vertex are equal. But whilst this is evinced, it is at the same time demonstrated, that the four angles which are formed, are equal to four right. For when we say let there be two right lines,  $a b, c d$ .



cutting each other in the point  $e$ : because  $a$  stands upon  $c d$ , it makes the successive angles equal to two right. And again, because  $b e$  stands upon  $c d$ , it also makes the successive angles equal to two right; then together with the object of enquiry we demonstrate, that the angles about the point  $e$ , are equal to four right. A corollary, therefore, is a theorem, unexpectedly emerging from the demonstration of another problem, or theorem. For we seem to fall upon corollaries, as it were, by a certain chance; and they offer themselves to our inspection, without being proposed, or investigated by us. Hence, we assimilate these also to gains. And perhaps those skilled in mathematical concerns, have imposed on them this appellation, shewing the vulgar, who rejoice in apparent gain, that these are the true gifts of divinity, and true gains, and not the objects of their sordid estimation. For this indeed produces that faculty resident in our nature, and adds the prolific power of science, to principal enquiries, manifesting the copious riches of theorems. And such is the property of corollaries.

But they are to be divided in the first place, according to sciences. For of corollaries, some are geometrical, but others arithmetical. Thus the present corollary is geometrical: but that which is added at the end of the second theorem of the seventh book of the arithmetical elements, is arithmetical. But afterwards they must be divided according to the principal objects of enquiry. For some things are consequent to problems, but others to theorems. Thus, the present is consequent to a theorem: but that which is placed in the second of the seventh book, is consequent to a problem. But in the third place, they must be divided according to their ostensions. For some are exhibited, together with ostensive methods, but others together with deductions to an impossible. Thus the present is shewn by a direct ostension: but that which is at the same time exhibited in the first of the third book, appears, together with a deduction to an impossible. But corollaries may also be divided in many other modes, but these may suffice our present purpose. The present corollary, however,

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teaching us that the place about one point is distributed into angles equal to four right, is subservient to that admirable theorem, which shews that the following three multangles about one point, can alone fill place, viz. the equilateral triangle, the quadrangle, and an equilateral, and equiangular sexangle. But the equilateral triangle must be six times assumed ; since six two-thirds, form four right angles. But the sexangle must be three times formed ; for every sexangular angle is equal to one right, and a third part of a right. And a quadrangle must be four times assumed : for every quadrangular angle is right. Hence, six equilateral triangles conjoined according to their angles, fill four right angles, as also three sexangles, and four quadrangles. But all other multangles, however composed, according to angles, are either deficient from four right, or exceed four right angles \* ; while these alone, according to the aforesaid numbers, are equal to four right. And this theorem is Pythagoric. But by the present corollary, if even more than two right lines should cut each other in one point, as for instance, three or four, or any other number, all the angles which they form, may be shewn to be equal to four right. For they will vindicate to themselves the place of four right angles. But it is manifest that the angles always become double to the number of right lines. And thus two right lines intersecting each other, there will be four angles equal to four right : but from the intersection of three lines, there will be six angles ; and from four, eight, and so on, in infinitum. For the multitude of the right lines is always doubled : but the angles increase according to multitude, and are diminished according to magnitude, because it is the same four right angles, which is perpetually divided.

\* That no other figure besides these can fill place, will be evident, if its angle, when found, is multiplied by any number : for, as Tacquet well observes, it will always either exceed, or be deficient from four right angles. For a more particular demonstration of this admirable theorem, see Tacquet Elementa Geometriae, p. 88.

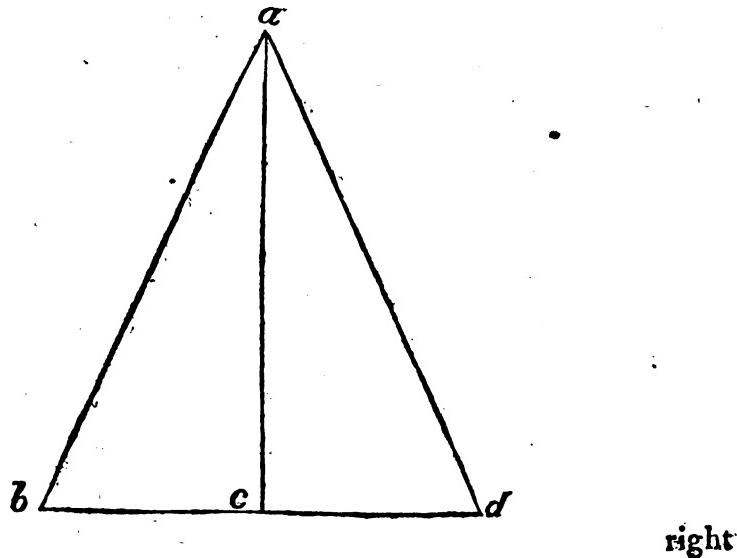
## PROPOSITION XVI. THEOREM IX.

In every triangle having one side produced, the external angle is greater than either of the internal and opposite angles.

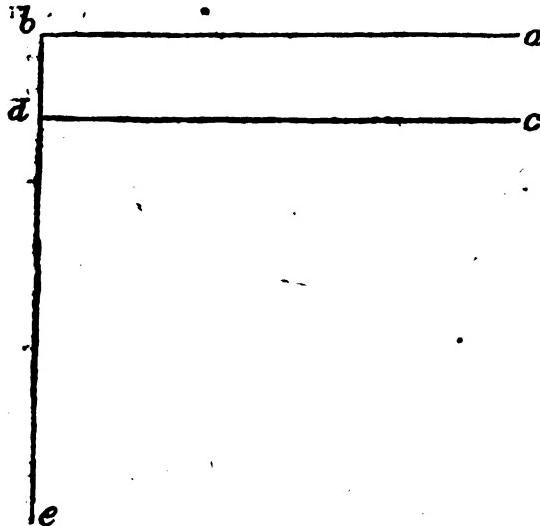
Those who enunciated this proposition, and at the same time omitted the particle, *having one side produced*, perhaps afforded an occasion of objection to many others, as well as to Philip, (according to the narration of the mechanist Heron.) But such as were desirous of entirely removing this calumny, enunciated the theorem, with the proposed addition, corresponding with the general manner of the geometrician. For in the fifth theorem, being desirous to shew, that the angles under the base of an isosceles triangle are equal, he adds, that when the equal right lines are produced, the angles under the base are equal. Hence we infer, that though this proposition might be defective and imperfect in various copies, yet it was perfect and written entire, by the institutor of the Elements. What then does the proposition assert? *That in every triangle, if you produce one of its sides, you will find the angle constituted external to the triangle, greater than either of the internal and opposite angles.* For a little after, this angle will be shewn equal to both, but it is proved to be greater than either in the present; and he necessarily compares it with the opposite angles, and not with the successive angle. For to this last it may be both equal and less: but it is greater than either of the former. Thus, if this triangle should be right angled, and you conceive one of the sides comprehending the right angle to be produced, the external will be equal to the successive angle. But if it should happen to be obtuse-angled, the internal angle may be greater than the external; and it is on this account that he does not compare the external with the successive angle, but with the opposite angles. For of the angles within a triangle, the successive angle borders on the external, but the two others are opposite. Hence, the external angle is greater than either

either of the successive, but may not exceed the successive angle to which it is proximate. But some conjoining these two theorems, I mean the present, and the following, enunciate the proposition thus. *In every triangle having one side produced, the external angle is greater than either of the internal and opposite angles; and any two of the internal angles, are less than two right.* But there is occasion for the connection of these theorems, because the geometrician himself, a little after, enunciates the proposition after this manner, in equal angles, for he says: *In any triangle having one of its sides produced, the external angle is equal to the two interior, and opposite angles; and the three internal angles of a triangle, are equal to two right.* Hence, they think it proper in the present similar case, to connect the objects of investigation, and to make the proposition a composite. But if the datum be enunciated with this addition, it also will be a composite, (since it is requisite to understand two things, viz. the subject triangle, and one side produced:) and if the datum be given without this, it will be a *composite* in capacity, but *simple* in energy; for this must be received at the same time as a datum; since while we suppose an external angle, we must pre-suppose the side as produced.

But we may assume from the present theorem, that it is impossible from the same point, for three equal right lines to fall on the same right line. Thus let there be drawn from one point  $a$ , three equal



right lines,  $a b$ ,  $a c$ ,  $a d$ , to the right line  $b d$ . Because, therefore,  $a b$  is equal to  $a c$ , the angles at the base are equal. Hence, the angle  $a b c$ , is equal to the angle  $a c b$ . Again, because  $a b$  is equal to  $a d$ , the angle  $a b d$ , is equal to the angle  $a d b$ . But the angle  $a b c$ , was equal to the angle  $a c b$ . Hence, the angle  $a c b$ , is equal to the angle  $a d b$ , the external, to the internal and opposite, which is impossible. From the same point therefore, to the same right line, three equal right lines cannot be drawn. But by the present theorem, we can also demonstrate, that if a right line falling on two right lines, makes the external angle equal to the internal and opposite, those right lines will by no means make a triangle, nor coincide, because the same thing would be both greater and equal, which is impossible. Thus for example, let  $a b$ ,  $c d$ , be right lines, and let the right line



$e b$  falling on them make the equal angles  $a b d$ ,  $c d e$ , the right lines  $a b$ ,  $c d$ , will not coincide. For if they coincide, the equal angles remaining, the angle  $c d e$ , will be equal to the angle  $a b d$ . And since it is external, it will be greater than the internal and opposite angle. Hence, it is necessary, if they coincide, that the angles remain no longer equal, but that the angle at the point  $d$ , be augmented. For whether  $a b$  remaining immovable, we conceive that

$\star d$ ,

$c d$  is moved towards it, so as to coincide in the point  $c$ , we shall produce a greater distance in the angle  $c d e$ ; since  $c d$  approaches to  $a b$ , in the same proportion as it recedes from  $d e$ . Or whether  $c d$ , abiding, we conceive that  $a b$  is moved towards it, in a similar manner, we shall by this means diminish the angle  $a b d$ ; for it is at the same time carried towards  $c d$ , and to  $b d$ . Or whether we conceive both of them tending to each other, we shall find that  $a b$  by tending to  $c d$ , contracts the angle  $a b e$ ; and that  $c d$ , by receding from  $d e$ , on account of the motion to the line  $a b$ , increases the angle  $c d e$ . Hence, it is necessary, if it be a triangle; and if the right lines  $a b$ ,  $c d$ , coincide, that the external angle must be also greater than the internal and opposite angle. For either the internal angle remaining, the external is increased, or the external abiding, the internal is diminished, or the internal is contracted, and the external is more dilated. But the cause of these consequences is the motion of the right lines, the one tending to those parts, where it diminishes the internal angle, but the other to the parts where it increases the external. And from this the reader should consider, how the origin of things produces the true causes of enquiries, which we have previously surveyed.

## PROPOSITION XVII. THEOREM X.

The two angles of every triangle, taken all possible ways, are less than two right.

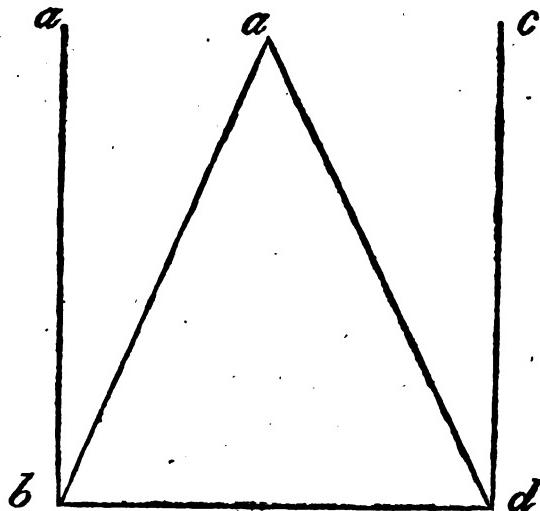
In the present theorem he shews indeterminately, that any two angles of a triangle, are less than two right, but in the following theorems he determines how much they are less, and that they are deficient by the remaining angle of the triangle: for its three angles are equal to two right; and on this account the two remaining angles are less than two right. And, indeed, the demonstration of the elementary institutor proceeds in a manifest order; for it uses the preceding theorem. But it is necessary, as in the last proposition, by regarding the origin of triangles, to find the cause of the present symptom. Let

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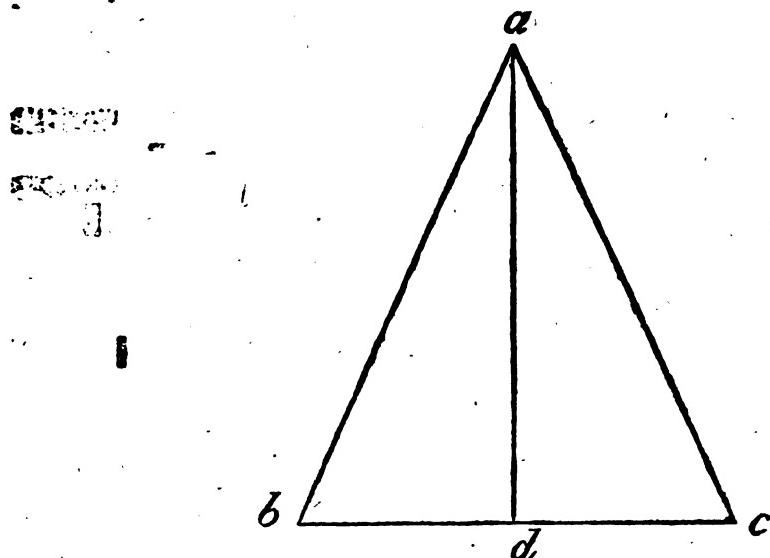
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then the right lines  $a b$ ,  $c d$ , be at right angles to  $b d$ . If these lines then



are to form a triangle, it is requisite they should incline to each other. But their inclination diminishes the internal angles, on which account they become less than two right : for they were right before their inclination. In like manner, if we conceive right lines standing at right angles, on the side  $a b$ , the same consequences will ensue respecting the inclination of the right lines ; and the angles at the points  $a$ ,  $b$ , will be less than two right; and so of the other side. This then is the cause of the proposition, and not the external angle being greater than either of the internal, and opposite angles : since it is not necessary that the side should be produced, nor that any angle should be constituted external to the triangle ; but it is necessary that any two of the internal angles should be less than two right. Hence, it is necessary, as I have said, that the cause of this theorem should be the inclination of the right lines diminishing the angles at the base. But as the institutor of the elements exhibits the object of enquiry, by the external angle, we may accomplish this, without producing any one of the sides. Thus let there be a triangle  $a b c$ , and let there be taken in the side  $b c$ , any point  $d$ , and let  $a d$  be connected. Because, therefore



fore, one side of the triangle  $a b d$ , is produced, viz.  $b d$ , the external angle  $a d c$ , is greater than the internal  $a b d$ . Again, because one side of the triangle  $a d c$  is produced, viz.  $c d$ , the external angle  $a d b$ , is greater than the internal  $a c d$ . But the angles about the right line  $a d$ , are equal to two right, by the thirteenth of this. Hence, the angles  $a b c$ ,  $a c b$ , are less than two right. In like manner, we may shew, that the angles  $b a c$ , and  $b c a$ , are less than two right, by taking a point in the side  $a c$ , and by connecting the point  $b$  with the assumed point. And again, we may affirm, that the angles  $c a b$ ,  $a b c$ , are less than two right, by taking a point in the side  $a b$ , and by connecting a right line, from the point  $c$ , and the received point. And thus the thing proposed, is exhibited by the same theorem, without producing any side of the triangle. Hence, it is possible, that by this, the theorem may be proved, which asserts, *that from the same point, two perpendiculars cannot be drawn, to one right line.* For let there be drawn, if possible, from the point  $a$ , two perpendiculars  $a b$ ,  $a c$ , to the right line  $b c$ . Then the angles  $a b c$ ,  $a c b$ , are right. But because  $a b c$  is a triangle, two of its angles are less than two right. The angles, therefore,  $a b c$ ,  $a c b$ , are less than two right. But they are also equal to two right, because they are perpendiculars, which is impossible. Hence, from the same point, to the same right line, two perpendiculars cannot be drawn.

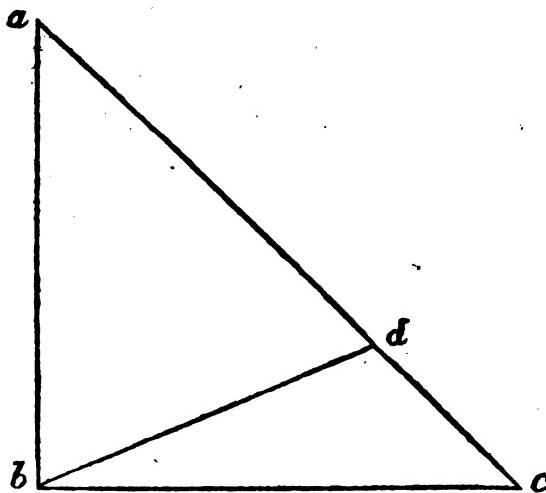
## PROPOSITION XVIII. THEOREM XI.

The greater side of every triangle, subtends the greater angle.

That the equality of the sides in every triangle, forms the equality of the angles which they subtend, and that in like manner the equality of the angles shews the equality of their subtending sides, we learn from the fifth and sixth theorems. But that the equality of those angles, which are subtended by the sides, follows the inequality of the sides, and the contrary we now learn by the present eighteenth and nineteenth theorems. For the one shews that the greater angle is contained under the greater side, but the other, that the greater side subtends the greater angle ; because these are mutually converted, but the same symptoms are contemplated in things contrary, as in the fifth and sixth theorems. But it is manifest, that we proportionally assume the greater and less side, in scalene triangles, that we distinguish the greatest, middle, and least, and the angles in a similar manner : but in isosceles triangles, the greater and less, simply assumed, are sufficient ; for there is one side which is unequal to two, because it is either greater or less, as these theorems cannot take place in equilateral triangles. And here you may observe, that the theorems which exhibit the equality of angles or sides, agree with equilateral and isosceles triangles : but those which exhibit inequality to such as are Isosceles and scalene. But the cause of this is, because of triangles, some are produced from equality alone, others from inequality alone, and others from the conjunction of both, which are partly constituted from equality, and partly through inequality. And some are allied to *bound*, others to *infinity*, and others are generated from the mixture of both. Hence the ternary permeates through all geometrical forms, as through lines, angles, and figures ; and among figures, through such as are trilateral, quadrilateral, and all the rest in a consequent order. But *bound*, likewise, must be considered as inherent in geo-

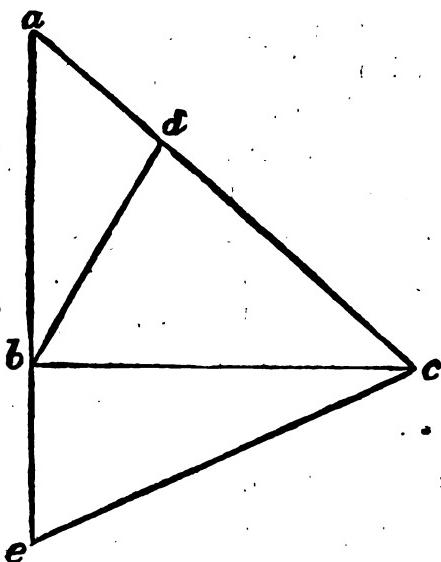
metrical forms, as well through similitude, as equality; and *infinite*, both by dissimilitude, and inequality; and that which is mixt, sometimes from the junction of similitudes, and dissimilitudes, and sometimes from the union of equalities, and inequalities. But the reason of this also, is because geometrical forms regard both quantity and quality. And we have assigned these, because, when we have determined these two, it will be manifest to us, that when the institutor of the Elements says, *of every triangle*, he does not also speak of the equilateral, but of that which has a greater and less side: for it is necessary to consider the object of enquiry, as consequent to the preceding datum; and that the triangle which has a greater and less side, contains a greater angle, under the greater side.

But because the geometrician, when in the construction he receives the triangle  $a b c$ , and the side  $a c$ , greater than the side  $a b$ , in order that he may shew, that the angle at the point  $b$  is greater than



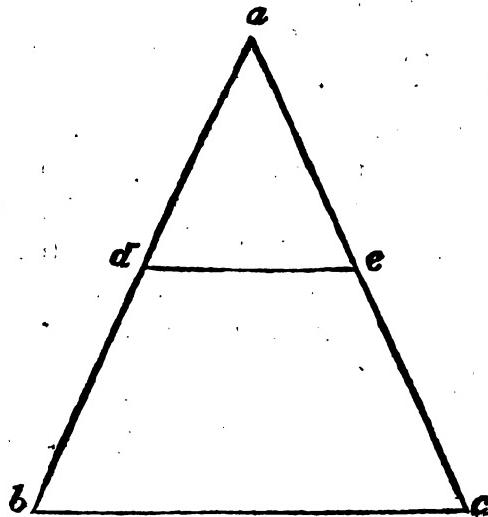
at the point  $c$ , from the side  $a c$ , he cuts off a right line  $a d$ , equal to the side  $a b$ : on this account it may be said that it is necessary to make the ablation at the point  $c$ , let us therefore exhibit the thing proposed upon this hypothesis, according to Porphyry, as follows. Let  $d c$  be equal to  $a b$ , and produce  $a b$  to the point  $e$ , and place  $b e$  equal to  $d a$ . The whole, therefore,  $a e$ , is equal to the whole  $a c$ .

Connect

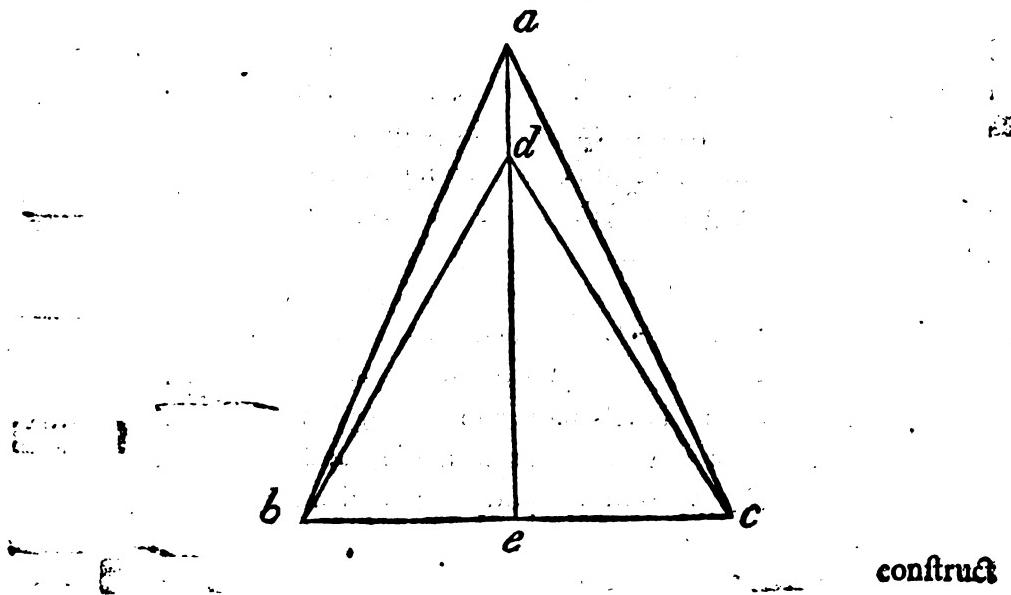


Connect  $e c$ . Because, therefore,  $a e$  is equal to  $a c$ , the angle, also,  $a e c$ , is equal to the angle  $a c e$ , (by the fifth). Hence, the angle  $a e c$  is greater than the angle  $a c b$ . But the angle, also,  $a b c$  is greater than the angle  $a e c$ ; because one side of the triangle  $c b e$  is produced, viz.  $b e$ , and so the angle  $a b c$ , since it is external, is greater than the internal and opposite angle. Much more, therefore, is the angle  $a b c$ , greater than the angle  $a c b$ , which was to be shewn. And such are the geometrical exhibitions of the present theorem.

But it is manifest that the cause of this symptom is the amplification, or diminution according to magnitude, of the side subtending the angle. For when it is greater, it more amplifies the angle; but when less, at the same time it diminishes, and gives a greater contraction to the angle. And this takes place on account of the right line being situated in its extremities: for through its being placed in its extremities, it changes likewise the magnitudes of the angles, according to the increase and decrease which it receives. And this we affirm in one triangle, since it is possible that the same angle may be subtended by a greater or less right line; and that the same right line may subtend a greater and less angle. For let the triangle happen to be an isosceles one,  $a b c$ , and let there be taken in the side  $a b$ , a point



a point  $a$ , and let  $ae$  be taken equal to  $ad$ , and connect  $de$ . The right lines therefore,  $de$ ,  $bc$ , subtend the angle at the point  $a$ , of which the one is greater, but the other less. And in the same manner infinite right lines, greater and less subtending the angle  $a$ . Again, let the triangle  $abc$ , be isosceles, and let  $bc$  be less than  $ba$ ,  $ac$ , and



construct

construct upon  $b c$ , an equilateral triangle  $b d c$ , and connect  $a d$ , and produce it to  $e$ . Because, therefore, the angle  $b d e$ , of the triangle  $a b d$ , is external, it is greater than the angle  $b a d$ . In like manner the angle  $c d e$ , is greater than the angle  $c a d$ . The whole, therefore,  $b d c$ , is greater than the whole  $b a c$ , and the same right line subtends both, viz. the greater and the less angle. But it is shewn, that likewise greater and less right lines subtend the same angle. But in one and the same triangle, one right line subtends one angle, and the greater always the greater, and the less always the less, the cause of which we have contemplated.

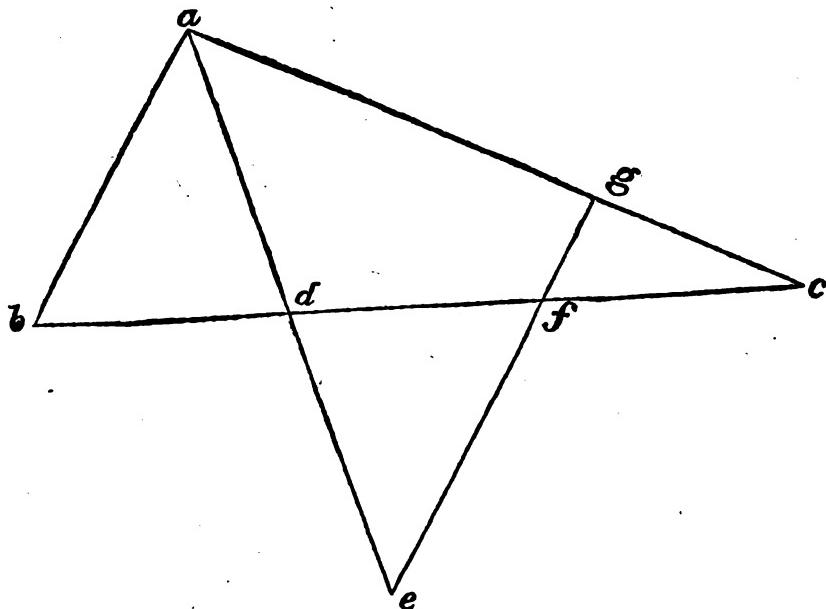
### PROPOSITION XIX. THEOREM XII.

The greater side of every triangle subtends the greater angle.

This is the converse of the preceding theorem; and the datum, as well as the object of enquiry, is simple in each. Add too, that what was conclusion there, is hypothesis here: and what was hypothesis there is conclusion in this. But the former precedes, because it has the inequality of the sides given; and this follows, because it supposes unequal angles. For *sides*, indeed, seem to contain right-lined figures, but the *angles* appear to be contained; and the mode of demonstration in the former is ostensive, but in this it concludes the thing proposed by a deduction to an impossibility. The geometrician, therefore, by division, reasons concerning that which is impossible: for the angles being unequal, *I say*, (says he) *that the sides also subtending the unequal angles are unequal; and the greater subtends the greater given angle*. For if that which subtends the greater angle is not greater, it is either equal, or less. But if it be equal, the angles also which they subtend, are equal by the fifth. But if less, the angle also which it subtends, is less by the preceding: for it was shewn that the greater side subtends the greater angle, and the less the lesser. But the angles have a contrary position; and hence, the one side is greater than the other.

But

But it is possible that we may exhibit the thing proposed, without this division. For if the angle of a triangle be bisected, and the right line drawn to the base, cutting the angle, divides it into unequal parts, the sides containing that angle will be unequal, and the greater will be that which coincides with the greater segment of the base, but the less that which coincides with the lesser. Let there be a triangle  $a b c$ , and let the angle at  $a$  be bisected, by the right line  $a d$ , and let  $a d$  cut



the base  $b c$ , into unequal parts, and let  $c d$  be greater than  $b d$ . I say that the side  $a c$  is greater than the side  $a b$ . Produce  $a d$  to the point  $e$ , and place  $d e$  equal to  $a d$ . And because  $d c$  is greater than  $d b$ , place  $d f$  equal to  $b d$ , and connect  $e f$ , and produce it to the point  $g$ . Because, therefore,  $a d$  is equal to  $d e$ , and  $b d$  to  $d f$ , the two are equal to the two, and they comprehend equal angles at the vertex. Hence, the base  $b a$ , is equal to the base  $e f$ , and all, therefore, are equal to all. On this account also the angle  $d e f$ , is equal to the angle  $d a b$ . But this is not unequal to  $d a g$ . Hence, the side  $a g$ , is equal to the side  $e g$ , by the sixth. The side, therefore,  $a c$ , is

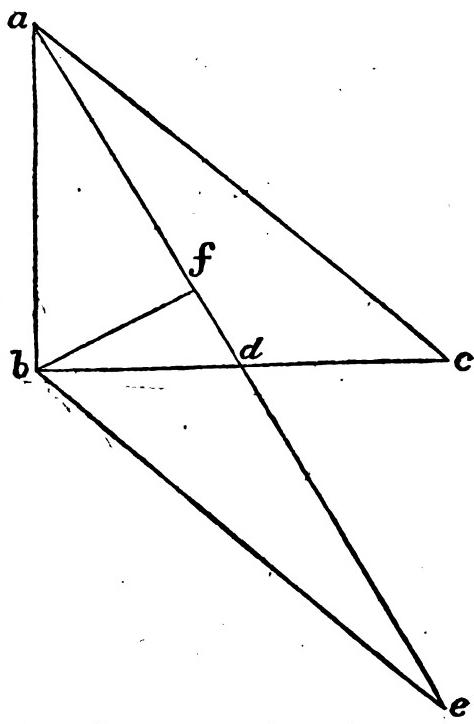
VOL. II.

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greater

greater than the side  $ef$ . But the side  $fe$ , is equal to the side  $ab$ ; and hence, the side  $ac$ , is greater than the side  $ab$ , which was to be demonstrated.

This being pre-assumed, we can shew that the greater side subtends the greater angle. Let there be a triangle  $abc$ , having the angle at



the point  $b$ , greater than the angle at the point  $c$ . I say that the side  $ac$ , is greater than the side  $ab$ . Let  $bc$  be bisected in the point  $d$ , and connect  $ad$ , and draw  $de$ , equal to  $ad$ , and connect  $be$ . Because, therefore,  $bd$ , is equal to  $dc$ , and  $ad$ , to  $de$ , the two are equal to the two, and they comprehend equal angles at the vertex. Hence, the base  $be$ , is equal to the base  $ac$ , and all are equal to all. Hence too, the angle  $dbe$ , is equal to the angle at the point  $c$ , but less than the angle  $abd$ . The angle, therefore  $abe$ , is bisected by the right line  $bf$ . Hence,  $ef$ , is greater than  $fa$ . Because, therefore, the angle

at

at the point  $b$ , of the triangle  $a b e$ , is bisected by the right line  $b f$ , and  $e f$  is greater than  $f a$ , it follows from what has been previously shewn, that the side  $b e$ , is greater than the side  $b a$ . But  $b e$  has been shewn to be equal to  $a c$ . The side, therefore,  $a c$ , is greater than the side  $a b$ ; and the object of enquiry is exhibited. And it is manifest that the institutor of the Elements, avoiding a variety of demonstration, refrains from this mode of demonstrating, and employs a method of proof, which leads from division to an impossibility, because he was willing to fabricate the converse to the preceding, without any intervening medium. For the eighth theorem, indeed, which is the converse of the fourth, brings great disturbance, because it makes conversion difficult to be known. For it is more excellent to exhibit converse theorems, by preserving the continuity through an impossible, than to destroy the continuity by a principal demonstration. And hence, Euclid shews almost all converse theorems by a deduction to an impossibility.

## PROPOSITION XX. THEOREM XIII.

Two sides of every triangle, however taken, are greater than the remaining one.

The Epicureans oppose the present theorem, asserting that it is manifest even to an ass; and that it requires no demonstration: and besides this, that it is alike the employment of the ignorant, to consider things manifest as worthy of proof, and to assent to such as are of themselves immanifest and unknown; for he who confounds these, seems to be ignorant of the difference between demonstrable and indemonstrable. But that the present theorem is known even to an ass, they evince from hence, that grass being placed in one extremity of the sides, the ass seeking his food, wanders over one side, and not over two, Against these we reply, that the present theorem is indeed manifest to sense, but not to reason producing science: for this is the case in a variety of concerns. Thus for example, we are indubitably

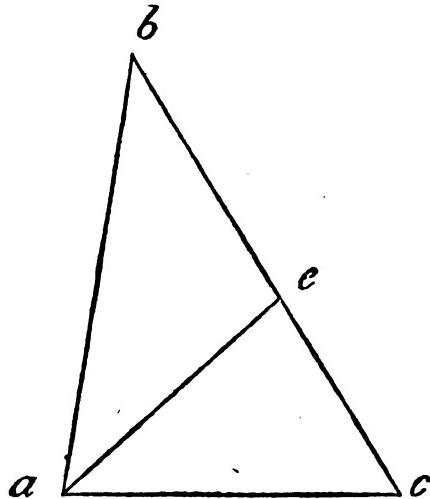
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certain

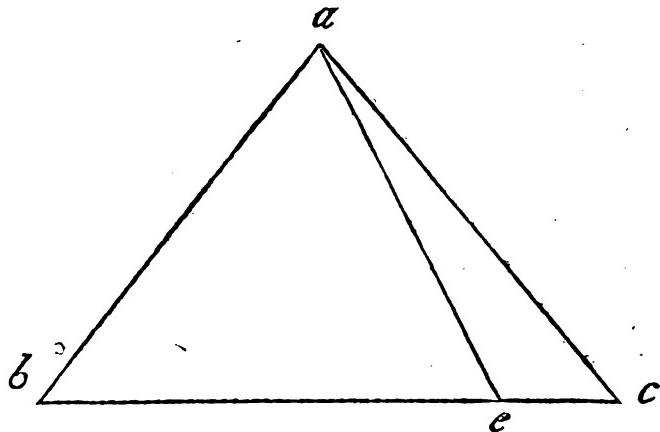
certain from sense, that fire warms, but it is the business of science to convince us how it warms ; whether by an incorporeal power, or by corporeal sections ; whether by spherical, or pyramidal particles. Again, that we are moved is evident to sense, but it is difficult to assign a rational cause how we are moved ; whether over an impartible, or over an interval : but how can we run through infinite, since every magnitude is divisible in infinitum ? Let, therefore, the present theorem, that the two sides of a triangle are greater than the remainder, be manifest to sense, yet it belongs to science to inform us how this is effected. And thus much may suffice against the Epicureans.

But it is requisite to relate the other demonstrations of the present theorem, such as Heron, and the familiars of Porphyry have fabricated, without producing the right line, after the manner of Euclid. Let there be a triangle  $a b c$ , it is requisite, therefore, to shew, that that the sides  $a b$ ,  $a c$ , are greater than the side  $b c$ . Bise&t the



angle at  $\alpha$ , by the right line  $a e$ . Because, therefore, the angle  $a e c$ , is external to the triangle  $a b e$ , it is greater than the angle  $b a e$ . But the angle  $b a e$ , was placed equal to the angle  $e a c$ . The angle, therefore,  $a e c$ , is greater than the angle  $e a c$ . Hence, the side also  $a c$ , is greater than the side  $c e$ . And for the same reason the side  $a b$ , is

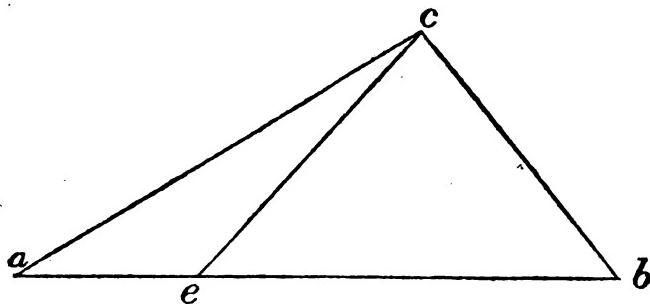
is greater than the side  $b e$ . For the angle  $a e b$  is external to the triangle  $a e c$ , and is greater than the angle  $c a e$ ; that is than the angle  $e a b$ . And on this account the side  $a b$ , is greater than the side  $b e$ . The sides, therefore,  $a b, a c$ , are greater than the whole side  $b c$ . And the like may be shewn of the other sides. Let there again be a triangle  $a b c$ . If therefore the triangle  $a b c$ , be equilateral, two sides will be doubtless



greater than the remaining one: for when there are three equal quantities, any two are double of the remainder. But if it be isosceles, it will have a base either less, or greater than each of the equal sides. If therefore the base be less, the two sides are given greater than the remainder. But if the base be greater, let it be  $b c$ , and cut off from it a part equal to either of the sides, which let be  $b e$ , and connect  $a e$ . Because, therefore, the angle  $a e c$ , is external to the triangle  $a e b$ , it is greater than the angle  $b a e$ . On the same account the angle  $a e b$ , is greater than the angle  $c a e$ . Hence, the angles about the point  $e$ , are greater than the whole angle about the point  $a$ , of which  $b e a$  is equal to  $b a e$ , since  $a b$  is equal to  $b e$ . The remainder, therefore,  $a e c$ , is greater than the remainder  $c a e$ . Hence, the side  $a c$ , is greater than the side  $c e$ . But the side  $a b$ , was also equal to the side  $b e$ . The sides, therefore,  $a b, a c$ , are greater than the side  $b c$ .

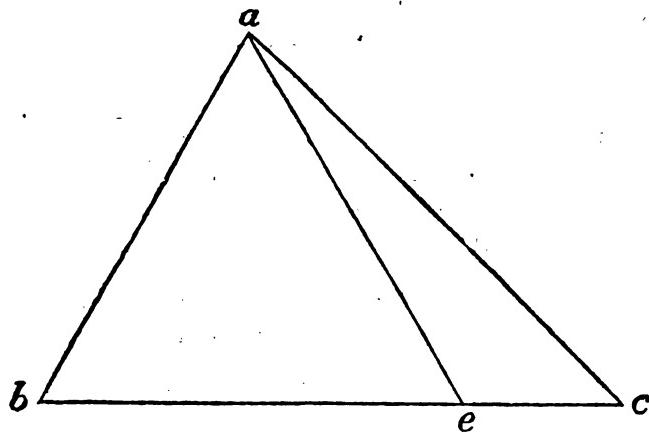
But

But if the triangle  $a b c$ , be scalene, let the greatest side be  $a b$ , the



middle  $a c$ , and the least  $c b$ . The greatest side, therefore, assumed with either of the others, exceeds the remainder : for by itself it is greater than either. But if we are desirous of shewing that the sides  $a c$ ,  $c b$ , are greater than the greatest side  $a b$ , we must employ the same construction as in the isosceles triangle, cutting off from the greater side, a part equal to one of the other sides, and connecting the line  $c e$ , and using the external angles of the triangles. •

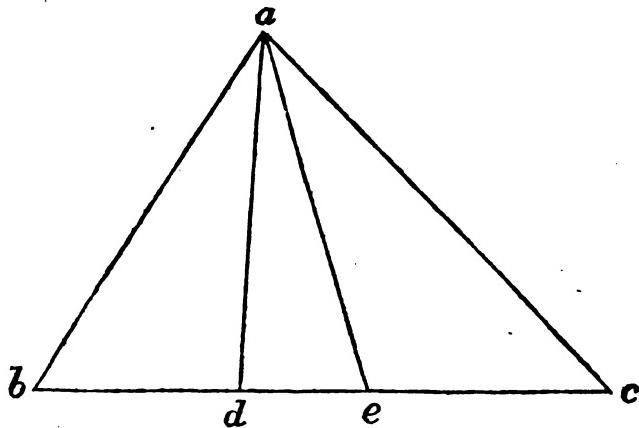
Let there be again any triangle  $a b c$ . I say that the sides  $a b$ ,  $a c$ , are greater than the side  $b c$ . For if they are not greater, they are



either equal or less. Let them be equal, and cut off  $b e$ , equal to  $a b$ . The remainder, therefore,  $e c$ , is equal to  $a c$ . Because then,  $a b$ ,  $b e$ , are equal, they subtend equal angles ; and this is likewise true of  $a c$ ,  $e c$ , because they are equal. Hence, the angles at the point  $e$ , are equal to the angles at the point  $a$ , which is impossible. Again let

the

the sides  $a b$ ,  $a c$ , be less than  $b c$ , and cut off  $b d$ , equal to  $a b$ , and



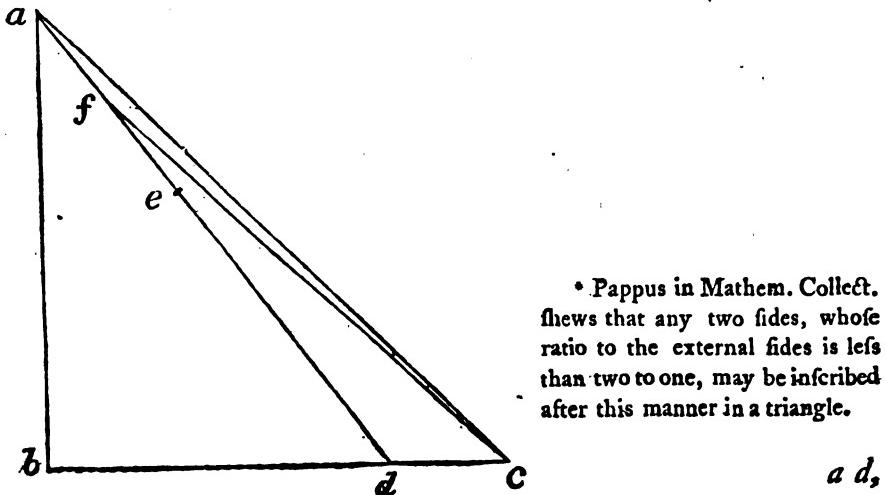
$e c$  to  $a c$ . Because, therefore,  $a b$  is equal to  $b d$ , the angle  $b d a$ , is not unequal to the angle  $b a d$ . And because  $a c$  is equal to  $c e$ , the angle  $c e a$ , is equal to the angle  $e a c$ . Hence, the two angles  $b d a$ ,  $c e a$ , are equal to the two  $b a d$ , and  $e a c$ . Again, because the angle  $b d a$  is external to the triangle  $a d c$ , it is greater than the angle  $e a c$ : for it is greater than  $c a d$ . By a similar reason also, because the angle  $c e a$ , is external to the triangle  $a b e$ , it is greater than the angle  $b a d$ : for it is greater than the angle  $b a e$ . Hence, the angles  $b d a$ ,  $c e a$ , are greater than the two  $b a d$ ,  $e a c$ . But they were also equal to them, which is impossible. The sides, therefore,  $a b$ ,  $a c$ , are neither equal to, nor less than the side  $b c$ , but greater. And the like may be exhibited in others.

#### PROPOSITION XXIV. THEOREM XIV.

If upon one side of a triangle, two right lines beginning from the extremities, are internally constituted, the constituted right lines will be less than the other sides of the triangle, but they will contain a greater angle.

That which is expressed by the proposition, is, indeed, manifest; and the demonstration adopted by the elementary institutor, is evident; and the theorem is consequent to the first principles, since it depends on two theorems, the one previously exhibited, and the fourteenth.

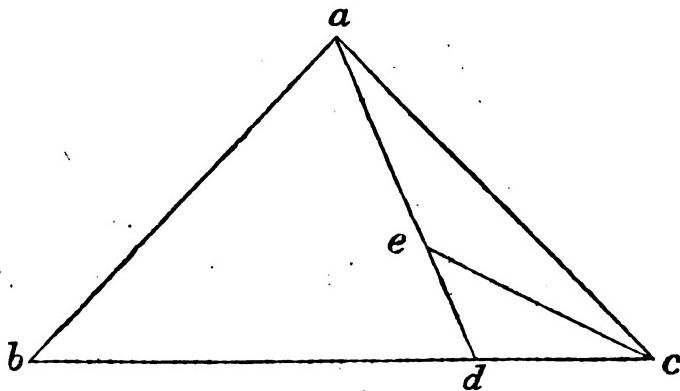
teenth. For in order to shew, that the lines internally constituted, are less than the external, the theorem is required, which says, *the two sides of every triangle, are greater than the remaining one*: but for the purpose of confirming that the angle comprehended by them is greater than that comprehended by the external sides, that theorem procures the greatest utility, which says, *the external angle of every triangle, is greater than the internal and opposite angle*. But you will receive at the same time, conviction of geometrical diligence, and a commemoration of things admirable in the mathematical disciplines, if we shall shew *that it is possible within a certain triangle, upon one of its sides, not upon the whole, but upon some one of its parts, to constitute two right lines greater than the external right lines*\*; and again, others comprehending a less angle, and comprehended in the angle made by the external lines. For this being exhibited, it will at the same time be manifest, that the institutor of the Elements necessarily adds, that the internally constituted lines must begin from the extremities of the common basis; and must be constituted upon one whole side, and not upon any one of its parts: but likewise, as I have said, one of the admirable things which geometry contains, will be manifest. For is it not, indeed, admirable, that the lines constituted upon the whole side, should be less than the external sides: but that those constituted upon a part should be greater? Let there be then a right angled triangle  $a b c$ , having the angle at the point  $b$  right, and take in the side  $b c$ , any point  $d$ , and connect  $a d$ . Hence,  $a d$  is greater than  $a b$ . Take from



\* Pappus in Mathem. Collect. shews that any two sides, whose ratio to the external sides is less than two to one, may be inscribed after this manner in a triangle.

$a d$ , a part equal to  $a b$ , which let be  $d e$ , and bisect  $e a$ , in the point  $f$ , and connect  $f c$ . Because, therefore,  $a f c$  is a triangle, the lines  $a f, f c$ , are greater than  $a c$ . But  $a f$  is equal to  $f e$ . The right lines therefore,  $f e, f c$ , are greater than  $a c$ . But  $d e$  is equal to  $a b$ . Hence, the right lines  $f c, f d$ , are greater than the right lines  $a b, a c$ , and they are internally constituted.

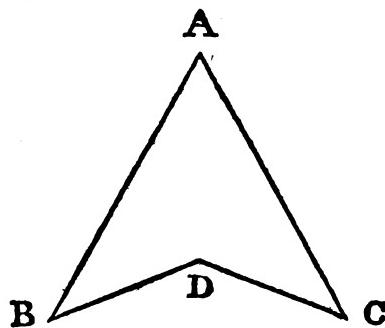
Let there be again an isosceles triangle  $a b c$ , having the base  $b c$ , greater than either of the equal sides. Then from  $b c$ , cut off  $b d$ ,



equal to  $a b$ , and connect  $a d$ , and take in  $a d$ , any point  $e$ , and connect  $e c$ . Because, therefore,  $a b$ , is equal to  $b d$ , the angle  $b a d$ , is also equal to the angle  $b d a$ . And because the angle  $b d a$  is external to the triangle  $e d c$ , it is greater than the internal and opposite  $d e c$ . Hence, the angle  $b a d$ , is greater than the angle  $d e c$ . Much more, therefore, is the angle  $b a c$ , greater than the angle  $d e c$ ; and  $b a c$  is contained by the external lines, but  $d e c$  by internal lines. Within a triangle, therefore, right lines  $d e, e c$ , comprehending a lesser angle, are constituted within the angle comprehended by the external lines; and the thing proposed is shewn without employing the parallel lines of expositors. Hence, it is necessary that the constituted right lines should begin from the extremities of the basis: for those which are constituted upon any one of its parts, are shewn to be sometimes greater than the external lines, and to comprehend a lesser angle. But when they are constituted in this manner, beginning from the extremities, the species of triangles, called (*ἀκαρνάνι*) or, similar to the point of a spear \*, presents itself to our view; and is one of the admirable things contained in geometry, viz. to find a quadrilateral tri-

\* See likewise concerning these triangles. Vol. I. p. 173 of these Commentaries.

angle. As for example, the triangle A B C. For it is contained by the four sides B A, A C, B D, D C; but it has three angles, one at B, the other at A, and the other at C. And hence, the present figure is a quadrilateral triangle.

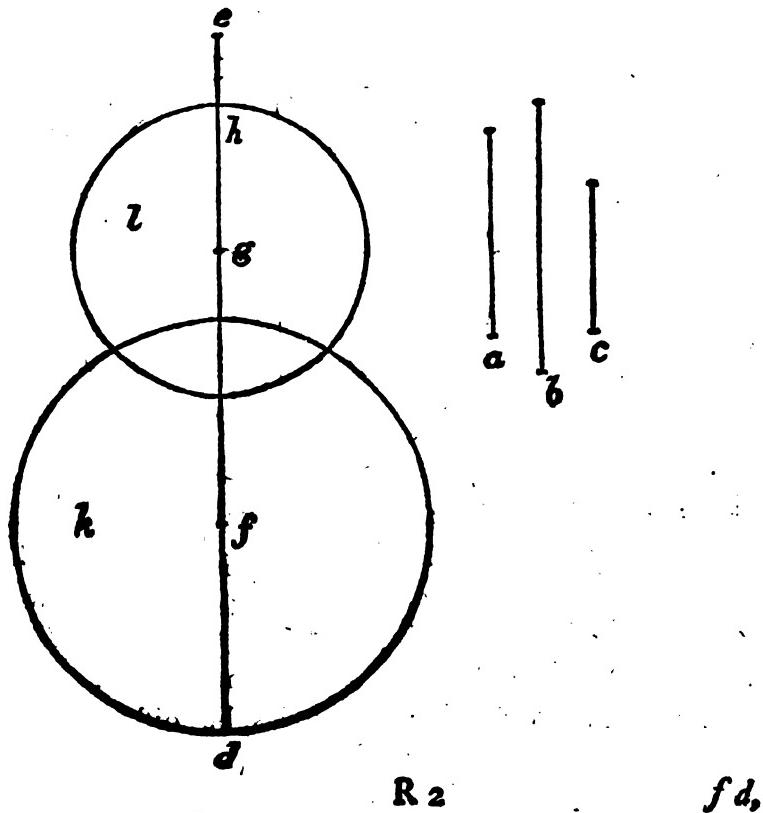


### PROPOSITION XXII. PROBLEM VIII.

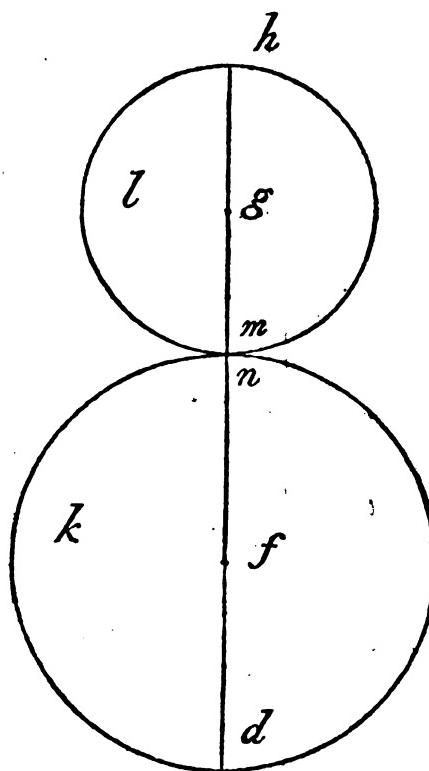
To construct a triangle from three right lines, which are equal to three given right lines. But it is requisite that two of the lines must be greater than the remaining one, in whatever manner they may be taken.

We again pass to problems, and Euclid commands us to construct a triangle from three proposed right lines, two of which are greater than the remaining one, equal to given right lines. Because he knew this in the first place, that it was impossible to construct a triangle from those same lines, which had already received the declared position: but that this was possible to be effected from their equals. In the next place, he knew it was necessary that two of the right lines about to complete the triangle, should be greater than the remaining one: for the two sides of every triangle are greater than the remaining one, however assumed, as we have shewn. On this account he adds, *that it is necessary the first right lines remaining, to construct a triangle from three equal to them: but that it is requisite, any two, however taken, should be greater than the remainder, or there will not be a triangle from three lines equal to the given right lines.* But by this means he also destroys all the objections which are urged against the construction, and which may be perfectly dissolved by this addition. Hence, the present problem ranks among things determined, and not

among such as are indetermined : For of problems as well as of theorems, some are indeterminate, but others without termination. Thus if we should simply say, from three right lines which are equal to three given right lines, to construct a triangle, the problem is indeterminate and impossible. But if we add, two of which, however assumed, are greater than the remainder, the problem is determined and possible. For as the division of theorems takes place, according to true and false, so that of problems according to a possible and impossible enunciation. But that the objections which are urged against the construction, may be from hence dissolved, we shall learn from a little inspection: for we shall follow the words of the geometrician. Let there be three right lines,  $a b c$ , of which any two, however taken, are greater than the third, and let it be required to accomplish the thing proposed. Let there be placed a certain right line  $d e$ , on one part finite, as at the point  $d$ : and on the other part infinite. Then place  $df$  equal to  $a$ , but  $fg$  to  $b$ : and  $gb$  to  $c$ . And from the centre  $f$ , but interval



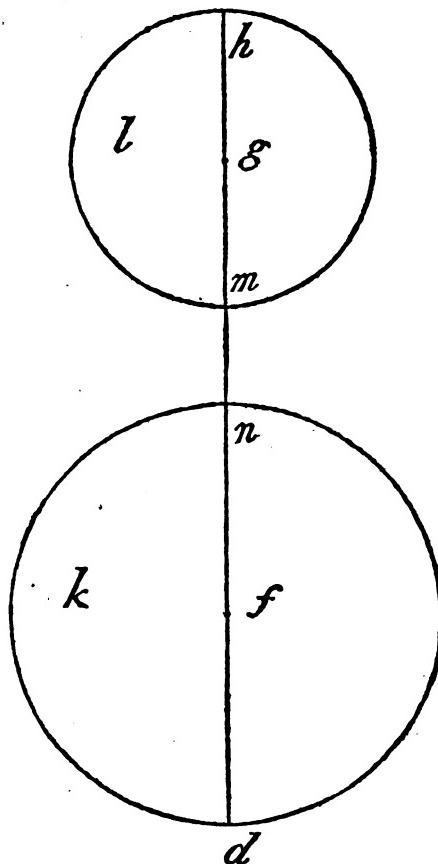
*fd*, let a circle *k* be described. Again, with the center *g*, but interval *gh*, let the circle *l* be designed; and the circles will intersect each other. For this is assumed by the institutor of the Elements. But it may be asked how this takes place? For perhaps they either only touch each other, or they do not even touch. Since it is necessary that they should suffer some one of three cases, I mean that they should either intersect or touch, or be distant from each other. I say, therefore, that they necessarily intersect each other. For let them in the first place, touch each other. Because, therefore, the point *f* is



the centre of the circle *k*,  $df$  is equal to  $fn$ . And because the point *g* is the centre of the circle *l*,  $bg$  is equal to  $gm$ . The two, therefore,  $df, gm$ , are equal to one, viz. to  $fg$ . But they were placed greater than one, as also  $a$ , together with  $c$ , is greater than  $b$ . They are therefore equal to it, and at the same time greater, which is impossible. Again, if it be possible, let the circles be distant from each other, as

*k* and

*k* and *l*. Because, therefore, the point *f*, is the centre of the circle *k*,



*df*, is equal to *fn*. And because the point *g*, is the centre of the circle *l*, *bg* is equal to *gm*. The whole, therefore, *fg*, is greater than the two, *df*, *bg*: for *fg*, exceeds *df*, *gb*, by *nm*. But it was supposed that *df*, *bg*, were greater than *fg*, in the same manner as *a* and *c* are greater than *b*. For *df* was placed equal to *a*, but *fg*, to *b*, and *bg* to *c*. It is necessary, therefore, that the circles *k l*, should intersect each other. Hence, the institutor of the Elements very properly receives them cutting one another: since of the three right lines, he supposes two greater than the third, however, they may be assumed, but neither equal to, nor less than one. But it is necessary that when the circles touch, two of the lines should be equal to the third; and that when they are distant from each other, two should be greater than the remainder.

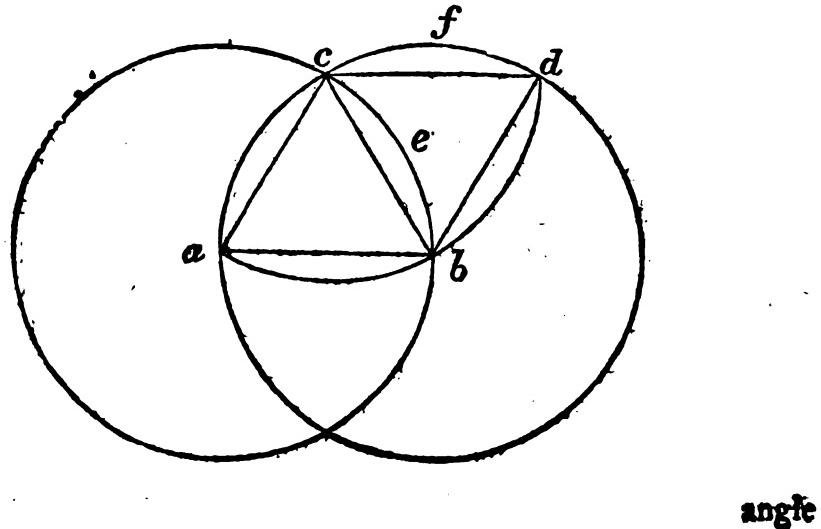
## PROPOSITION XXIII. PROBLEM IX.

On a given right line, and at a given point in it, to constitute an angle equal to a given right lined angle.

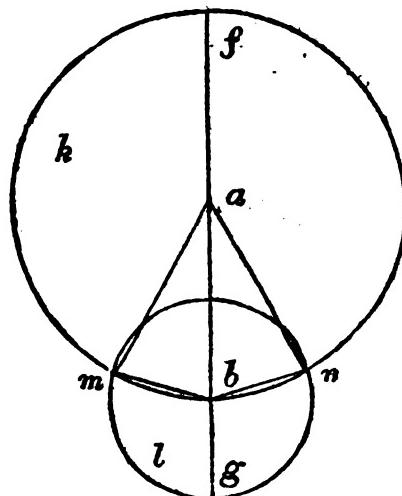
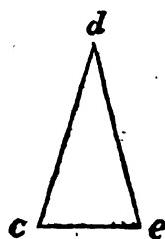
This also is a problem, whose invention according to Eudemus is rather the gain of Oenopides than of Euclid : but it requires the construction of an angle, on a given right line; and at a given point in it equal to another right lined angle. This, then, Euclid necessarily adds, that the given angle must be rectilineal ; because it is impossible that an angle can be constructed on a right line equal to every angle. For it has been shewn † that there are only two curve-lined angles equal to right-lined angles, viz. the angle of a lunular figure, which we have proved equal to every right-lined angle ; and the angle of that figure similar to an axe \*, which is equal to two thirds of a right

\* In the second comment of this book.

† This will be manifest from the following figure. Let the circles  $a c$ ,  $b d$ , be drawn passing through their respective centers  $a$ ,  $b$ ; and from the centre  $c$ , with the radius  $c b$ , equal to  $a b$ , describe the arch  $a b d$ , and draw the lines  $c b$ ,  $c d$ ,  $c a$ . Then because  $a c b$  is an equilateral triangle, as also  $c b d$ , each of the angles  $a c b$ ,  $b c d$ , shall be equal to  $\frac{2}{3}$  of one right angle ; and because the biline  $c d$ , is equal to the biline  $c b$ , hence, the angle formed by the arch  $c b$ , and the arch  $c d$ , viz. the angle  $a c f$ , shall be equal to the angle formed by the right line  $c b$ , and the right line  $c d$ , i. e. to  $\frac{2}{3}$  of one right angle. Q. E. D.



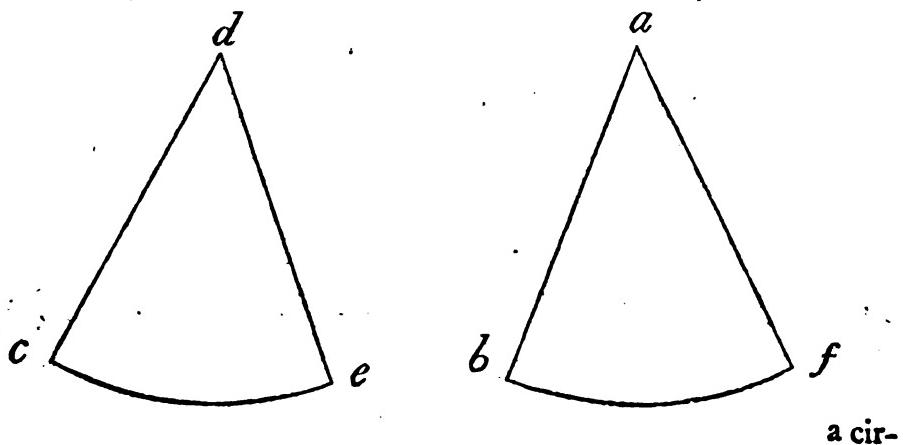
angle. But a lunular figure of this kind, which is called (*πελκοειδές*) Pelecoides, is formed from two circles cutting each other through their centres. However, the construction of an angle on a certain right line, causes the constituted angle to become determinate, and not indifferent in species, but forms it either right-lined, or mixt. But since no mixt can be equal to a right-lined angle, it is manifest that this must be perfectly rectilineal. The institutor of the Elements, therefore, simply using the present problem, and constructing a triangle from three right lines, equal to three given lines, accomplishes the thing proposed. But you may receive a more exquisite construction of the triangle, by the following method. Let there be a



given right line  $a b$ , and a given point in it  $a$ , and a given right lined-

lined angle  $cde$ . It is required, therefore, to accomplish the problem. Connect  $ce$ , and produce  $ab$  on both sides to the points  $f, g$ . Then place  $fa$ , equal to  $cd$ , and  $de$  to  $ab$ , and  $bg$  to  $ec$ . And with the centre  $a$ , but interval  $af$ , describe the circle  $k$ . And again, as in the preceding, with the centre  $b$ , but interval  $bg$ , describe the circle  $l$ . The circles, therefore, will cut each other, as we have shewn in the last proposition. Let them cut each other in the points  $m, n$ , and from these points draw right lines to the centres as in the figure. Because, therefore,  $fa$  is equal to  $am$ , and  $an$ , but  $cd$  is equal to  $fa$ ;  $am$ , and  $an$ , will be each equal to  $cd$ . Again, because  $bg$  is equal to  $bm$ , and  $bn$ , but  $gb$  is not unequal to  $ce$ ;  $bm$ , and  $bn$ , will be also equal to  $ce$ . But  $ab$  is equal to  $de$ . The two therefore,  $ab, am$ , are not unequal to the two  $de, dc$ , and the base  $bm$ , is equal to the base  $ce$ . Hence, the angle  $ma b$ , is equal to the angle at the point  $d$ . And again, the two  $na, ab$ , are equal to the two  $cd, de$ , and the base  $nb$ , is equal to the base  $ce$ . The angle, therefore,  $na b$ , is equal to the angle  $cde$ , and the thing proposed is doubly accomplished: for we have not only constituted one, but two angles, equal to the given angle, on each side of the right line  $ab$ ; so that in whatever part we may desire the construction to be made, it will be indubitable, and without contradiction. And this we have added to the construction of the elementary institutor.

But we cannot praise the method of Apollonius, because it requires the assistance of the third book. For he receives any angle  $cde$ , and a right line  $ab$ , and with the centre  $d$ , but interval  $cd$ , he describes



a circumference  $c e$ . In like manner with the centre  $a$ , but interval  $a b$ , he describes a circumference  $b f$ ; and intercepting a circumference  $c e$ , equal to  $b f$ , he connects the right line  $a f$ , and affirms that the angles  $a$  and  $d$ , insisting on equal circumferences, are equal. But it is necessary to pre-assume that  $a b$  is equal to  $c d$ , in order that the circles may be also equal. We therefore think that a demonstration of this kind requiring posterior propositions, is foreign from an elementary institution; and we give the preference to that of the geometer, as consequent to principles.

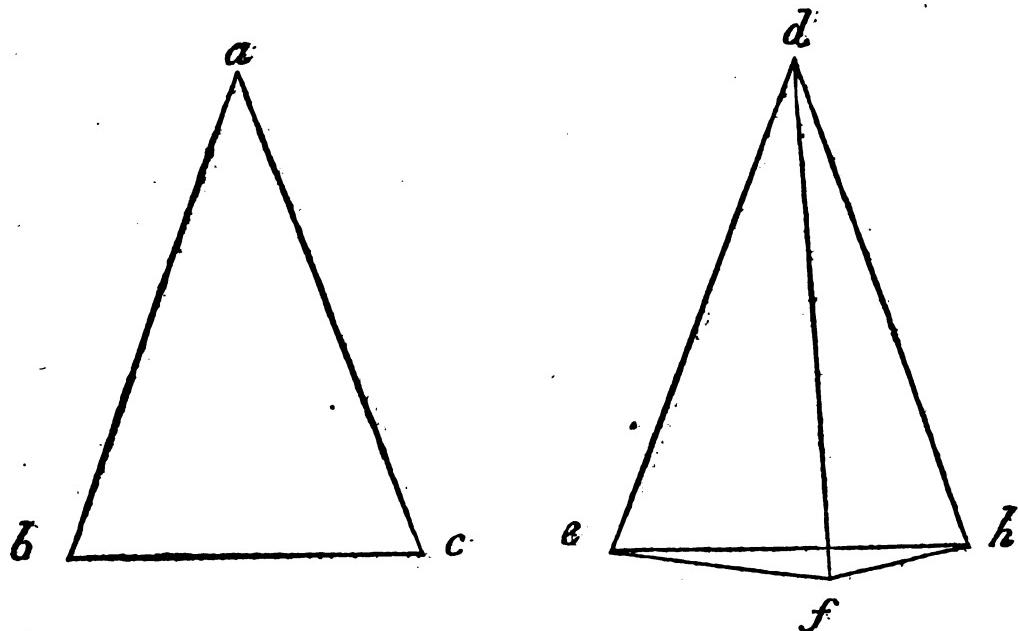
### PROPOSITION XXIV. THEOREM XV.

If two triangles have two sides equal to two, each to each, but the one angle contained by the equal right lines greater than the other: they shall also have the base of the one greater than the base of the other.

Euclid again passes on to theorems, and speaks concerning inequality in two triangles, in a manner similar to his discourse concerning equality. For supposing two triangles, having two sides equal to two, each to each, he sometimes places the vertical angle equal in each, and sometimes unequal; and he proceeds in a similar manner with respect to the base. Besides this, he demonstrates that the equality of the bases is consequent to the equality of the vertical angle, and that the equality of the vertical angles, is consequent to the equality of the bases: but he now shews that the inequality of the one, follows the inequality of the other. The present theorem, therefore, is opposite to the fourth: for that, indeed, supposes the vertical angles of the triangles equal, but this supposes them unequal. And that demonstrates the equality of their bases; but this proves them unequal, in the same manner as their angles. It precedes, however, the following theorem: for that deduces its proof of inequality from the bases to the angles subtending the bases: but this, on the contrary, reasons from the angles to the bases, which are under the angles. Hence it is, after this manner, the converse of its consequent proposition, but opposite to the eighth theorem. For the one from the equality of the

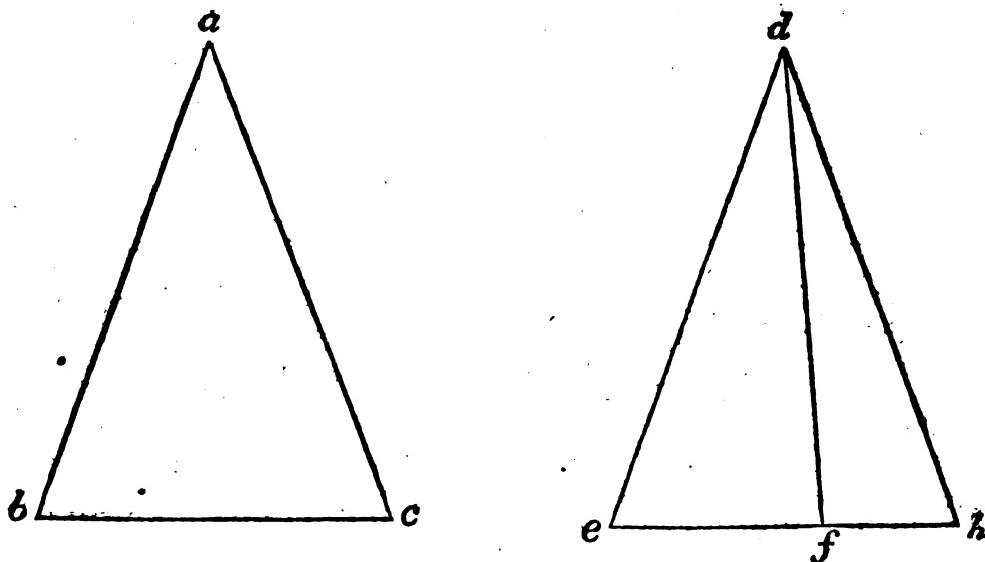
bases, demonstrates the equality of the vertical angles, but the other from the inequality of the bases, shews that the vertical angles are unequal. It is, however, common to these four (two of which are conversant with equality, I mean the fourth, and the eighth, but two about inequality, the present and the following; and two begin from angles, viz. the fourth, and the object of investigation in the present, but two from bases, viz. the eighth, and the following proposition); it is common, I say, to all these four, as well to the fourth and the eighth, as to the twenty-fourth and twenty-fifth, to have two sides equal to two, each to each. For these being unequal, all enquiry is superfluous, and subject to deception. And thus much for a universal speculation concerning the present theorem.

But let us now consider the construction of the elementary institutor, and add to it where deficient. For Euclid receiving two triangles,  $a b c$ ,  $d e f$ , having the sides  $a b$ ,  $a c$ , equal to the sides  $d e$ ,



$d f$ , each to each, and the angle at the point  $a$ , being greater than the angle at the point  $d$ , and willing to shew that the base  $b c$ , is greater than the base  $e f$ , on the right line  $e d$ , and at a point in it  $d$ , constitutes an angle  $e d b$ , equal to the angle at the point  $a$ . For the  $d f$ ,

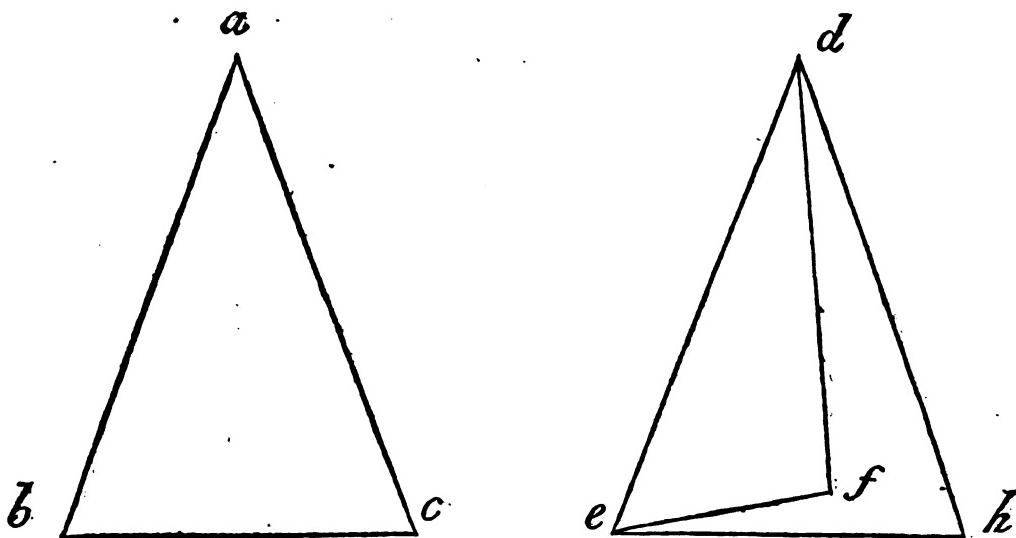
angle at the point  $a$ , is greater than the angle at the point  $d$ , and he connects  $db$ , equal to  $ac$ . The right line, therefore,  $eb$ , produced to the point  $b$ , either falls above, or upon, or beneath the line  $ef$ . The institutor of the Elements, indeed, considers it as lying above the line. But let it be upon the right line. Again, therefore, we may exhibit the same. For the two  $ab, ac$ , are equal to the two  $de, db$ , and



they contain equal angles. Hence, the base  $bc$ , is equal to the base  $eb$ . But  $eb$  is greater than  $ef$ ; and on this account  $bc$  is greater than  $ef$ . Again, let it be placed beneath  $ef$ . Connecting, therefore,  $eb$ , we must say, that since  $ab, ac$ , are equal to  $de, db$ , and they comprehend equal angles,  $bc$  is also equal to  $eb$ . Because, therefore, within the triangle  $deb$ , two right lines  $df, fe$ , are constructed on the side  $de$ , they are less than the external sides. But  $db$ , is equal to  $df$ : for it is equal to  $ac$ . Hence  $be$  is greater than  $ef$ : But  $be$  is equal to  $bc$ . And therefore,  $bc$  is greater than  $ef$ . The theorem, therefore, is exhibited according to every position. Why then, as is the fourth theorem, he at the same time demonstrated

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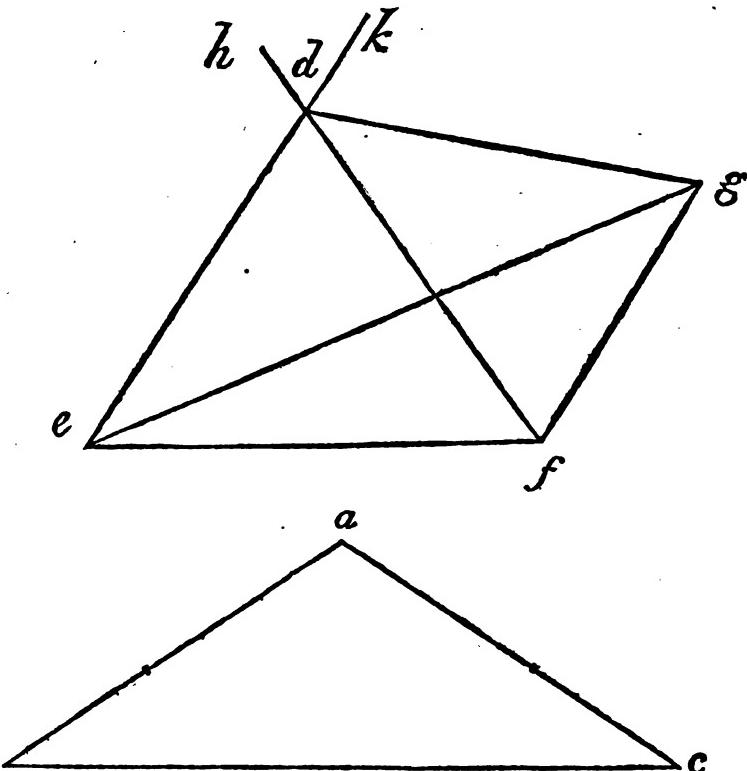
that,



that the areas of triangles are equal, does he not add in the present, that besides the inequality of the bases, the areas also are unequal? Against this doubt we must say, that there is not the same proportion in equal, as in unequal angles and bases. For when the angles and bases are equal, the equality also of the triangles follows: but when they are unequal, it is not necessary that the inequality of the areas should be consequent; since the triangles may as well be equal, as unequal; and that may be greater, and likewise less, which contains the greater angle, and the greater base. On this account, therefore, the institutor of the Elements leaves the comparison of the triangles; to which we may add, that the contemplation of these, requires the doctrine of parallels.

But if it be requisite, that anticipating things which are afterwards exhibited, we at present make a comparison of areas, we must say, that if the angles  $\alpha, d$ , are equal to two right, the triangles may be shewn to be equal: but when they are greater than two right, the lesser triangle will be that which contains the greater angle; and when they are less than two right, this will be the case with the greater triangle. For let the construction in the element be given, and produce-

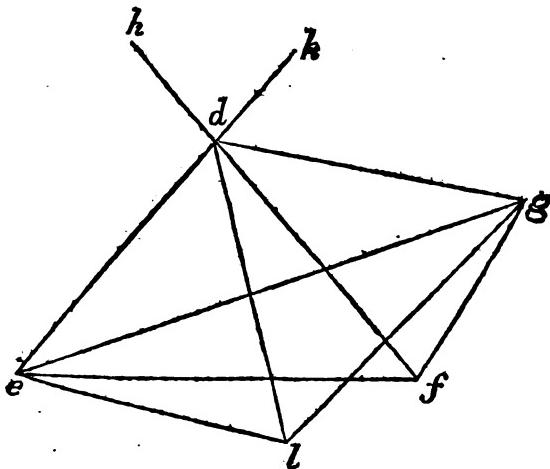
$e d$ ,



$e d, f d$ , to the points  $k, b$ ; and let us suppose the angles  $b a c, e d f$ , equal to two right. Because, therefore, the angle  $b a c$ , is equal to the angle  $e d g$ , the angles  $e d g, e d f$ , are equal to two right. But the angles  $e d g, k d g$ , are also equal to two right. Let the common angle  $e d g$  be taken away, and the remainder  $e d f$ , will be equal to the remainder  $k d g$ . But  $e d f$  is equal to  $b d k$ ; for they are vertical angles. Hence, the angle  $k d g$ , is equal to the angle  $b d k$ . And because the angle  $g d b$ , is external to the triangle  $g d f$ , it is equal to the two internal and opposite angles at the points  $g$  and  $f$ . But these angles are equal to each other, because  $d g$  is equal to  $d f$ . Hence, the angle  $g d b$ , is double of the angle at the point  $g$ , and of the angle at the point  $f$ . The angle, therefore, at the point  $g$ , is equal to the angle  $g d k$ , and they are alternate; and consequently  $d e$  is parallel to  $f g$ . The triangles, therefore,  $g d e, f d e$ , are upon the same base  $d e$ , and between the same parallels  $d e, g f$ ; and are consequently equal. But the triangle  $g d e$ , is equal to the triangle  $a b c$ ; and so the

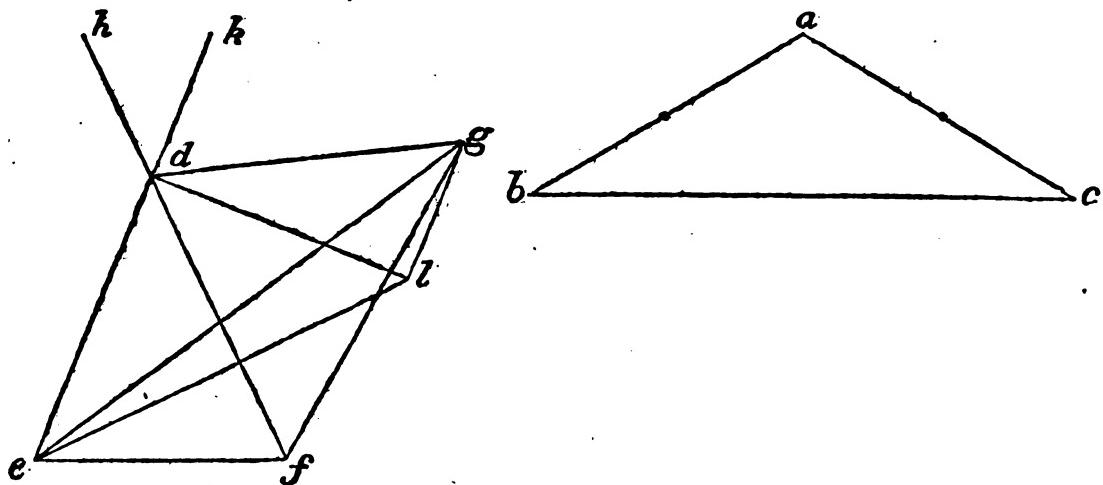
the triangle  $def$ , is not unequal to the triangle  $abc$ . And here you may observe, that we require three theorems belonging to the doctrine of parallels; one, indeed, affirming, that the external angle of every triangle is equal to the two internal and opposite angles: but the other, that if a right line falling upon two right lines, makes the alternate angles equal, the right lines are parallel; and the third, that triangles constituted upon the same base, and between the same parallels, are equal, which the institutor of the Elements also knowing, omits the comparison of triangles.

But let the angles  $bac, edf$ , be greater than two right, and let the same things be constructed. Because, therefore, the angles  $bac, edf$ , i. e. the angles  $edg, edf$ , are greater than two right; but the angles  $edg, gdk$ , are equal to two right, by taking away the common angle  $edg$ , the angle  $edf$ , is greater than the angle  $gdk$ . Hence, the angle  $gdh$ , is more than double of the angle  $gdk$ ; and so the angle  $gdk$ , is less than the angle at the point  $g$ . Let  $gdk$  be placed equal to  $dgl$ , and let  $el$ , and  $dl$ , be connected:



$gl$ , therefore, is parallel to  $de$ ; and hence, the triangles  $gde, lde$ , are equal. But the triangle  $lde$ , is less than the triangle  $fde$ . The triangle, therefore,  $gde$ , is less than the triangle  $fde$ . But the triangle  $gde$ , is equal to the triangle  $abc$ ; and hence, the triangle  $abc$ , is less than the triangle  $fde$ , viz. is less than the triangle which contains the greater angle.

In the third place, let the unequal angles be less than two right, and let the same things be constructed. Because, therefore, the angles  $edg, gdk$ , are equal to two right, by taking away the common angle  $edg$ , the whole  $gdh$ , is less than double of  $gdk$ . But it is double also of the angle at the point  $g$ . Hence, the angle  $gdk$ , is greater than the angle at the point  $g$ . Let the angle  $dgl$ , be placed equal to the angle  $gdk$ , and let  $gl$  coincide with  $el$ , in the point  $l$ , and connect  $dl$ . Hence,  $gl$  is parallel to  $de$ ; and consequently the triangles  $gde, lde$ , are equal to each other. But the triangle  $ldc$ , is greater than the triangle  $fde$ ; and the triangle  $gdc$ , is equal to the triangle  $abc$ . Hence, the triangle  $abc$ , is greater than the triangle  $dfe$ . It is shewn, therefore, that the triangle  $abc$ , is both equal to,



and is also greater and less than the triangle  $dfe$ , the angles at the points  $a$  and  $d$ , being either equal to, or greater or less than two right. And thus, all the hypotheses may be accomplished. For what if the angle at the point  $a$ , should be one right, and the half of a right angle, but the angle at the point  $b$ , the half of one right, would not those two angles be equal to two right? But what if the angle at the point  $a$ , should be one right, and the half of a right, but the angle at the point  $b$ , two thirds of one right, would they not be greater than two right angles? And lastly, if the angle at the point  $a$ , should be one right, and the half of a right angle, but the angle at the point  $b$ ,

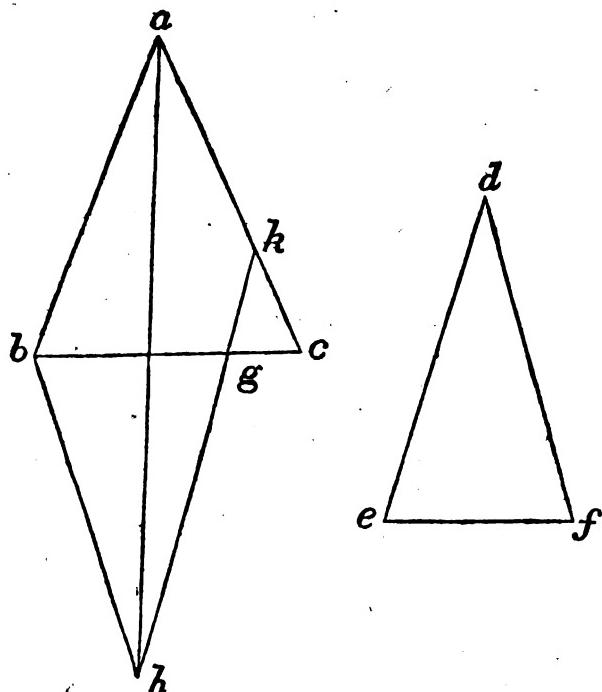
a third.

a third part of a right angle, would they not be less than two right, and the angle  $\alpha$  be greater than the angle  $\delta$ ? All these comparisons, therefore, are produced by the assistance of parallels; and hence, they are necessarily not found in the present elementary institution.

### PROPOSITION XXV. THEOREM XVI.

If two triangles have two sides equal to two, each to each, but have the base of the one greater than the base of the other; they shall likewise have the angle contained by the equal sides in the one, greater than the angle contained by the equal sides in the other.

The present theorem is the opposite to the eighth, but the converse of the preceding. For the institutor of the Elements produces theorems concerning the equality and inequality of angles and bases, according to *conjunction*; in each of the *conjunctions*, receiving some as precedents, but others as converse. And in such as are precedent indeed, he employs direct ostensions: but in such as are converse, he uses deductions to an impossibility. After this manner he proceeds in some particular triangle, sometimes from the equality of the sides which it contains, shewing the consequent equality of the angles which they subtend: but sometimes from their inequality evincing inequality. And again, on the contrary, affirming that equality of sides is consequent to equality of angles, but inequality to inequality. However, that we may proceed to the thing proposed, we refer those who are desirous of learning how the geometrician shews when this is manifest, to his books on this subject. But we shall briefly relate the demonstrations which others produce of this proposition; and in the first place, that which Menelaus Alexandrinus invented and delivered. Let there be two triangles  $\alpha b c$ ,  $d e f$ , having the two sides  $\alpha b$ ,  $\alpha c$ , equal



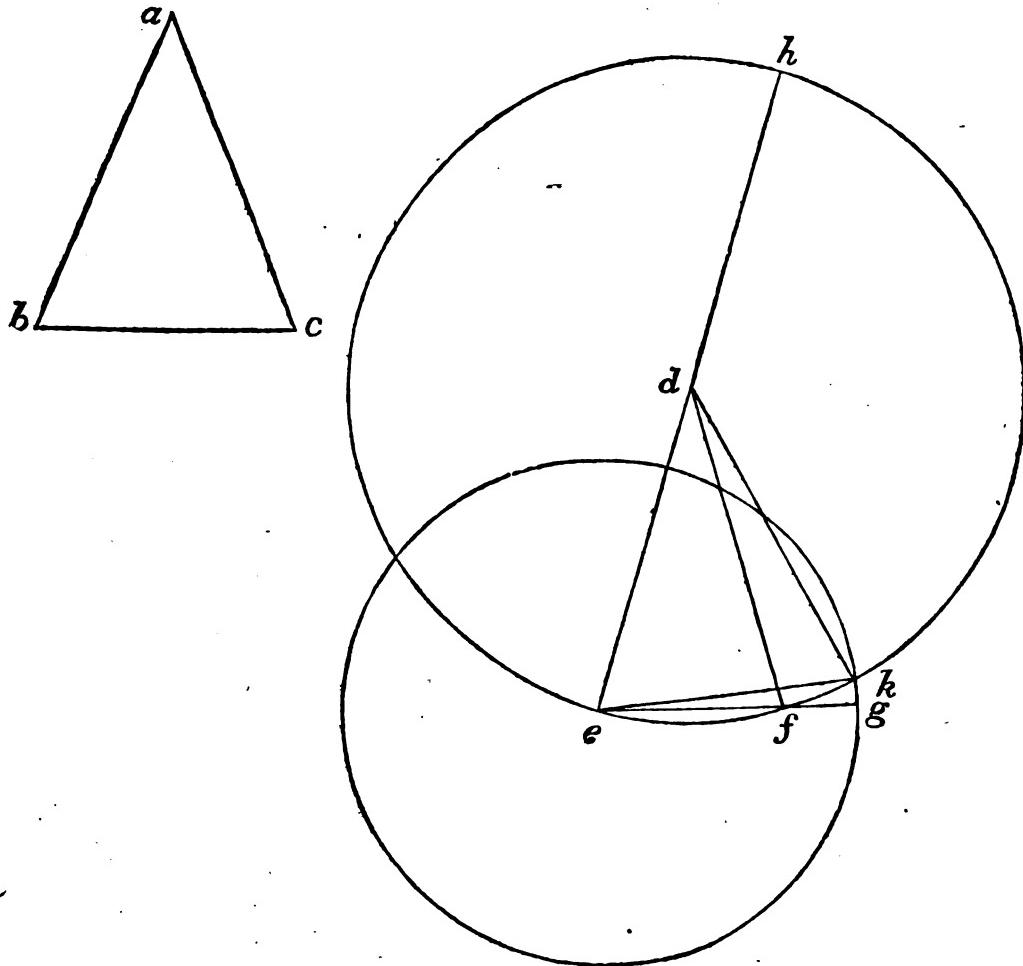
equal to the two  $d e$ ,  $d f$ , each to each, and the base  $b c$ , greater than the base  $e f$ , I say that the angle at the point  $a$ , is greater than the angle at the point  $d$ . For let there be cut from the base  $b c$ , a line  $b g$ , equal to the base  $ef$ , and construct at the point  $b$ , an angle  $g b b$ , equal to the angle  $d e f$ , and place  $b b$  equal to  $d e$ . Lastly, connect  $b g$ , and produce it to the point  $k$ , and connect  $a b$ . Because, therefore,  $b g$  is equal to  $ef$ , but  $b b$  to  $d e$ , the two are equal to the two, and they contain equal angles. Hence,  $g b$  is equal to  $d f$ , and the angle  $b b g$ , is not unequal to the angle  $e d f$ . And because  $g b$  is equal to  $d f$ , but  $d f$  to  $a c$ ,  $g b$ , also, is equal to  $a c$ . Hence  $b k$  is greater than  $a c$ , and consequently is much greater than  $a k$ . The angle, therefore,  $k a b$ , is greater than the angle  $k b a$ . Again, because  $b b$ , is equal to  $a b$ , for it is equal to  $d e$ , the angle  $b b a$ , is equal to the angle  $b a b$ . Hence, the whole angle  $b b k$ , is less than the whole,  $b a c$ , but is shewn to be equal to the angle at the point  $d$ . The angle, therefore,  $b a c$ , is greater than the angle at the point  $d$ . And such is the demonstration of Menelaus.

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But Heron, the mechanist, shews the same thing, in the following manner, without leading to an impossibility, as is the case with the demonstration of Euclid. Let there be two triangles  $a b c$ ,  $d e f$ , with the same hypotheses as above. And because  $b c$  is greater than  $e f$ ,



let  $e f$  be produced, and place  $e g$  equal to  $b c$ ; and in like manner extend  $d e$ , and place  $d b$  equal to  $d f$ . The circle, therefore, which is described with the centre  $d$ , and interval  $d f$ , will pass also through the point  $b$ . Let it be described as  $f k b$ . And because  $a c$ ,  $a b$ , are together greater than  $b c$ , but these are equal to  $e b$ , and  $b c$  is equal to  $g e$ , hence the circle which is described with the centre  $e$ , but interval

terval  $eg$ , will cut  $eb$ . Let it cut  $eb$ , as the circle  $gk$ , and connect from the common section of the circles to the centres, the right lines  $kd$ ,  $ke$ . Because, therefore, the point  $d$ , is the centre of the circle  $bkf$ ,  $dk$  is equal to  $db$ , i. e. to  $ac$ . Again, because the point  $e$ , is the centre of the circle  $gk$ , the line  $ek$ , is equal to  $eg$ , i. e. to  $bc$ . Hence, since the two  $ab$ ,  $ac$ , are equal to the two  $de$ ,  $dk$ , and the base  $bc$ , is equal to the base  $ek$ , the angle, also,  $bac$ , is equal to the angle  $edk$ . And thus the angle  $bac$ , is greater than the angle  $fde$ .

### PROPOSITION XV. THEOREM XVII.

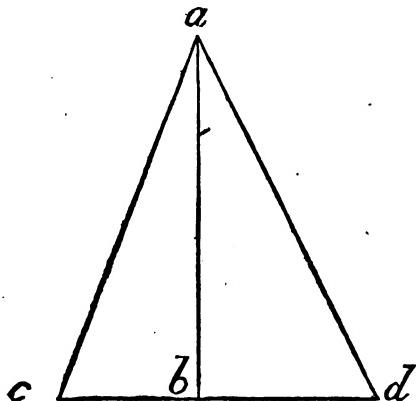
If two triangles have two angles equal to two, each to each, and one side equal to one side, either that which is adjacent to the equal angles, or that which subtends one of the equal angles: then they shall have the remaining sides equal to the remaining sides, each to each, and the remaining angle equal to the remaining angle.

It is necessary, that he who wishes to compare triangles with each other, according to sides, angles, and areas, should either, by receiving the sides alone equal, enquire after the equality of angles; or by assuming the angles alone equal, investigate the equality of the sides; or by mingling the angles and sides, scrutinize the equality of angles and sides. Since, therefore, Euclid alone receives the angles equal, he could not likewise shew that the sides of the triangles are equal. For the least triangles are equiangular with the greatest, though at the same time they are excelled by them, both according to sides and comprehended space: but the angles of the former are separately equal to the angles of the latter. However, as he supposes the sides alone to be equal, he demonstrates that all are equal, by the eighth theorem, in which there are two triangles having two sides

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equal

equal to two, each to each, and the base to the base, and these are shewn to be equiangular, and to possess a power of comprehending equal spaces. And the institutor of the Elements omits this addition, as necessarily following from the fourth; and requiring no demonstration. But when receiving sides and angles, he ought to receive either one side equal to one, and one angle equal to one; or one side, and two angles of the triangles, equal to two; or on the contrary, one angle and two sides; or one angle and three sides; or one side and three angles; or more than one side, and more than one angle. But when he had received one angle, and one side, he could by no means shew the thing proposed. I mean, the equality of the rest. For it is possible that two triangles which are equal, according to one side only, and one angle, may be entirely unequal as to the rest. Thus let there be a right line  $a b$ , perpendicularly erected upon the right line  $c d$ , but let  $b d$  be greater than  $b c$ , and connect  $a c$ ,  $a d$ . In these tri-

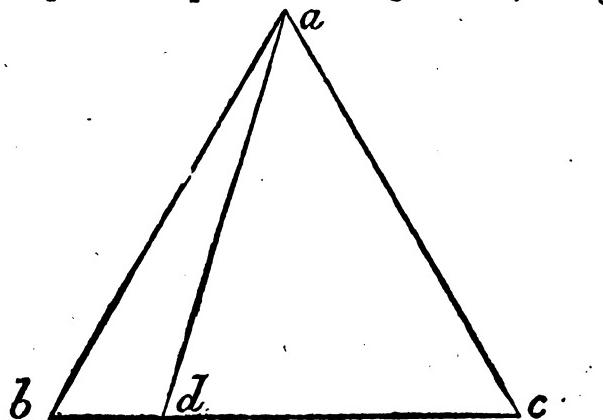


angles, therefore, there is one common side, and one angle equal to one, but all the rest are unequal. But it is lawful to receive one side, and two angles, and to prove the rest equal, and this he performs by the present theorem: though again, to suppose one side, and three equal angles, is superfluous; since from the equality of two alone, the equality of the rest is exhibited. Again, receiving one angle, and two equal sides, he demonstrates that the rest are equal in the fourth theorem. But it is superfluous to receive one angle, and three equal sides: for two equals being alone assumed, conclude the equality

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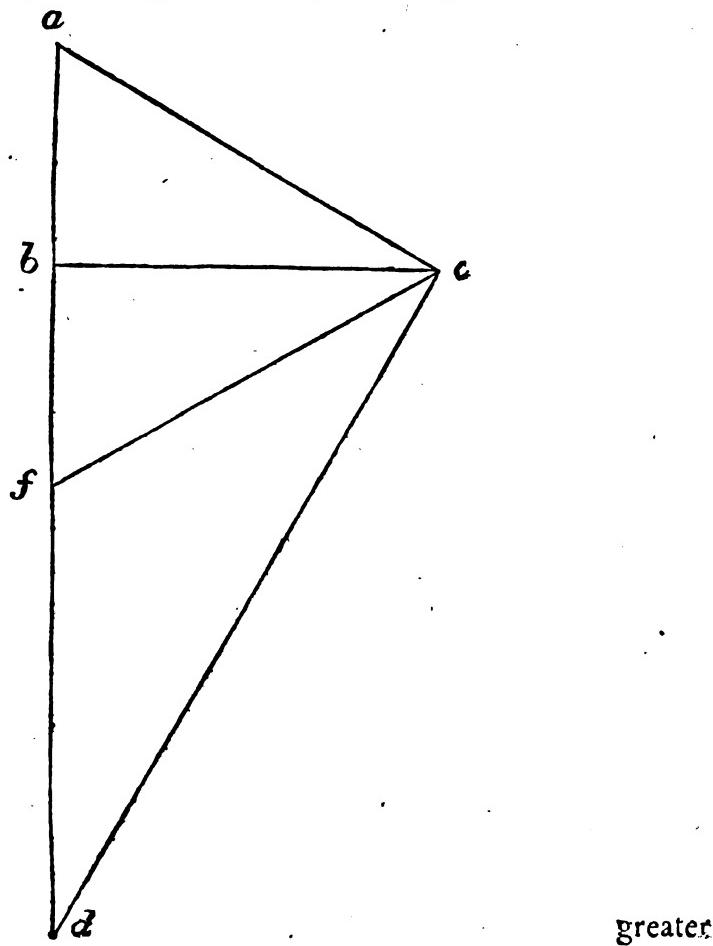
lity of the rest. Besides, it is superfluous to assume two sides, and two equal angles ; or two sides, and three equal angles ; or two angles and three sides ; or three angles and three sides. For the consequents to fewer hypotheses attend likewise a greater multitude, while the hypotheses are received with proper conditions. Hence, three hypotheses requiring demonstration, present themselves to our view, one, which alone receives three sides ; and another which assumes one side, and two angles, which the geometrician now proposes ; and a third, the opposite to this. On this account, we have only these three theorems, concerning the equality of triangles, which are conversant in sides and angles ; since all the other hypotheses are either invalid for the purpose of shewing the object of enquiry ; or they are valid indeed, but superfluous, because the same things may be readily procured by fewer hypotheses. As, therefore, when he assumed two sides equal to two, and one angle equal to one, he did not, indeed, assume every angle, but (as it was proposed by him) that contained by equal right lines, in the same manner when he assumes two angles equal to two, and one side to one, he does not assume any side, but either that which is adjacent to the equal ang'es, or that which subtends one of the equal angles. For neither is it possible in the fourth theorem, by assuming any equal angle, nor in the present by assuming any side, to shew the equality of the rest.

Thus for example, an equilateral triangle  $a b c$ , being given, let



the

the side  $b c$  be divided into unequal parts, by the line  $a d$ . Hence, there will be formed two triangles, having two sides  $a b$ ,  $a d$ , equal to the two  $a c$ ,  $a d$ , and one angle at the point  $b$ , equal to one angle at the point  $c$ , but the remaining sides will not also be equal, as for instance, the side  $b d$ , to the side  $d c$ : for they are unequal. But neither are the remaining angles equal: the reason of which is, because we receive an angle equal to an angle, but not the angle which is contained by equal sides. After the same manner, indeed, the present theorem also will appear dubious, unless we assume, according to the aforesaid condition, an equal side subtending one of the equal angles, or adjacent to the equal angles. For let there be a right angled triangle  $a b c$ , having the angle at the point  $b$  right, and the side  $b c$ ,



greater than the side  $b a$ , and let there be constructed on the right line  $b c$ , and at a point in it  $c$ , an angle  $b c d$ , equal to the angle  $b a c$ , and let  $b d, c d$ , produced, coincide in the point  $d$ . There are two triangles, therefore  $a b c, b c d$ , having one side  $b c$  common, and two angles equal to two, viz.  $a b c$ , to  $c b d$  (for they are right), and  $b a c$  to  $b c d$ , according to construction. Hence, as it appears the triangles are equal, and yet it may be shewn that the triangle  $b d c$ , is greater than the triangle  $a b c$ . But the reason of this is, because in the triangle  $a b c$ , we assume the common side  $b c$ , subtending one of the equal angles, viz. the angle at the point  $a$ : but in the triangle  $b c d$ , we assume the equal side, adjacent to the equal angles. It was requisite, therefore, in each, either to subtend one of the equal angles, or to be adjacent to the equal angles. But not observing this, we affirmed that triangle to be equal, which is necessarily greater: for is not the triangle  $b c d$ , greater than the triangle  $a b c$ ? To be convinced of this, let there be constructed on the right line  $b c$ , and at a given point in it  $c$ , an angle  $f c b$ , equal to the angle  $a c b$ : for the angle  $b c d$ , as well as the angle at the point  $a$ , is greater than the angle  $a c b$ . Because, therefore, there are two triangles,  $a b c, b c f$ ; having two angles  $a b c, b c a$ , equal to two  $c b f, b c f$ , each to each, and one side common, adjacent to the equal angles, viz.  $b c$ , the triangles are equal. But the triangle  $b c d$  is greater than the triangle  $b c f$ , and consequently it is also greater than the triangle  $a b c$ . But it was formerly shewn to be equal, on account of the assumption of any side: And thus much the diligence of Porphyry has supplied us on the present occasion. But Eudemus, in his Geometrical Narrations, refers the present theorem to Thales. For he says it is necessary to use this theorem in determining the distance of ships at sea, according to the method employed by Thales in this investigation. But from the preceding division we may briefly assume all the contemplation concerning the equality of triangles, and are enabled to relate the causes of things omitted, confuting those hypotheses, as either false, or superfluous. And thus far we determine the limits of the first section of the elementary institutor, because he forms the constructions

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constructions and comparisons of triangles, according to equal and unequal. And by construction, indeed, he delivers their essence : but by comparison, their identity and diversity. For there are three things which are conversant about being, *essence*, *same*, and *different* \*, as well in quantities, as in qualities, according to the propriety of subjects. From these, therefore, as images it may be shewn, that every thing is the *same* with itself, and *differs* from itself, on account of the multitude which it contains ; and that all things are the *same* with one another, and *different* from themselves. For both, in every triangle, and in more triangles than one, equality and inequality has been found to reside.

\* See more concerning these universal genera in the third section of the Dissertation, Vol I. of this work.

BOOK

## B O O K IV.

WHATEVER can be said in an elementary institution, concerning the origin and equality of triangles, we may learn from the preceding discourse. But after this, the narration of Euclid is concerning quadrilateral figures, and he particularly teaches us concerning parallelograms, together with the contemplation of these delivering the doctrine of trapeziums. For a quadrilateral figure, (as we have formerly observed in our discourse on hypotheses,) is divided into parallelogram and trapezium; and a parallelogram into other certain species, and in like manner a trapezium. But because a parallelogram, on account of its participation of equality, possesses disposition and order, but a trapezium has neither the same, nor a similar order; Euclid's principal discourse, is with propriety, concerning parallelograms, but he also contemplates together with these a trapezium. For from the section of parallelograms, the origin of trapeziums will appear, as will be manifest as we proceed. But because again, it is not possible that any thing can be said of the construction or equality of parallelograms, without the consideration of parallels, (for as it is manifest from the very name, that is, a parallelogram, which is circumscribed by parallel right lines in an opposite position,) hence, he necessarily assumes from parallels the beginning of his doctrine, but having advanced a little from these, he enters on the doctrine of parallelograms, employing one middle theorem, between the elementary institution of each, because he appears to contemplate a certain symptom inherent in parallels: but he delivers the first origin of a parallelogram. For such is the proposition, which says, that *right lines which join equal and parallel right lines towards the same parts, are themselves equal and parallel.* For in this theorem, indeed, a certain accident to equal and parallel right lines is considered: but from the connection a parallelogram appears, having its sides opposite and parallel. And from hence it is manifest that the discourse con-

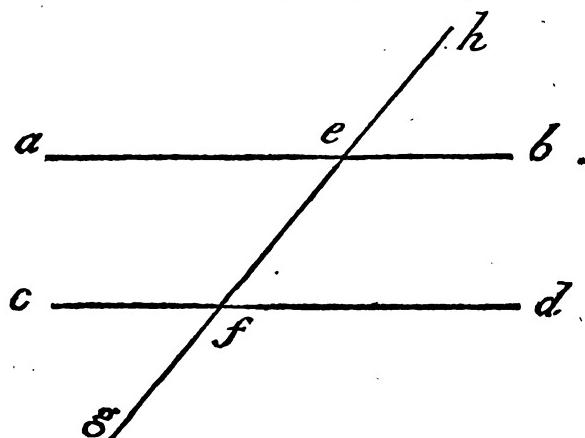
cerning parallels, was necessarily pre-assumed. But three things are to be assumed, essentially inherent in parallels, which they essentially express, and are converted with them; not only the three together, but every one separately assumed from the rest. Of these, one is, *that when a right line cuts parallel lines, the alternate angles are equal*; but the second, *that when a right line cuts parallel lines, the internal angles are equal to two right*; and the third; *that in consequence of a right line cutting parallel lines, the external is equal to the internal and opposite angle*. For when any one of these *symptoms* is demonstrated, we have sufficient authority to affirm that the right lines are parallel. But other mathematicians, also, have been accustomed to discourse after this manner concerning lines, delivering the symptoms of every species. For Apollonius, in each of the conical lines, shews what a *symptom* is, as also Nicomedes in his Treatise on Conchoids, and Hippas in his Quadratics, and Perseus in his Spirals. Since after their origin, that which is essentially inherent in these lines, and according to what it is inherent, being assumed, distinguishes a constructed form from all others. After the same manner, therefore, the institutor of the Elements, first of all investigates, the symptoms of parallels.

### PROPOSITION XXVII. THEOREM XVIII.

If a right line falling upon two right lines, makes the alternate angles equal to each other, those right lines shall be parallel to each other.

In the present theorem it was not pre-assumed as evident that the right lines are in one plane, but this ought rather to be previously admitted in all theorems which are considered in a plane. This, however, is added, because it does not universally follow, that when the alternate angles are equal, the right lines will be parallel, unless they are in the same plane. For nothing hinders, but that a right line falling on right lines disposed in the shape of the letter X, one of

which is situated in one plane, but the other in a different one, may make the alternate angles equal; and yet the right lines thus disposed will not be parallel. It was pre-assumed \*, therefore, that in a treatise on planes, we conceive every thing described in one and the same plane: and on this account, he does not require this addition in the present proposition. But it is requisite to know that the geometricalian considers the particle *alternate*, in a twofold respect, sometimes, indeed, according to a certain situation, but sometimes according to a certain consequence of proportions. And according to this last signification, the particle *alternate* is used in the fifth book, and in such as are arithmetical: but agreeable to the former, both in this, and in all the other books concerning parallel right lines, and that which falls upon these. For he calls the angles *alternate*, which are not formed at the same parts, and are not successive to each other, which are distinct; indeed, from the incident line, but both of them exist within parallels, and differ in this, that the one has an upward, but the other a downward position. I say, for example, that when a right line  $ef$ , falls on the right lines  $ab$ , and  $cd$ , he calls the angles  $aef$ ,  $dfe$ , and also the angles  $cfe$ ,  $bef$ , *alternate*, or *altern,*



because they have an alternate, or changed order, according to their position. But this too must be known, that from such a situation

\* In Book II. Comment 2. of this work.

of right lines, all the symptoms become by division, six ; three of which the geometrician alone receives ; and three he omits. For we either assume the angles at the same parts, or not at the same. And if at the same parts, either both within the right lines, which shews them to be parallels ; or both without, or one without, and the other within. And if not at the same parts, again, after the same manner, they are either both without the right lines, cutting the lines it is necessary to receive ; or within ; or one within, and the other without. But what we have said will become manifest by the same description as above. For let there be certain right lines  $a b$ ,  $c d$ , and let a right line  $e f$ , fall upon them, and let it be produced to the points  $b$  and  $g$ . If then you assume angles at the same parts, you will either place them both within, as  $b e f$ , and  $e f d$ , or as  $a e f$ , and  $e f c$ ; or both without, as  $b e b$ , and  $d f g$ , or as  $b e a$ , and  $c f g$ ; or one within, and the other without, as  $b e b$ , and  $e f d$ , or as  $g f d$ , and  $f e b$ , or as  $b e a$ , and  $e f c$ , or as  $g f c$ , and  $a e f$ : for these last are received in a quadruple respect. But if you assume the angles not at the same parts, you will either place both within, as  $a e f$ , and  $e f d$ , or as  $c f e$ , and  $f e b$ ; or both without, as  $a c b$ , and  $d f g$ , or as  $b e b$ , and  $c f g$ ; or one within, and the other without, and this again in a quadruple respect. For they will either be the angles  $a e b$ , and  $e f d$ ; or  $b e b$ , and  $e f c$ ; or  $g f c$  and  $f e b$ ; or  $g f d$ , and  $f e a$ . And besides these, there is no other assumption.

As, therefore, angles are assumed according to six modes, the geometrician combines three assumptions alone ; and these consequent symptoms, are naturally adapted to express parallels. But of these three assumptions, one belongs to those angles which are not at the same parts, viz. to those which are only assumed within ; and these he calls alternate, so that those, which are both external, and those, one of which is external, but the other internal, are omitted : but two of these assumptions belong to angles at the same parts, to those, indeed, which are both internal, which he says are equal to two right, and to those, one of which is internal, but the other external, which he says are equal, leaving indeed one assumption which supposes both the

angles to be external. We therefore affirm that the same things will be consequent to the three omitted hypotheses. Thus, in the preceding figure, let both the external angles  $b \epsilon b$ ,  $dfg$ , be at the same parts, I say that these are equal to two right angles. For the angle  $dfg$ , is equal to the angle  $b \epsilon b$ , and the angle  $b \epsilon f$ , to the angle  $dfg$ . But if the angles  $b \epsilon f$ ,  $efd$ , are equal to two right, the angles  $dfg$ ,  $b \epsilon b$ , are equal to two right. Let again the angles  $a \epsilon b$ ,  $efd$ , not be towards the same parts, of which the one is within, but the other external, I say that these also are equal to two right angles. For if the angle  $a \epsilon b$ , is equal to the angle  $b \epsilon f$ , but the angles  $b \epsilon f$ , and  $efd$ , are equal to two right, the angles, also,  $a \epsilon b$ , and  $efd$ , are equal to two right. Again, let them not be at the same parts, but both without the right lines as  $a \epsilon b$ ,  $dfg$ . I say that these are equal to one another. For if the angles  $a \epsilon b$ , and  $b \epsilon f$ , are equal to each other, but the angle  $dfg$ , is equal to the angle  $b \epsilon f$ , hence the angle  $a \epsilon b$ , is not unequal to the angle  $dfg$ . If, therefore, the things assumed by the geometrician, in three hypotheses are verified, all the same follow in the remaining three as indisputably true. Besides this too is to be observed, that in such as the geometrician receives these, according to two assumptions, the angles are supposed equal to each other, but when according to one assumption, equal to two right: but in these last on the contrary, according to two assumptions, they are supposed equal to two right angles, but according to one equal to each other. For since all the assumptions are six, it happens, indeed, from three, that the angles are equal to two right, but from the other three, that they are equal to each other. Hence, those which are omitted are not undeservedly contrary to the assumptions which are reckoned worthy of relation. But the geometrician appears to have chosen such hypotheses as either abound in affirmation, or are more simple, and on this account of those angles which are not at the same parts, he assumed alone the internal, which he calls alternate: but of those at the same parts, he assumes as well the internal, as well as one internal and the other external: but he avoids the rest, either because they are more declared by negation,

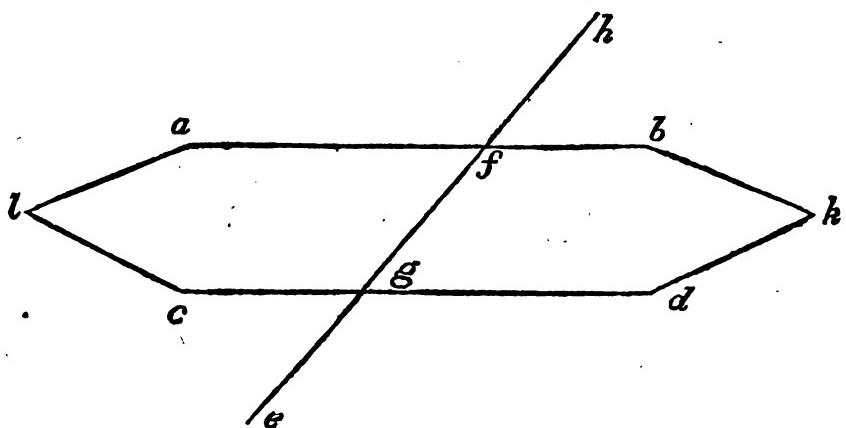
tion, or because they are more various. However, whether this or some other be the cause, the number of the consequents to those hypotheses is from hence sufficiently manifest.

### PROPOSITION XXVIII. THEOREM XIX.

If a right line falling upon two right lines, makes the external equal to the internal angle, placed opposite, and at the same parts, or makes the angles internally situated, and at the same parts equal to two right, those right lines shall be parallel to each other.

The preceding theorem receiving the angles, not at the same parts, but situated within right lines, shews that the right lines are parallel among themselves : but the present theorem proposes the two remaining hypotheses, of which one separates the angles according to the particles *without* and *within*, but the other supposes them both within, and exhibits the same conclusion. But it may seem, perhaps, that the institutor of the Elements has inconveniently distributed the theorems. For it was necessary either to receive three hypotheses in a divided manner, and to make three theorems ; or to collect all into one theorem, as Æneas Hierapolites does, who wrote a compendium of the Elements ; or willing to divide them into two, to make an orderly division, and to assume the hypotheses separately, which contain equal angles, and separately that in which the angles are equal to two right. But in the present propositions, in one theorem he supposes the alternate angles equal, but in the other, the external to the internal, and the internal angles situated at the same parts equal to two right. What then is the cause of this division ? Does he regard the equality of the angles to each other, or to two right, and on this account does not separate the proposed theorems from each other ; or does he respect the angles being received at the same, or not at the same parts ? For the preceding theorem does not respect angles at the same parts, since such as these are alternate : but the

the present regards such as are situated at the same parts, as is perspicuous from the proposition. But how the institutor of the Elements shews, that from the internal angles being equal to two right, the right lines are parallel, appears from his writings on this subject. Ptolemy, however, in the theorems in which he proposes to demonstrate that right lines produced from angles less than two right, coincide at the same parts, in which the angles less than two right are situated, shewing before all his theorems, that from the internal angles being equal to two right, the right lines are parallel, proves it in the following manner. Let there be two right lines  $a b, c d$ ,



and let a certain right line  $egfb$ , so cut them, that it may make the angles  $b f g$ , and  $f g d$ , equal to two right, I say that those right lines are parallel, that is, will never coincide. For if it be possible, let them coincide while the right lines  $b f, g d$ , are produced in the point  $k$ . Because, therefore, the right line  $ef$ , stands upon the right line  $a b$ , it makes the angles  $a f e, b f e$ , equal to two right. In like manner because  $fg$  stands upon  $c d$ , it makes the angles  $c g f, d g f$ , equal to two right. Hence, the four angles  $b f e, a f e, c g f, d g f$ , are equal to four right, two of which  $b f g, f g d$ , are supposed equal to two right. The remainders, therefore,  $a f g, c g f$ , are equal to two right. If then the right lines  $f b, g d$ , when produced, coincide, the internal angles being equal to two right,  $f a$ , and  $g c$ , also,

also, shall coincide when produced : for the angles  $a f g$ ,  $c g f$ , are also equal to two right. Either therefore the right lines shall coincide in both parts, or in neither, since these, as well as the former, are equal to two right. Let the right lines then  $f a$ ,  $g c$ , coincide in the point  $l$ . But if this be admitted two right lines  $l a f k$ ,  $l c g k$ , will comprehend space, which is impossible. It is not therefore possible, that the internal angles being equal to two right, the right lines can coincide. They are therefore parallel.

### PROPOSITION XXIX. THEOREM XX.

A right line falling upon parallel right lines, makes the alternate angles equal to each other; and the external equal to the internal angle, oppositely situated, and at the same parts; and the internal angles at the same parts equal to two right.

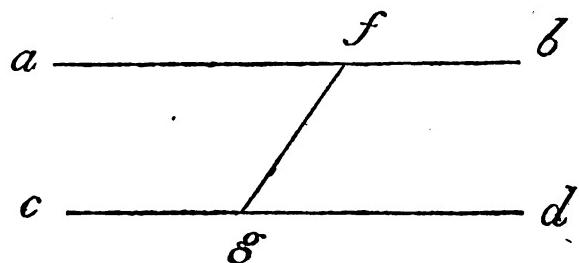
The present theorem is converted in both the preceding. For that which is the object of investigation, in each of them, forms the hypothesis : but what are data in the preceding, he proposes to shew in the present. And this difference of converse theorems is not to be passed over in silence, I mean that every thing which is converted, is either converted as one to one, as the sixth proposition to the fifth ; or as one to a many, as the present to the preceding ; or as many to one, as will shortly be manifest \*. But in the present theorem, the institutor of the Elements first employs the petition, which says : *If a right line falling upon two right lines, makes the angles situated internally, and at the same parts less than two right, those right lines whilst they are infinitely produced, will coincide at those parts in which the angles less than two right are situated.* But in our exposition of things prior to theorems †, we have asserted, that this petition is not allowed by all to be indemonstrably evident. For how can this be

\* In the Comment on the 32d proposition.

† In book III. chap. I. and in Com. 3.

the

be the case when its converse is delivered among the theorems as demonstrable? For the theorem which says that the two internal angles of every triangle are less than two right, is the converse of this petition. Besides, the perpetual inclination of right lines, more and more, while they are produced, is not a certain sign of coincidence, because other lines are found perpetually inclining, and never coinciding, as we have already observed. Formerly, therefore, some, when they had pre-ordained this as a theorem, considered that which is assumed by the institutor of the Elements as a petition, to be worthy of demonstration. But this seems to be shewn by Ptolemy himself, in a book entitled: *That right lines which are produced from less than two right angles, coincide.* And this he proves by pre-assuming many things, which as far as to the present theorem, are already demonstrated by the elementary institutor; and he supposes that all are true (lest we should also superadd another confusion) and that this, as a small assumption, may be exhibited from the preceding. But this also is one of the things previously exhibited, which says, *that the right lines produced from two angles equal to two right, will never coincid.* I say, therefore, that the converse also is true, which says, *that right lines being parallel, if they are cut by one right line, the angles situated internally, and at the same parts, shall be equal to two right angles.* For it is necessary that a line cutting parallels, should either make the angles internally situated, and towards the same parts, equal to two right, or less, or greater than two right. Let the lines then,  $a b$ ,  $c d$ , be parallel, and let the right line,  $g f$ ,



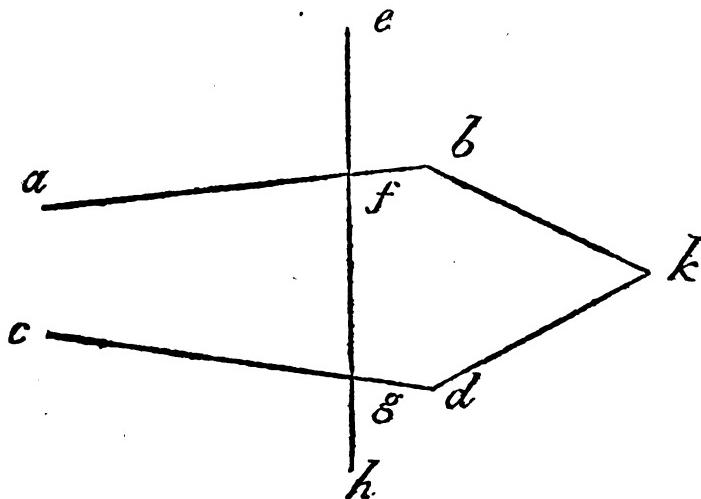
fall upon them, I say that it will not make the angles internal, and  
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at the same parts greater than two right. For if the angles  $a f g$ ,  $c g f$ , are greater than two right, the remainders  $b f g$ ,  $d g f$ , are less than two right. But the same are also greater than two right. For  $a f$ , and  $c g$ , are not more parallel than  $f b$ , and  $g d$ . Hence, if the line which falls upon  $a f$ ,  $c g$ , makes the internal angles greater than two right, that also which falls upon  $f b$ ,  $g d$ , will make the internal greater than two right. But they are also less than two right (since the four,  $a f g$ ,  $c g f$ ,  $b f g$ ,  $d g f$ , are equal to four right) which is impossible. In like manner we may plainly shew, that the right line which falls upon parallels, does not make the angles internal, and at the same parts, less than two right. But if it makes them neither greater nor less than two right, it remains that the incident line must make the angles internal, and at the same parts equal to two right. This then being previously shewn, the thing proposed, is doubtless demonstrated. For I say, that if a right line falling upon two right lines, makes the angles situated internally, and at the same parts, less than two right, if those right lines are produced they will coincide at those parts in which the angles less than two right are situated. For let them not coincide. But if they are non-coincident at those parts in which the angles less than two right are situated, much more will they be non-coincident at the other parts, in which the angles greater than two right are situated. Hence, the right lines will be non-coincident at both parts; and if this be true, they will be parallel. But it was shewn that the right line which falls on parallels, makes the angles internal and at the same parts equal to two right. The same, therefore, are both equal to, and less than two right, which is impossible.

Ptolemy having previously shewn this, and proceeding to the thing proposed, wishes to add something more accurate, and to shew that if a right line falling upon two right lines, makes the angles internal, and at the same parts, less than two right, the lines are not only coincident as has been shewn, but likewise that their coincidence takes place at those parts, in which the angles less than two right, and not at those in which the angles greater than two right are situ-

ated. For let there be two right lines  $a b$ ,  $c d$ , and let a right line

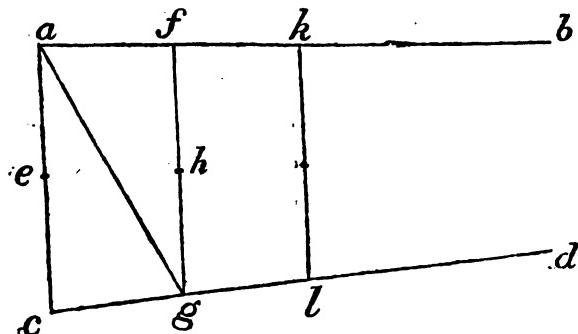


$\alpha$   $\beta$   $\gamma$   $\delta$ , falling upon them make the angles  $\alpha f g$ , and  $\gamma g f$ , less than two right. The remainders, therefore, are greater than two right; and thus it is shewn that the right lines coincide. But if they coincide, they will either coincide at the points  $a$  and  $c$ , or at the points  $b$  and  $d$ . Let them coincide at the points  $b$  and  $d$  in the point  $k$ . Because, therefore, the angles  $\alpha f g$ , and  $\gamma g f$ , are less than two right, but the angles  $\alpha f g$ ,  $\beta f g$ , are equal to two right, by taking away the common angle  $\alpha f g$ , the angle  $\gamma g f$ , will be less than the angle  $\beta f g$ . The external angle, therefore, of the triangle  $g f k$ , is less than the internal and opposite angle, which is impossible. Hence then, they do not coincide at these parts. But they do coincide; and consequently they will be coincident at the other parts, in which the angles less than two right are situated. And thus far Ptolemy.

But it is necessary to scrutinize this demonstration, lest perhaps there should be any perverse and captious reasoning in the assumed hypotheses, in those, I say, in which he affirms, that a right line cutting non-coincident right lines, by forming four internal angles, forms the angles at the same parts on each side, either equal to two right, or greater, or less than two right. For the division is not per-

fect; since nothing hinders our calling those lines non-coincident, which are produced from angles less than two right, denominating, indeed, the two angles at the same parts, greater than two right, but the two at the remaining parts less than two right, and not admitting in these, one and the same proportion. But the division being imperfect, the thing proposed is by no means demonstrated. Besides this, also, is not to be passed over in silence against his demonstration, that he does not essentially shew that which is impossible. For it is not because a certain right line cutting parallels, makes the angles at the same parts on each side, greater or less than two right, that an absurdity on this account follows these hypotheses. Nevertheless, because the four angles within the lines which are cut, are equal to four right, on this account each of these hypotheses is impossible; since, if parallel right lines are not assumed, yet, when the same hypotheses are assumed, the same consequences will be the result. And such are our animadversions against the demonstration of Ptolemy: for the imbecility of his demonstration appears from what has been said.

Let us now consider those, who say it is impossible that lines produced from angles less than two right, should coincide. For when they have assumed two right lines  $a b$ ,  $c d$ , and a right line  $a c$ , fall-



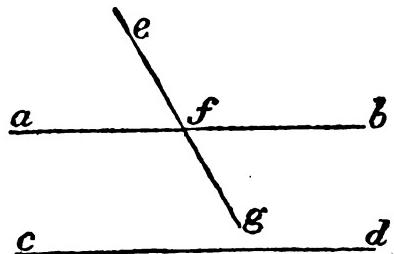
ing upon them, and making the two internal angles less than two right, they say it is possible that the right lines  $a b$ ,  $c d$ , may be shewn to be non-coincident. For let  $a c$  be bisected in  $e$ , and cut off from  $a b$ , a part  $a f$ , equal to  $a e$ : but from  $c d$ , a part  $c g$ , equal to

e.c.

e.c. It is manifest, therefore, that the right lines  $a f$ ,  $c g$ , will not coincide in the points  $f$  and  $g$ . For if they coincide, these two in the triangle will be equal to  $a c$ , which is impossible. Let again  $f g$  be connected, and bisected in  $b$ , and cut off equal parts. These, therefore, will not coincide on the same account, and this will be the case, in infinitum, by connecting the non coincident points, and bisecting the connecting line, and by cutting from the right lines, lines equal to the halves of the connecting lines; for by this means they say, that the right lines  $a b$ ,  $c d$ , will never coincide. To such as these we reply, that they indeed affirm that which is true, but not so much as they imagine. For it is not true that the point of coincidence is simply determined by this means, nor is it true that the lines by no means coincide. Thus, when the angles  $b a c$ , and  $d c a$ , are determined, the lines  $a b$ , and  $c d$ , will not coincide in the points  $f$  and  $g$ , yet nothing hinders their coinciding in the points  $k$  and  $l$ , though  $f k$  and  $g l$  should be equal to  $f b$ , and  $g d$ . For when  $a k$  and  $c l$  coincide, the angles  $k f b$ ,  $l g d$ , will not remain the same, and a certain part of the right line  $f g$ , will be left external to the right lines  $a k$  and  $c l$ ; and so again the two lines  $f k$ , and  $g l$ , are so much greater than the base, as the interior parts of the right line  $f g$ , which they intercept. Besides this also is to be said to such as affirm the non-coincidence of lines extended from angles less than two right, that they destroy what they are unwilling to destroy. For let the same description be given. Whether, therefore, is it possible, or impossible to connect a right line from the point  $a$ , to the point  $g$ ? For if it be impossible, besides destroying the fifth petition, they also destroy that which says, *that a right line may be drawn from every point to every point*: but if possible let it be connected. Because, therefore, the angles  $f a c$ ,  $g c a$ , are less than two right, it is manifest that the angles also,  $g a c$ ,  $g c a$ , are much less than two right. The right lines, therefore,  $a g$ ,  $c g$ , will coincide in the point  $g$ , being produced from angles less than two right. Hence, it is not possible to affirm indeterminately, that lines produced from angles less than two right, will not coincide. It is however manifest, that some right lines

lines produced from angles less than two right will coincide, though the present discourse seems to investigate this in all. For it may be said, that when the diminution of two right lines is indefinite, the lines will remain non-coincident according to such a diminution: but will coincide according to another less than this. But he who desires to behold a demonstration of this affair, must be informed that it is requisite for this purpose to pre-assume such an axiom as is employed by Aristotle\* in proving the world to be finite, viz. *If from one point two right lines forming an angle are produced in infinitum, the distance of the lines infinitely produced will exceed every finite magnitude.* For he shews that when infinite right lines are produced from the centre to the circumference, the interval also contained between them will be infinite: since, if it be only finite, it is possible that the distance may be increased; and on this account the right lines will not be infinite. Right lines, therefore, infinitely produced, are distant from each other by an interval greater than every finite magnitude.

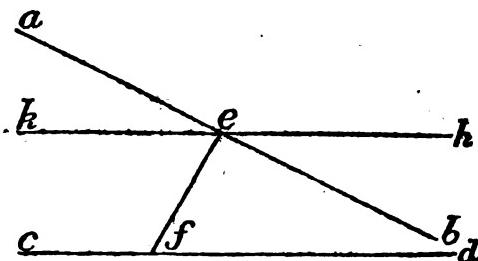
This being pre-supposed, I say that if any right line cuts the one of parallel right lines, it will also cut the other. For let  $a b$  and



$c d$  be parallels, and let the right line  $efg$  cut  $a b$ . I say that it will also cut  $c d$ . For since there are two right lines, which are produced infinitely from the point  $f$ , viz.  $b f$ , and  $f g$ , they shall have a distance greater than every magnitude. Hence, they shall exceed the quantity of the interval contained between the parallel lines. Since, therefore, their distance from each other is greater than that of the parallels,  $f g$  shall cut  $c d$ . But this being demonstrated, we can exhibit the thing proposed in a consequent order. For let there be

\* In lib. i. de Cælo, tex. 35.

two right lines  $a b$ ,  $c d$ , and let a right line  $e f$ , fall upon them, making the angles  $b e f$ ,  $d f e$ , less than two right. I say that the right



lines will coincide in those parts, in which the angles less than two right are situated. For since the angles  $b e f$ ,  $d f e$ , are less than two right, let the angle  $b e b$ , be equal to the excess of two right angles above these angles, and produce  $b e$ , to the point  $k$ . Because, therefore, a right line  $e f$ , falls upon the right lines  $b k$ ,  $c d$ , and makes the internal angles equal to two right, viz. the angles  $b e f$ ,  $d f e$ , the right lines  $b k$ ,  $c d$ , are parallel; and  $a b$  cuts  $k b$ . It will therefore also cut  $c d$ , by the assumption previously exhibited. Hence, the right lines  $a b$ ,  $c d$ , will coincide in those parts, in which the angles less than two right are situated. And on this account the thing proposed, is evinced \*.

### PROPOSITION XXX. THEOREM XXI.

Right lines parallel to the same right line, are parallel to each other.

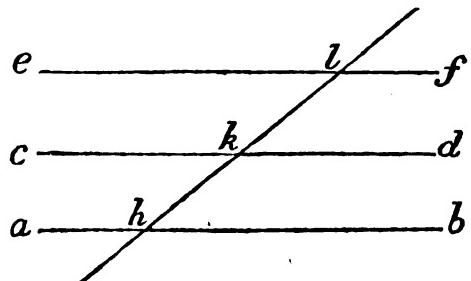
The geometer in these discourses which are conversant with relation, is accustomed to shew identity permeating through all quantities, having the same relation to the *same*. Thus among the axioms also he says, *things equal to the same, are equal to each other*: and in the following books he says, *things similar to the same, are similar to each other*, and *things having the same proportion to the same, have the*

\* Clavius and Simson have employed a multitude of propositions in the demonstration of this petition; but their demonstrations fall far short in my opinion of the elegance of the present.

*same*

*same proportion to each other.* After this manner, therefore, he now also demonstrates, that right lines parallel to the same, are parallel to each other. But it happens that this is not true in all respects. For quantities double of the same, are not also double of each other: nor are those which are sesquialter of the same, sesquialter likewise to each other, but it appears to take place in those alone, which are univocally converted in equality, similitude, identity, and parallel position. For that which is parallel to a parallel, is itself also parallel. As that which is equal to an equal, is itself equal; and that which is similar to a similar, is itself similar. For the relation of parallels to each other, is similitude of position. He affirms, therefore, and shews, in the present theorem, that lines parallel to the same, are entirely so related, that they are also parallel to each other. And he also exhibits the parallels with an external position, and likewise a medium, to which these have a similar relation, that what he asserts may become manifest from a common conception. For if they coincide with each other on either side, and coincide with that which is situated in the middle, they will no longer be parallel to it.

But it is possible that he who changes the position may shew the same thing, and by the same methods which the geometrician employs in exhibiting his proposition. For instance, he assumes both  $c d$ , and  $e f$ , parallel to  $a b$ , both of them situated above, and  $a b$



being beneath, and not in the middle. For a right line  $b k l$ , falling upon them, makes each of the angles  $b k d$ ,  $k l f$ , equal to  $a b k$ , because they are alternate; and on this account it makes the angles  $b k d$ ,  $k l f$ , equal to each other. The right lines, therefore,  $c d$ ,  $e f$ , are

are parallel. But if any one should say that  $a b$ ,  $b b$ , are parallel to  $c d$ , and are therefore parallel to each other, we reply that  $a b$ ,  $b b$ , are parts of one parallel, and are not two parallel lines. For parallels are conceived to be infinitely produced, but  $a b$ , when produced, falls upon  $b b$ . It is therefore the same with  $b b$ , and not a different line. Hence, all the parts of a parallel, are parallel both to the right line, to which the whole was parallel, and to its parts. As for example,  $a b$  is as well parallel to  $k d$ , as  $b b$ , to  $c k$ . For if they are infinitely produced, they will never coincide. And these remarks must be considered as not foreign from the purpose, both on account of sophistical importunities, and the juvenile habits of mathematical auditors. For the vulgar rejoice to find captious reasonings of this kind, and to procure vain molestation to the possessors of science. But it is not requisite to convert the present theorem, and to shew that lines parallel to each other, are also parallel to the same. For if we again suppose one line parallel to some other, the remainder also of these shall be parallel to it, and they will be parallel to the same, and we shall again return to the same proposition.

### PROPOSITION XXXI. PROBLEM X.

Through a given point to draw a right line parallel to a given right line.

It is requisite that we should not only learn the essential accidents of parallels, in the discourses of the elementary institutor, but also that we should relate their origin, and know how one right line becomes parallel to another : for origin every where renders the essence of subjects more known to us. And this the institutor of the Elements effects by the present problem. For having received a point and a right line, he draws through that point a line parallel to a right line. But we ought to pre-assume as necessary, that the point should entirely be placed external to the right line : for we must not place it in the right line, because it is said, *through a given point* ; since no

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other,

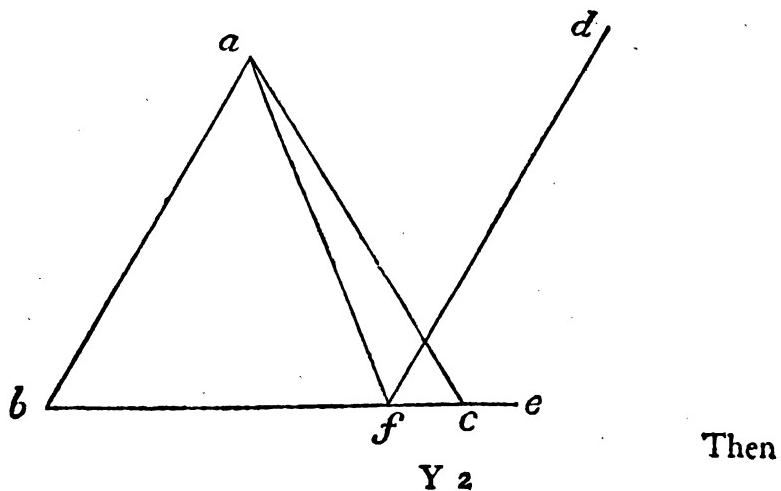
other, besides the given line, can be that which is drawn parallel through the point. Since, therefore, the point and the right line is divided, it indicates that the point is to be received external to the right line, which he manifests in a perpendicular by addition, commanding, *upon a given infinite right line, and from a given point which is not in it, to let fall a perpendicular.* One thing, therefore, which is common to both these problems, is, the external position of the point : but the other, that from the same point two perpendiculars cannot be let fall to the same right line, and that through the same point, two lines cannot be drawn parallel to the same right line. Hence, the institutor of the Elements commands in the singular number *to draw a right line*, in the former problem, *a perpendicular*, but in the present *a parallel*. And, *that* indeed, has been shewn, but *this* is manifest, from what is previously demonstrated. For if through the same point two parallels are drawn to the same right line, they would be parallel to each other, and coincide in the given point, which is impossible. But it is requisite to observe the differences of these two propositions, *from a given point*, and *through a given point*. For sometimes the point is the beginning of the right line which is drawn, and on this account the deduction is made from it : but sometimes the point is in the drawn right line, and on this account the drawing is made through the point. For the particle *through*, was not asserted, because the right line cuts a given point, but because it coincides with it, and terminates its own interval, in respect of that right line, by the distance of the point and the right line. Since as much as the given point is distant from the given right line, so much also is the interval of the parallel between itself and the right line.

### PROPOSITION XXXII. THEOREM XXII.

One side of every triangle being produced, the external angle of the triangle is equal to the two internal and opposite angles ; and the three internal angles of a triangle are equal to two right angles.

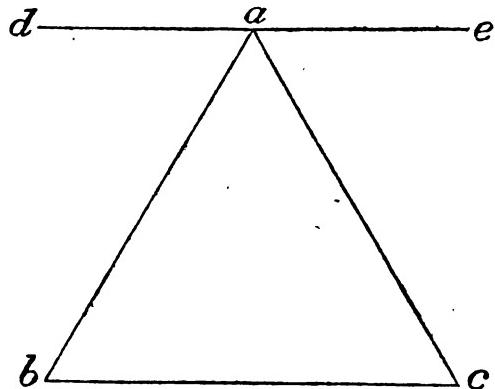
As

As much as was deficient in the sixteenth and seventeenth theorem, so much Euclid adds in the present. For we not only learn by this theorem that the external angle of a triangle is greater than either of the internal and opposite angles, but likewise how much it is greater; since as it is equal to both, it is greater than either of the remaining angles. Nor do we alone know from this theorem, that any two angles of a triangle, are less than two right; but by how much they are less: for they are deficient by the remaining third. The former, therefore, were more indefinite theorems: but this brings with it, on both sides, a boundary to science. We must not, however, call them on this account superfluous: for they are of the greatest utility in many demonstrations; and the present is proved by their assistance. And besides this, it is necessary that our knowledge, proceeding from the imperfect to the perfect, should pass from indeterminate apprehensions, to determinate and certain propositions. But the institutor of the Elements, by drawing a parallel externally, exhibits each of the objects of investigation. It is, however, possible that the same thing may be shewn without drawing the parallel externally; and this, by only changing the order of the things exhibited. For Euclid first shews, that the external angle is equal to the internal and opposite, and from this he proves the remainder. But we shall demonstrate this by a contrary mode of proceeding. Let there be then a triangle  $a b c$ , and let the side  $b c$  be produced to the point  $e$ .



Then take a point  $f$  in  $b c$ , and connect  $a f$ , and through the point  $f$ , let  $f d$  be drawn parallel to  $a b$ . Because, therefore,  $f d$  is parallel to  $a b$ , and a right line  $a f$ , falls upon these parallels, as also a right line  $b c$ , hence, the alternate angles are equal, and the external is equal to the internal angle. The whole, therefore,  $a f c$ , is equal to  $f a b$ , added to  $a b f$ . In like manner we may shew by drawing a parallel, that the angle  $a f b$ , is equal to the angles  $f a c$ ,  $a c f$ . The two, therefore,  $a f b$ ,  $a f c$ , are equal to the three angles of the triangle : and hence, the three angles of a triangle are equal to two right, viz. to  $a f b$ , added to  $a f c$ . But  $a c f$ ,  $a c e$ , are also equal to two right ang'es. Let, therefore, the common angle  $a c f$ , be taken away ; and then the remaining external angle will be equal to the internal and opposite angles. And after this manner may the present theorem be exhibited.

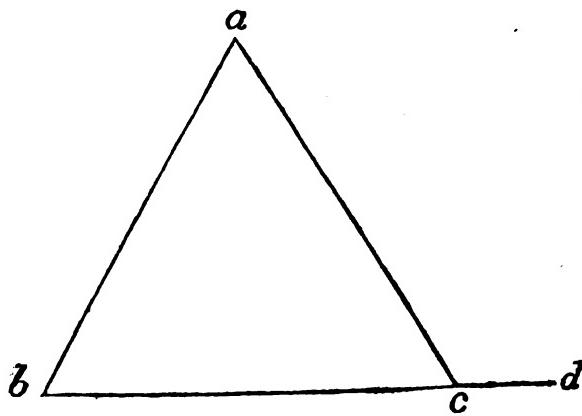
But Eudemus, the Peripatetic, ascribes the invention of this theorem to the Pythagoreans, I mean that every triangle has its internal angles equal to two right, and says that they demonstrate it in the following manner. Let there be a triangle  $a b c$ , and let there be



drawn through the point  $a$ , a line  $d e$ , parallel to  $b c$ . Because, therefore, the right lines  $d e$ ,  $b c$ , are parallel, the alternate angles are equal. Hence, the angle  $d a b$ , is equal to the angle  $a b c$ ; and the angle  $e a c$ , to the angle  $a c b$ . Let the common angle  $b a c$ , be added. The angles, therefore,  $d a b$ ,  $b a c$ ,  $c a e$ , that is, the angles  $d a b$ ,  $b a e$ , and that is two right, are equal to the three angles of the triangle. And such is the demonstration of the Pythagoreans.

But

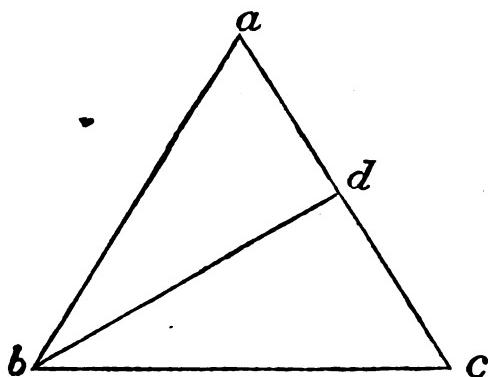
But it is here requisite to deliver such theorems as are converse to the present theorem of the elementary institutor. For two are converted to one, since this is a composite, both, according to *the object of enquiry*, and the *datum*: for the *datum* is two-fold, viz. the triangle, and one of its sides produced; and in like manner *the object of enquiry*. For one part says, that the external angle is equal to the internal and opposite angles: but the other, that the three internal angles are equal to two right. If therefore, we suppose that the external is equal to the internal and opposite angles, we may shew that one side is produced, and that the right line externally situated, is in a direct position with one of the sides of the triangle: but if the three internal angles are equal to two right, we may shew that the given figure is a triangle. And so *the whole object of enquiry*, is converse to *the whole datum*. Let there be then a triangle  $a b c$ , and let the external angle  $a c d$ , be equal to the internal and opposite angles, I say that the side  $b c$ , is produced to the point  $d$ , and that  $b c d$ , is one



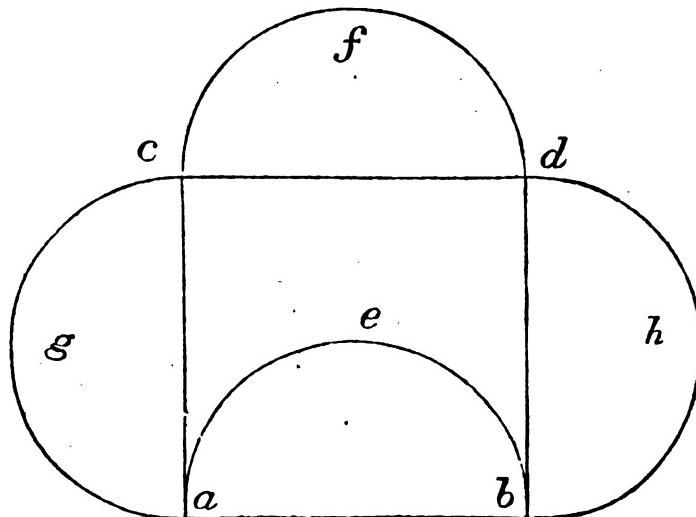
right line. For since the angle  $a c d$ , is equal to the internal and opposite angles, let the common angle  $a c b$  be added. The angles, therefore,  $a c d$ ,  $a c b$ , are equal to the three angles of the triangle  $a b c$ . But the three angles of the triangle  $a b c$ , are equal to two right.

right. And hence, the angles  $a c d$ ,  $a c b$ , are equal to two right. But if two right lines being consequently placed, and not at the same parts to any right line, and at a point in it, make the successive angles equal to two right, those right lines shall be in a direct position to each other. The right line, therefore,  $b c$ , is in a direct position to  $c d$ .

Let there be again, a certain right lined figure  $a b c$ , having three angles alone equal to two right, viz.  $a$ ,  $b$ , and  $c$ , I say that the figure



is a triangle, and that  $a c$ , is one right line. For let the right line  $b d$  be connected. Because, therefore, the three angles of each of the triangles  $a b d$ ,  $b d c$ , are equal to two right, of which the angles of the figure  $a b c$ , are equal to two right, the remainders  $a d b$ ,  $c d b$ , are equal to two right, and they are placed about a right line  $b d$ . Hence  $d c$ , is in the same direction with  $d a$ ; and so the side  $a c$ , is one right line. In like manner we may shew that the side  $a b$ , and the side  $b c$ , are each of them one right line. And consequently the figure  $a b c$ , is a triangle. If then a figure having internal angles equal to two right, is right-lined, it is perfectly a triangle: but it does not follow that a figure is a triangle merely because it has internal angles equal to two right. For you will find a figure constructed from circumferences, having its internal angles equal to two right. For let there be a quadrangle  $a b c d$ , and upon the side  $a b$ , let a semicircle  $a e b$ , be internally described: but upon the other sides,



sides, let the semicircles be externally described, as  $f, g, b$ . The figure, therefore, which is comprehended by the semicircles, has two angles  $gae, ebb$ , equal to two right, viz. to  $cab, dba$ . For this was shewn in the petitions \*, and these angles alone are in this figure. There is, therefore, a certain figure not a triangle, which has its internal angles equal to two right. And thus much may suffice concerning converse theorems.

But as we have discovered that the three angles of every triangle are equal to two right, we ought to determine a certain method, by which we may find how many angles, of all other multangles, are equal to so many right angles ; as for instance, of a quadrangle, quinquare, and of all consequent multilateral figures. In the first place, therefore, it must be known, that every right-lined figure may be resolved into triangles, since a triangle is the principle of the constitution of all things, which Plato also asserts in the *Timæus*, when he teaches us that the rectitude of a plane basis is composed from triangles. But every figure is resolved into triangles less in number, by the binary, than its proper sides. If a quadrilateral figure, into two triangles : if a figure of five sides, into three : if of six sides, into

\* In Lib. III. Com. 2:

four :

four. For two triangles composed together, immediately form a quadrilateral figure. But the number of composite triangles by which the first constituted figure differs from its sides, is the measure of difference to the rest. Hence, every multilateral figure possesses more sides, by the binary, than the triangles into which it may be dissolved. But every triangle has been shewn to contain angles equal to two right.. And hence, if the number of the angles be made double to that of the composite triangles, it will afford a multitude of right angles, to which the angles of every multangle will be equal. On this account every quadrilateral figure has angles equal to four right, since it is composed from two triangles : but every figure of five sides has angles equal to six right ; and after the same manner of the rest in a consequent order. This one thing, therefore, is to be assumed from the present theorem, concerning all multangular and, right-lined figures.

But there is another consequent to this, which is summarily as follows. In every right-lined figure, each of its sides being at the same time produced, the angles externally constituted are equal to four right. For it is requisite, indeed, that the successive right angles should be double of the multitude of the sides; because, in each they are constituted equal to two right. But the right angles equal to the internal angles being taken away, the remaining external angles are equal to four right. As for example, if the figure is a triangle, while every one of its sides is produced, at the same time internal and external angles are constructed equal to six right angles, of which the internal angles are equal to two right, but the remaining external angles to four right. But if the figure be quadrilateral, the angles are in all eight, since they are double of the sides, of which the internal are equal to four right, and the external to the four remaining angles, and the consequences will be similar in infinitum. But after these observations we may also collect, that by this theorem every angle of an equilateral triangle is two thirds of a right angle: but that an isosceles triangle, when the vertical angle is right, has each of its remaining angles the half of one right, as a semiquadrangle;

drangle; and that a scalene triangle, when it is the half of an equilateral triangle, formed by a perpendicular drawn from any angle to its opposite side, has one angle right, but the other (which likewise belonged to the equilateral triangle) two thirds of a right angle, and the remainder by a necessary consequence, a third part of a right angle. For it is requisite that the three should be equal to two right. But I do not conceive that these remarks are foreign from our purpose, since they prepare us for the doctrine of Timæus. This also must be observed, that the possession of internal angles equal to two right, is inherent essentially, and answering to the predication *according to what*, in a triangle. And on this account, Aristotle in his 'Treatise on Demonstration\*', employs this as an example, considering it *according to what*. As therefore *to be terminated*, is essentially and primarily inherent in every figure, so likewise the possession of internal angles equal to two right, is essentially and primarily inherent in a triangle, though not in every figure. And the truth of this theorem seems to present itself to us according to common conceptions. For if we conceive a right line, and two right lines standing on its extremities, and inclining to each other, so as to form a triangle, we shall find that in proportion to their inclination they diminish the right angles, which they form with the right line. Hence, obtaining as much angular quantity, by their inclination at the vertex, as they take away from the base, they necessarily form three angles equal to two right.

### P R O P O S I T I O N XXXIII. THEOREM XXIII.

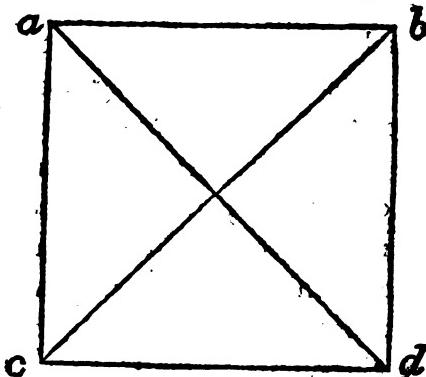
The right lines which join equal and parallel right lines at the same parts, are themselves also equal and parallel.

The present theorem is, as it were, the confine of the consideration of parallels and parallelograms: for it seems to declare a certain symptom of parallel right lines, and delivers the latent origin of parallelo-

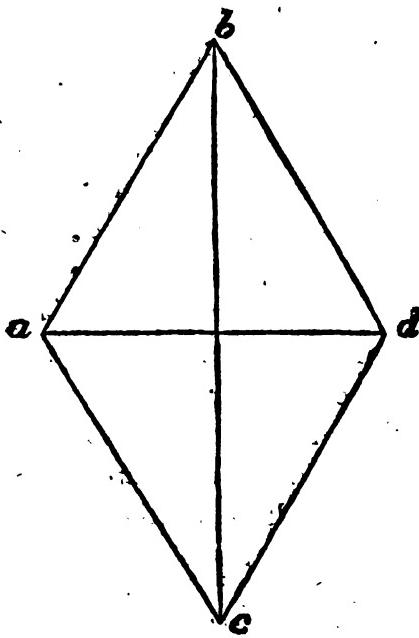
\* i. e. In his last analytics. See the second section of the Dissertation, Vol. I. of this work.

grams. For a parallelogram is formed, as well from those equal and parallel right lines, which are drawn in the beginning, as from those which conjoin them, and which are in like manner shewn to be equal and parallel. Hence, the proposition which immediately follows the present, contemplates the properties essentially inherent in these spaces, in a parallelogram as it were already constructed. And these things are indeed manifest. But it is requisite to consider the diligence which this proposition contains. In the first place, indeed, that it is not sufficient, that the lines which are conjoined should be equal: for the lines which connect equals, are not entirely equal, unless they are also parallel. For a triangle being isosceles, and a point being assumed in one of the equal sides, and through this a line being drawn parallel to the basis, equal lines shall indeed conjoin parallels to the basis, and the basis itself, yet these parallels shall not also be equal; and the sides will not be parallel, because they coincide at the vertex of the triangle.

In the second place, he considers that the subject right lines being parallel, is not sufficient to constitute the equality of the lines which conjoin them. For this is evident from the preceding construction of the isosceles triangle; since the drawn right line, and the basis, are parallel, and yet the lines which connect them are not parallel, because they are parts of the sides of the isosceles triangle. The parallel position, therefore, of the lines which are conjoined, is requisite to the equality of the connecting lines: but the equality of the latter is necessary to the parallel position of the former. On this account the institutor of the Elements assumes each, in those which are conjoined, for the purpose of exhibiting, that the connecting lines are as well equal, as parallel to one another. But in the third place, he intimates, that right lines being supposed both equal and parallel, their connecting lines will not be universally equal and parallel. For unless we make the conjunctions at the same parts as in this case, the connecting lines cannot be parallel (since they will cut each other), so they may be sometimes equal, and sometimes not. For if you assume a quadrangle, or oblong, as  $a b c d$ , and connect the



the right lines  $a\,d$ ,  $b\,c$ , the diameters are indeed equal, but not parallel, and they conjoin the equal and parallel opposite sides of the aforesaid spaces. But if the figure be a Rhombus, or a Rhomboides, the diameters of these, are not only non-parallels, but also unequal. For since  $a\,b$ , is equal to  $c\,d$ , but  $a\,c$  is common, and the angle  $b\,a\,c$ ,



is unequal to the angle  $a\,c\,d$ , the bases also are unequal. The insti-  
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tutor of the Elements, therefore, very properly considered, that the lines which conjoin equal and parallel lines, ought to make the conjunction at the same parts, left  $a c$ ,  $b d$ , being supposed equal and parallel, we should assume  $a d$ ,  $b c$ , as the connecting lines, and not  $a b$ , and  $c d$ . For he shews that these latter are equal and parallel : but that the former are, indeed, never parallel, but equal, as we have observed in a quadrangle and oblong, but never in a rhombus and rhomboides ; as the opposite to this has been proved to be true, because they are unequal, on account of the inequality of the angles internal, and situated at the same parts.

### PROPOSITION XXXIV. THEOREM XXIV.

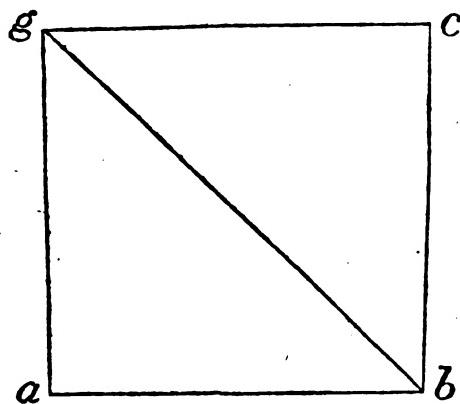
The opposite sides and angles of parallelogrammic spaces are equal to each other an they are bisected by the diameter.

As from the preceding theorem, he had assumed a parallelogram already constructed, he now contemplates its primarily inherent properties, and such things as express its peculiar constitution. But these are the following : *that the sides and angles which are opposite, are equal, and that the spaces themselves are bisected by the diameter.* For that part of the proposition relates to the spaces, which says : *and they are bisected by the diameter.* So that the area itself, is that whole which is bisected, and not the angles through which the diameter passes. These three properties then, are essentially inherent in parallelograms, the equality of the opposite sides and angles, and the bisection of the spaces by the diameter. And you may observe that the properties of parallelograms are investigated from all these, viz. from the sides, from the angles, and from the areas. But as there are four kinds of parallelograms, which Euclid defines in the hypotheses \*, viz. a quadrangle, oblong, rhombus, and rhomboides, it deserves to be re-

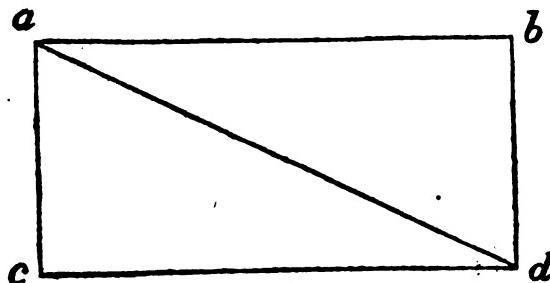
\* In the definitions which are with great propriety called by the Platonists hypotheses, because their evidence is admitted without proof, which at the same time they are capable of receiving from the first philosophy.

marked,

marked, that if we divide these four into rectangles, and non-rectangles, we shall find, that not only the diameters bisect these spaces, but that the diameters themselves, are, indeed, in rectangles equal, but in non-rectangles unequal, as was observed in the preceding theorem. But if we divide them into equilateral, and non-equilateral, we shall again find that in the equilateral figures, not only the spaces are bisected by the diameters, but likewise the angles through which they are drawn: but in non-equilaterals this is never the case. For in a quadrangle, and a rhombus, the diameters bisect the angles, and not the spaces only: but in an oblong, and a rhomboides, they alone bisect the spaces. For let there be a quadrangle, or a rhombus,  $g c a b$ ,



and a diameter  $g b$ . Because, therefore, the sides  $g c, c b$ , are equal to the sides  $g a, a b$  (for they are equilateral), and the angles  $g c b, g a b$ , are equal (for they are opposite), and the basis also is common, hence, all are equal to all; and on this account the angles  $c g a, a b c$ , are bisected. Again, let there be an oblong, or rhomboides



given. If, therefore, the angle  $b a c$ , and the angle  $c d b$ , is bisected by

by the diameter, but the angle  $c \alpha d$ , is equal to the angle  $a \alpha b$ , the angle also,  $b \alpha d$ , will be equal to the angle  $a \beta b$ . Hence, the side also,  $a b$ , will be equal to the side  $b d$ . But they are unequal; and consequently the angle  $b \alpha c$ , is not bisected by the diameter, nor its equal the angle  $c \beta b$ . That I may therefore comprehend the whole in a few words, in a quadrangle the diameters are equal, on account of the rectitude of the angles, and the angles are bisected by the diameters, on account of the equality of the sides, and the areas are bisected by the diagonal, on account of the common property of parallelograms: but in an oblong, the diameters are indeed equal, because it is a rectangle, but the angles are not bisected by the diameters, because it is not equilateral; though the division of spaces into equal parts, is also inherent in this figure, so far as it is a parallelogram: but in a rhombus the diameters are unequal, because it is not a rectangle, but the spaces are not only bisected by these, because it is a parallelogram, but the angles also, because it is equilateral; and in the remaining figure, i. e. a rhomboides, the diameters are unequal, because it is not a rectangle, and the angles are cut by these into unequal parts, because it is not equilateral, and the spaces alone situated at each part of the diagonals, are equal, because it is a parallelogram. And thus much concerning observations of this kind, which exhibit the diversity found in the four divisions of parallelograms.

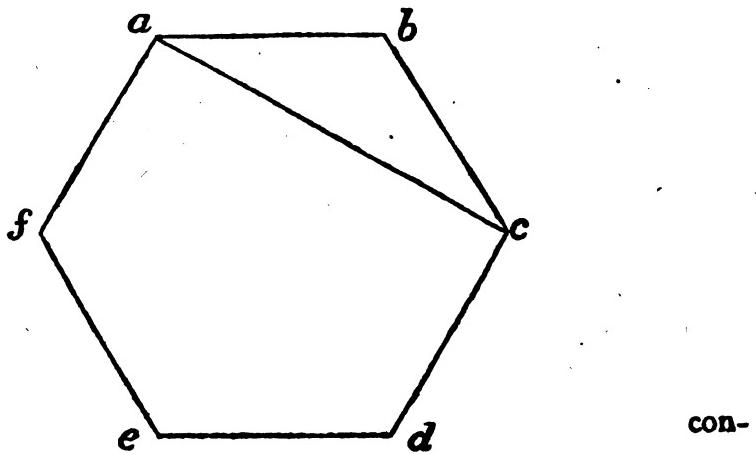
But we must not pass over in silence, the artificial consequence appearing in this theorem, that of theorems, some are universals, but others non-universals. But we shall speak concerning each of these, when we divide *the object of investigation*, which has, indeed, one part universal, but the other non-universal. For though every theorem may seem to be universal, and every thing exhibited by the elementary institutor may appear to be of this kind (as in the present he may not only seem to assert, that in all parallelograms universally, the opposite sides and angles are equal, but likewise that each is bisected by the diameter), yet we must say that some things are universally exhibited, but others not universally. For it is customary to call the *universal* which affirms the truth concerning every thing of which it is predicated, differently from *that universal*, comprehending all things in which

which the same symptom is inherent. Thus it is universal, that every isosceles triangle has three angles equal to two right, because it is true of all isosceles triangles: and it is universal that every triangle has three angles equal to two right, because it comprehends all things, in which this is essentially inherent. On which account we affirm that the possession of three angles equal to two right, is to be primarily manifested of a triangle. According to this signification, therefore, of theorems, calling some universal, but others non-universal, we must affirm that the present theorem, has, indeed, one of its objects of investigation universal, but the other non-universal. For the possession of opposite sides and angles that are equal, is a universal, since it is alone inherent in parallelograms: but that the diameter bisects the space, is not universal, because it does not comprehend all things in which this symptom is beheld; for this is inherent in a circle and ellipsis. And it appears, indeed, that primary conceptions of such like concerns, are more particular, but that in their progress they comprehend the whole. For when the ancients had contemplated that a diameter bisects an ellipsis, circle, and parallelogram, they afterwards surveyed that which was common in these. But we are deceived (says Aristotle †) when a non-universal is exhibited as universal, because that common something in which the symptom is primarily inherent, is nameless. For we cannot say what that is, which is common to numbers and magnitudes, motions and sounds; and it is likewise difficult to express what is common to an ellipsis, circle, and parallelogram. For one of these figures is right-lined, but the other circular, and the third mixt; and on this account we conceive that he exhibits universally, who demonstrates that a diameter bisects every parallelogram, because we do not at the same time perceive that common something, on account of which, this is true. This then in parallelograms, is not an universal of this kind, on account of the aforesaid cause; but the proposition is universal, which afferts, that every parallelogram has its opposite sides and angles equal. For if any figure is supposed, having its opposite sides and angles equal, it may

† In his last Analytics. See page 49, of the Dissertation, Vol. I. of this work.

be shewn to be a parallelogram. Thus let such a figure be  $a b c d$ \*, and its diameter  $a d$ . Because, therefore, the sides  $a b, b d$ , are equal to the sides  $a c, c d$ , and the angles comprehended by them are equal, and the base common, all will be equal to all. The angle, therefore,  $b a d$ , is equal to the angle  $a d c$ , and the angle  $a d b$ , to the angle  $c a d$ . Hence,  $a b$ , is parallel to  $c d$ , and  $a c$  to  $b d$ . And on this account the figure  $a b c d$ , is a parallelogram. And thus much may suffice for observations of this kind.

But the institutor of the Elements seems to have composed the name of parallelograms, by taking an occasion from the preceding theorem. For when he had shewn that right lines, which conjoin equal and parallel right lines at the same parts, were themselves also equal and parallel, it is evident that he pronounces as well the opposite sides which conjoin, as those which are conjoined, to be parallel : but that he very properly calls the figure which is contained by parallels, a parallelogram, in the same manner as he denominates that which is comprehended by right lines rectilineal. And it is evident that the institutor of the Elements places a parallelogram among quadrilateral figures. But it is worthy our observation and enquiry, whether every right-lined figure, which is composed from equal sides, since it is equilateral and equiangular, is to be called a parallelogram. For a figure of this kind also, has its opposite sides equal and parallel, as likewise the opposite angles equal. As for example, a sexangle, and an octangle, and a decangle. Thus, if you conceive a sexangle  $a b c d e f$ , and



\* See the last preceding figure.

con-

connect a right line  $a c$ , you may shew that  $a f$  is parallel to  $c d$ . For the angle at the point  $b$ , is one right, and the third part of a right angle ; and this is true of every angle of a sexangle, since it is equangular. Besides the side  $a b$ , is equal to the side  $b c$ , for it is placed equilateral. Each of the angles, therefore,  $b a c, b c a$ , is a third part of a right angle. Hence, the angles  $f a c, a c d$ , are right angles. And on this account  $a f$ , is parallel to  $c d$ . In like manner we may shew that the other opposite sides are parallel, and the same may be evinced in an octangle, and in the remaining figures of this kind. If, therefore, that is a parallelogram which is comprehended by parallels oppositely situated, a parallelogram will likewise subsist among non-quadrilateral figures. But it appears that with the institutor of the Elements a parallelogram is quadrilateral. And this is particularly perspicuous in that theorem, in which he says, *that a parallelogram which has the same base with a triangle, and is between the same parallels, is double of the triangle*: for this is alone true in quadrilateral figures.

### P R O P O S I T I O N XXXV. THEOREM XXV.

Parallelograms which are upon the same base, and between the same parallels, are equal to each other.

As we have said that of theorems, some are universal, but others particular, and as dividing these we have subjoined, that some are also simple, but others composite, and have shewn the nature of each, so according to another distinction, we assert that some of these are local, but others non-local. But I call those local, to which the same symptom happens in a certain place ; and I denominate the place of a line or a superficies, that situation, which produces one and the same symptom. For of local theorems some are constructed in lines, but others in superficies. And because of lines, some are plane, but others solid, the plane being those of which there is a simple conception in a plane, as of a right line : but the solid those whose origin appears from a certain section of a solid figure, as of cylindric, spiric, and co-

nic lines, I should say, that of the local theorems which are constructed in lines, some have a plane, but others a solid place. The present theorem, therefore, is both local, and local in lines, and a plane. For the whole space which lies between the parallels, is the place of the parallelograms constructed upon the same base ; and which the institutor of the Elements shews to be equal to each other. But of those local theorems which are called solid, let the following be an example\*. *The parallelograms which are inscribed within the asymptotes and the hyperbola, are equal* : for it is evident that the hyperbola is a solid line.

But Chrysippus, as we are informed by Geminus, assimilates theorems of this kind to ideas. For as ideas comprehend the origin of infinites in terminated limits, so in these also there is a comprehension of infinites, in terminated places, and by this boundary equality appears, since the altitude of the parallels remaining the same, if infinite parallelograms are conceived upon the same base, they may all be shewn to be equal to each other. The present, therefore, is with the institutor of the Elements, the first local theorem. And he appears, when, agreeable to an elementary mode, he had distinguished theorems by a variety, according to all possible divisions, with great propriety not to have omitted, considering their idea of this kind. Nevertheless, as his discourse, for the present, is concerning right lines, he delivers local plane theorems in right lines : but in the third book, as he treats concerning things which may be contemplated of circles, and their symptoms, he likewise teaches the particulars, which are constructed in circumferences belonging to local, and at the same time, plane theorems. And such, among these, is the theorem, which says, *that angles in the same segment, are equal to one another*. Also this which asserts, *that the angles in a semicircle are right*. For

\* This is a well known property of the hyperbola, and its asymptotes ; and is thus expressed by Mr. Simpson, in his Conic Sections, Lib. 3. Prop. 16. " If from a point in the hyperbola, any two lines are drawn to the asymptotes, and if from any point, in the same or opposite hyperbolas, there are drawn to the asymptotes other right lines parallel to the former ; then the rectangle contained by the lines first drawn, shall be equal to the rectangle contained by the other drawn lines."

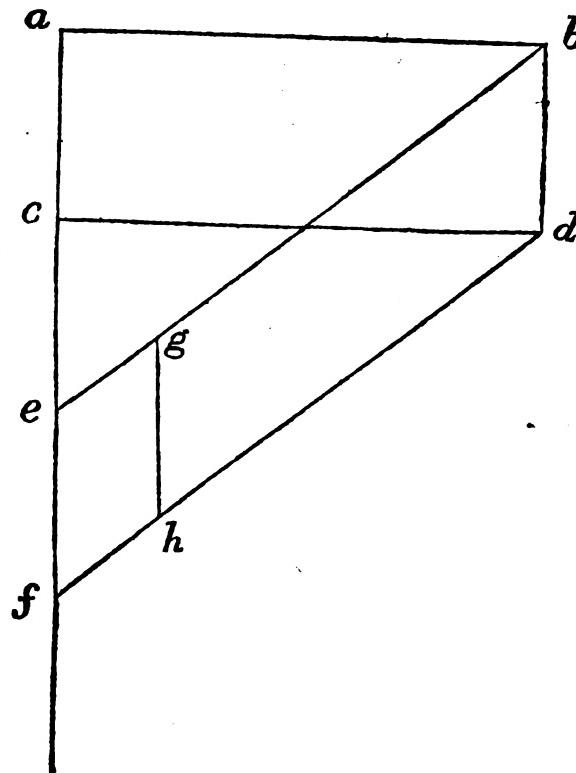
if

## COMMENTARIES OF PROCLUS. 479

if infinite angles are constructed in a circumference, the same base remaining, they are all shewn to be equal: but if that which is comprehended by the base and the circumference, is a semicircle, they are all shewn to be right. And these, indeed, correspond in proportion to triangles and parallelograms upon the same base, and between the same parallels. And such is the species of theorems called local, by the ancient mathematicians.

But perhaps it may seem perfectly worthy of admiration, to such as are unskilled in contemplations of this kind, that parallelograms constructed upon the same base, and between the same parallels, should be equal to each other. For it may be asked, how is this possible, since the longitude of the spaces, constructed on the same base, increases in infinitum? Since as much as we produce the parallels, by so much we may also increase the longitudes of the parallelograms. But some one may not improperly enquire how, while this takes place, the equality of the spaces remains. For if the breadth is the same (since the base is one), but the length is greater, will not the space also be greater? The present theorem, therefore, and that which follows concerning triangles, are among the number of mathematical theorems, which are denominated admirable. For mathematicians in theorems, as the Stoicks in arguments, have established a *place*, which is called admirable, and they place the present among theorems of this kind. The vulgar, therefore, are immediately astonished, when they hear that the multiplication of length does not destroy the equality of spaces on the same base. We must nevertheless assert, that equality and inequality possess the greatest power in increasing or diminishing the spaces of angles. For in proportion as we make angles unequal, in such proportion we diminish the space, if the length and breadth remain the same. Hence, the increase of length is necessary, that we may preserve equality. Thus, for example, let there be a parallelogram  $a b c d$ , and let the side  $a c$  be produced in infinitum, and let it be a right-angled parallelogram; and lastly, on the base  $b d$ , construct another parallelogram  $b e f d$ . That the length, therefore, is increased is evident: for the side  $b e$ , is greater than the side  $a b$ ,

A a 2 since



since the angle at the point  $a$ , is right. But this necessarily takes place, as the angles of the parallelogram  $b e f d$ , are unequal, and some of them are acute, but others obtuse: and this happens, because the side  $b e$ , approaches after a manner to the side  $b d$ , and contracts the space. For let  $b g$  be taken equal to  $a b$ , and through  $g$ , draw  $g b$  parallel to  $b d$ . The length, therefore, of the parallelogram  $b d g b$ , is equal to the length of the parallelogram  $a b c d$ , and the breadth is the same, and yet one space is less than the other; for it is less than  $b e f d$ . Hence, the inequality of angles diminishes the area, but the increment of length adding as much as the inequality of angles takes away, preserves the equality of the spaces. But the boundary of the increase of length, is the place of the parallel lines. For when both the parallelograms are rectangular, and have an equal ambit,

ambit, the quadrangle is shewn to be greater than the oblong \* : but when they are both equilateral, and have consequently an equal ambit, that which is rectangular, is shewn to be greater than that which is non-rectangular †. For the rectitude of angles, and the equality of sides, possesses universal power in the augmentation of spaces. It is on this account that a quadrangle is the greatest of all figures with an equal ambit, and a rhomboides the least. And these observations we shall demonstrate in another place ‡: for they more properly belong to the hypotheses of the second book.

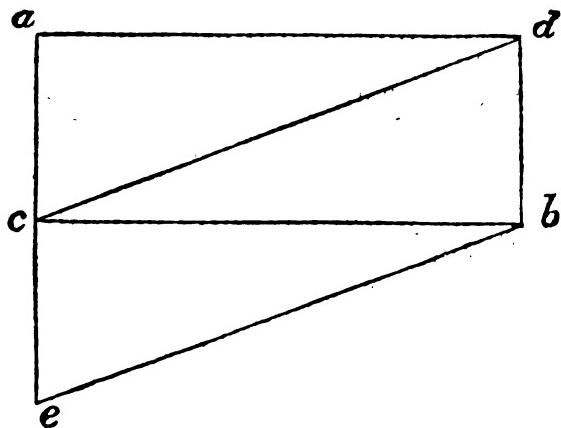
But with respect to the present theorem, it is requisite to know, that when Euclid calls parallelograms equal, he means the spaces, and not the sides : for he now discourses of areas. And we must likewise observe, that he first mentions trapezia in the demonstration of this theorem : from whence also it is manifest, that he does not improperly teach us concerning a trapezium, in the definitions, when he informs us that it is indeed of a quadrilateral species, but is not a parallelogram. For the figure which has not its opposite sides and angles equal, falls from the order of parallelograms. The institutor of the Elements, therefore, as he had chosen a more difficult case, demonstrates the thing proposed. But if any one should say, let the parallelograms  $a b c d$ , and  $b d c e$ , be upon the same base  $d b$ , so that the side  $c d$  may be the diameter of the parallelogram  $a b$ , we can shew that according to this position they are equal. For the triangle  $b c d$ , is the half of each parallelogram : because  $c d$  is the diameter of  $a b$ , but  $c b$  of  $d e$ ; and diameters bisect parallelograms.

\* Thus let there be a square, whose side is equal to three, and a parallelogram whose longest side is equal to four, and its shortest to two ; the ambit of each figure will indeed be equal to twelve, but the area of the square will be equal to nine, and of the parallelogram to eight.

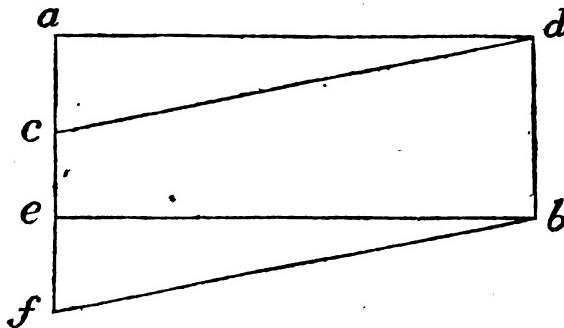
† This will be evident by conceiving a rectangular parallelogram equal to that which is non-rectangular, described on the same base : for the ambit of the former will be less than that of the latter, and consequently less than the parallelogram, with an ambit equal to the non-rectangular parallelogram.

‡ From hence it is evident that it was the intention of Proclus to comment on the whole of Euclid : but it does not appear that he ever carried this design into execution.

Hence,



Hence,  $a b$  is equal to the parallelogram  $d e$ . Again, if any one should suppose that the side  $a c$ , of the parallelogram  $a b$ , is cut by the side  $d c$ , and that the parallelograms are situated as  $a d b e$ ,  $b d c f$ , we



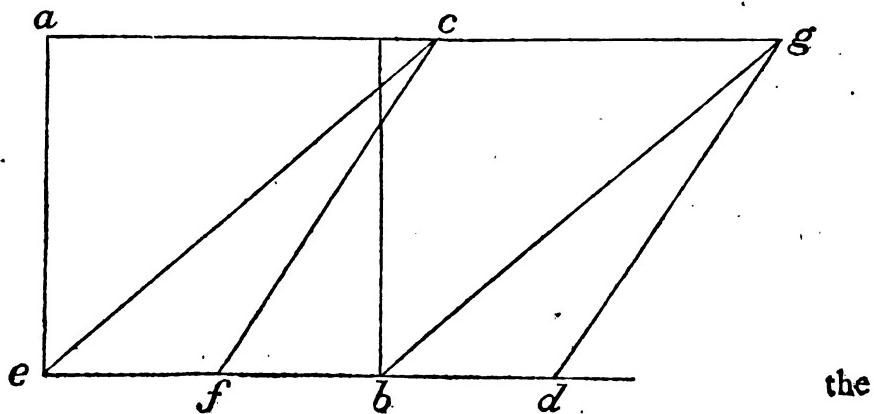
can shew that these also are equal. For since the side  $a e$ , is equal to the side  $c f$  (each because opposite being equal to  $d b$ ), let the common right line  $c e$  be taken away. Hence,  $a c$  is equal to  $e f$ . But  $a d$ , also, is equal to  $e b$ , and the angle  $c a d$ , to the angle  $f e b$ . For  $a d$  is parallel to  $e b$ ; and hence, the base  $c d$ , is equal to the base  $f b$ , and the whole triangle  $a d c$ , is equal to the whole triangle  $e b f$ . Let the common trapezium  $c b$ , be added. The whole, therefore,  $a b$ , is not unequal to the whole  $d f$ . And here you may observe that these are the only three cases. For the side  $d c$ , either cuts the side  $e b$ , according to the position of the elementary institutor; or it falls on the

point  $c$ , as in the penultimate description : or it cuts the line  $a\ e$ , according to the present supposition. And thus the theorem is shewn to be true according to all its cases. Lastly, as there is a two-fold difference of trapeziums, and one kind has neither of its opposite sides parallel, but the other has one side parallel to one, this latter species of trapeziums is alone employed by the geometrician throughout the elements, and in the present description : for  $c\ e$  is parallel to  $d\ b$ .

### PROPOSITION XXXVI. THEOREM XXVI.

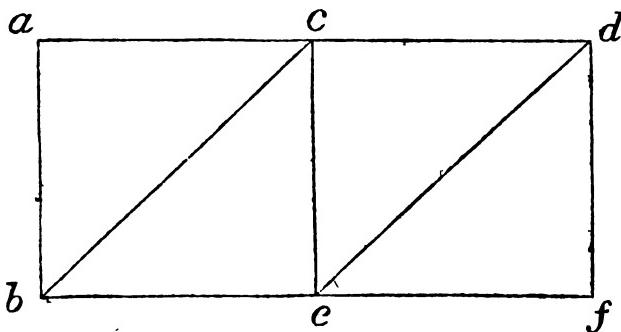
Parallelograms which are upon equal bases, and between the same parallels, are equal to each other.

The preceding theorem assumed, indeed, the same bases, but this receives them equal, and different from each other. But it is common to both, to suppose the parallelograms between the same parallels. It is requisite, therefore, that they should neither fall within, nor without their subject parallel lines. For parallelograms are said to be between the same parallels, when their bases and opposite sides are adapted to the same parallels. As to the rest, the institutor of the Elements, as he had assumed the bases entirely separate, exhibits the theorem. But nothing hinders our receiving them with this hypothesis, so that they may have a common part. For let  $a\ b$ ,  $c\ d$ , be parallelograms upon equal bases  $e\ b$ ,  $f\ d$ , having a common part, and constructed between the same parallels, I say that they are equal. Let



the lines  $ec$ ,  $bg$ , be connected. Because, therefore,  $ef$ , is equal to  $bd$  (for the base  $eb$ , was supposed equal to the base  $fd$ ), but the side  $cf$ , is equal to the side  $dg$ , and the angle  $cfe$ , is equal to the angle  $gd$ , and hence,  $ce$  is equal to  $bg$ . But it is also parallel to it. Hence,  $cb$  is a parallelogram, and has the same base with each of the parallelograms  $ab$ ,  $cd$ , and is between the same parallels. The parallelogram, therefore,  $ab$ , is equal to the parallelogram  $cd$ .

But if any one should suppose that the bases of the parallelograms have neither a common part, nor are separate from each other, but (which is the only remaining hypothesis) that they touch each other in one point, as in the parallelograms  $ae$ ,  $ed$ , we must say that the



base  $be$  is equal to the base  $ef$ , and to the side  $cd$ . Hence, also, the right line  $cb$ , is equal to the right line  $de$ , and is parallel to it. For the lines which join equal and parallel lines, are themselves also equal and parallel. Hence,  $bd$  is a parallelogram, and is upon the same base, and between the same parallels, with the parallelograms  $cb$ ,  $de$ . The parallelograms, therefore,  $cb$ ,  $de$ , are equal. But according to the first conception of a theorem, we may divide the constructions by asserting that the bases have either a common part, or touch each other, or are distant from each other. It is however possible, that though they may touch each other, as  $be$ ,  $ef$ , yet the whole parallelogram  $de$  may be supposed external to the side  $ce$ ; or one side of the parallelogram  $cf$ , may be the diameter of the parallelogram  $ae$ ; or the side  $ce$ , may cut the side  $ac$ ; or the side  $ac$ , being produced beyond  $a$ , the side  $ce$ , may fall as the diameter of the parallelogram increased

increased towards  $a$ , when the side  $df$  becomes the same as a line drawn from  $a$  to  $f$ ; or the side  $ce$ , may cut the side  $ac$ , produced beyond  $a$ ; or the side  $ac$ , may be still farther produced beyond  $a$ , so that the side  $ce$  may fall beyond the point, to which  $ac$  was extended in the preceding case, and the side  $df$ , may cut the line produced beyond  $a$ . \* \* \*

## PROPOSITION XXXVII. THEOREM XXVII.

Triangles which are upon the same base, and between the same parallels, are equal to each other.

*The beginning of this Commentary is wanting.*

\* \* \*

\* \* for those being equal, the spaces are unequal; and when these are unequal, those are shewn to be equal. And this is the case with Chorographers, when they reason concerning the magnitudes of cities, from their ambits. But formerly, certain persons deceived their partners, in the distribution of their possessions, deluding them by an excess of ambit, so as to make them believe that they received a greater portion of land, when they received a greater ambit; and that they were gainers, by changing spaces into areas of less ambit. ¶ Thus two isosceles triangles being proposed, one of which has each of its equal sides, containing five parts, but the base six: and the other has each of its equal sides five parts, but the base eight; and let these parts be, for instance, cubits, or digits, these triangles will very much deceive the ignorant in their choice. For the ambit of the one is eighteen, and of the other sixteen measures. But a geometrician is not ignorant that the spaces are equal, though the ambits are unequal; since the area of each is twelve measures. For if you draw a per-

† The present Commentary is imperfect, both in the Greek, and the translation of Barocius; who observes that the conclusion is wanting in all the copies which he had an opportunity of perusing. Those who are curious may consult his scholium, in which he has endeavoured to complete it.

‡ See Comment 8, of the third book, with its note.

pendicular from the vertex, you will bisect the bases, and cause the half of the one to be three but of the other four measures: but the perpendicular on the contrary, will be there equal to four, but here equal to three; since it is requisite that the square from the quinary, should be equal to the squares from the perpendicular, and the half of the base. But if the base of the one is equal to three, the perpendicular must be four; and if the base of the other is equal to four, its perpendicular must be three. When, therefore, you have multiplied the half of the base with the perpendicular, you will have a space equal to the triangle: but this is the same in each, whether you multiply the quaternary with the ternary, or the ternary with the quaternary. And we have made these observations for the purpose of shewing that the equality of spaces is not to be entirely received from the ambit. Nor should we wonder, that though triangles upon the same base, may be infinitely increased between the same parallels, according to the remaining sides, yet the equality of the spaces immutably remains. But those triangles are said to be between the same parallels, which have their bases upon one of the parallel lines, and fix their vertices on the remainder; and whose vertices being connected, form one right line, parallel to the bases on the same right line.

### PROPOSITION XXXVIII. THEOREM XXVII.

Triangles which are upon equal bases, and between the same parallels, are equal to each other.

The present theorem also is local, because it corresponds in proportion with parallelograms, and supposes the situation of triangles upon equal bases. But Euclid seems, to me, to have delivered one demonstration by the first proposition of the sixth book of these four theorems, two of which are exhibited in parallelograms, and two in triangles: and two of which are on the same base, and the other two on equal bases. But that Euclid has performed this is unknown to the vulgar. For after he had shewn that triangles and parallelograms, which are under the same altitude, have the same proportion to each other

other as their bases, nothing demonstrates all these four theorems more universally, from proportion, than this theorem : since to possess the same altitude, is nothing else than being constituted between the same parallels. For all figures between the same parallels, are under the same altitude, and the contrary : since the altitude is the perpendicular, which extends itself from one parallel to the rest. In that proposition, therefore, it is shewn by proportion, that triangles and parallelograms, under the same altitude, that is, situated between the same parallels, are to each other as their bases, and so when the bases are equal, the spaces are equal ; and when those are double, these will be double ; and when the bases have any other proportion, the spaces also will have to each other the same proportion. But for the present, because it is not proper that he should use proportion, who has not yet explained its nature, he is content with equality and identity alone : for the identity of bases is collected from equality. Hence, these four theorems are comprehended in that one ; not only because he shews by one demonstration, whatever are contained in these four, but likewise, because he adds what was wanting to their perfection, viz. identity of proportion, though the bases are unequal. But that this theorem, also, has many cases, and that it is possible that the bases of the triangles may be assumed, either having the same part as in parallelograms ; or possessing no common part, but touching each other according to one point ; or entirely separate, so that a line may intervene between them, is manifest, even to such as are endued with slender capacities. And this too is evident, that according to all cases, however the bases or vertices may be situated, the same method of proceeding must be adopted as in parallelograms ; viz. parallels to the sides must be drawn, and produced both ways, and the equality of the triangles exhibited.

### PROPOSITION XXXIX. THEOREM XXXIX.

Equal triangles, which are upon the same base, and at the same parts, are between the same parallels.

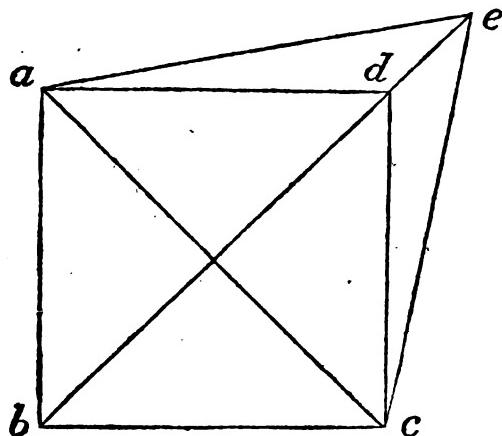
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When it was proposed to exhibit equality to us, then it was requisite to make four theorems, receiving two in parallelograms, but the other two in triangles, situated either upon the same, or upon equal bases. But now by conversion, we neglect the theorems which are converse in parallelograms, and esteem such as are converse in triangles worthy of relation. And the reason of this is, because the mode of demonstration in parallelograms, is the same indifferently, by a deduction to an impossibility, and the construction is similar. But we are content when we have exhibited the way in more simple figures, I mean triangles, to leave to the more curious the same mode of reasoning in the rest: since it is easy, at the same time, to perceive that there is the same method in these. For when we assume equal parallelograms, upon the same base, or upon equal bases, we must say that they are also between the same parallels. For if they are not, either one of them falls within, when the parallels which are in the other are produced; or without. But which ever case is assumed, when we receive it and its parallels, we may exhibit the same consequences as in triangles, I mean that the whole will be equal to its part: but this is impossible. It is however manifest, that the institutor of the Elements very properly adds the particle, *and at the same parts*. For it is possible that equal triangles, may be assumed upon the same base, one, indeed, at these parts, but the other at different parts, and yet these will not be entirely between the same parallels: for neither will they be contained under the same altitude. And on this account he added the particle.

But since a parallel may be drawn in a two-fold respect, according to an absurd hypothesis, i. e. either within or without, Euclid draws it within: but we can exhibit the same consequences, by drawing it without. For let the equal triangles  $a b c$ ,  $d b c$ , be upon base, and at the same parts, I say that they are between the same parallels, and that the right line connected at their vertices, is parallel to the base. Let the right line  $a d$  be connected. But if this is not parallel, let the line, external to this, i. e.  $a e$  be parallel, and let  $b d$  be produced to the point  $e$ , and connect  $e c$ . The triangle, therefore,

$a b c$ ,

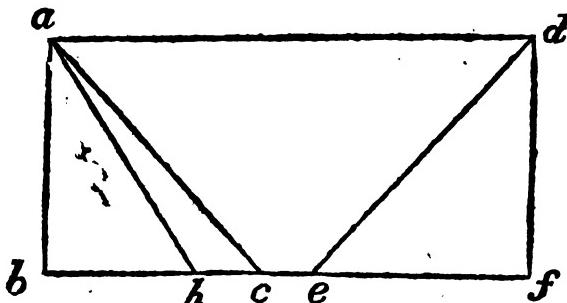


$a b c$ , is equal to the triangle  $e b c$ , the whole to the part. But this is impossible ; and hence, the parallel line does not fall external to  $a d$ . But it is shewn by the institutor of the Elements, that neither does it fall within : and hence  $a d$ , is parallel to  $b c$ . Hence too, equal triangles, which are at the same parts, and upon the same base are parallel to each other. And thus the remaining part of the deduction to an impossibility is demonstrated. But it is worthy of observation, that since the conversion of theorems is triple (for either the whole is converted to the whole, as we have noticed, in the eighteenth and nineteenth theorems ; or the whole to the part, as the sixth and fifth ; or the part to the part, as the eighth and the fourth : for the whole is not a *datum*, in the one, and *an object of investigation* in the other : nor is *the object of investigation*, a *datum*, but a part) these triangular theorems appear to be of this kind. For, that the triangles are equal, is an object of investigation in the preceding ; but this is not a *datum* alone in these, because it assumes, besides this, a part of that which was hypothesis in those. For to stand upon the same, or upon equal bases, is a *datum* in these, as well as in those, except that in these hypotheses he adds something which was neither an object of investigation, nor a *datum* in these ; since the particle *at the same parts*, is over and above extrinsically assumed.

## PROPOSITION XL. THEOREM XXX.

Equal triangles which are upon equal bases, and at the same parts, are between the same parallels.

There is the same mode of conversion too in the present theorem, and a similar demonstration; and that part of the deduction to an impossibility, which is omitted by the institutor of the Elements, is demonstrated after the same manner, and there is no occasion for repetition. But since these three conditions are in the aforesaid propositions, *situation upon equal, or on the same bases; position between the same parallels; and equality of triangles and parallelograms*, it is manifest that we may variously convert, by always connecting two, and leaving one. For we either supposed the bases the same, or equal, and triangles and parallelograms between the same parallels, and thus we form four theorems; or we consider the triangles and parallelograms equal, and the bases the same, or equal, and thus we produce another four, two of which the elementary institutor omits, viz. those which respect parallelograms, but the other two relative to triangles, he exhibits; or lastly, when we have assumed them equal, and between the same parallels, we prove the remainder, that they are either upon the same, or upon equal bases, and produce another four, which the institutor of the Elements entirely neglects. For there is the same demonstration in these, except that two of these four are not essentially true. Thus, equal parallelograms or triangles, between the same parallels, are not necessarily upon the same base: but all this is true in these hypotheses, that they are upon the same or equal bases; but the other does not entirely follow the assumed hypotheses. Hence, as all these theorems are ten, the geometrician speaks of six, and neglects four, lest he should labour in vain, by repetition, since the demonstration is the same. For it may be shewn in triangles, that if they are equal, and between the same parallels, they will either be upon the same, or upon equal bases. For let it be denied, and if possible, let the triangles  $a b c, d e f$ , have these conditions, upon unequal bases  $b c, e f$ . Let too,  $b c$ , be the greater, and cut off  $b b$ , equal to  $e f$ ,



$ef$ , and connect  $ab$ . Because, therefore, the triangles  $abb$ ,  $def$ , are upon equal bases,  $bb$ ,  $ef$ , and between the same parallels, they are equal. But the triangles also,  $abc$ ,  $def$ , are supposed equal. Hence, the triangles  $abc$ ,  $abb$ , are equal, which is impossible. The bases, therefore, of the triangles  $abc$ ,  $def$ , are not unequal. And the mode of demonstration will be the same in parallelograms. Since, therefore, the ostensive method is the same, and the impossibility the same, viz. that the whole is equal to its part, it is not improperly omitted by the elementary institutor. And thus we have shewn, that there are necessarily ten theorems, and have enumerated what are omitted, and shewn the reason of their omission. But let us now pass to the following propositions.

#### PROPOSITION XLI. THEOREM XXXI.

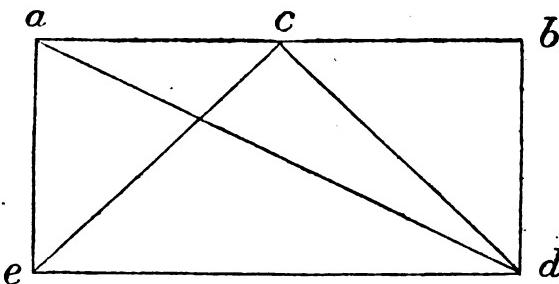
If a parallelogram has the same base with a triangle, and is between the same parallels, the parallelogram shall be double the triangle.

The present theorem also is local, but it minglesthe constructions of triangles and parallelograms, situated under the same altitude. As, therefore, we have separately surveyed parallelograms and triangles, so when we assume each of them in conjunction, and with the same condition, we contemplate their proportion to each other. In the former, therefore, an equality of proportion is apparent, since all upon the same bases, and between the same parallels, have a mutual equality,

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whether

whether they are triangles, or parallelograms. But in these latter, the first of unequal proportions, I mean the duple, is exhibited : for he demonstrates that a parallelogram is double of a triangle, on the same base, and possessing the same altitude. But the elementary instructor shews the thing proposed, by supposing the vertex of the triangle external to the parallelogram. We can, however, demonstrate the consequence, by assuming the line which is parallel to their common base, in the other side of the parallelogram : for these are two cases of the theorem. Since in consequence of the two having the same base, it is necessary that the vertex of the triangle should either be within, or without the parallelogram. Let there be, therefore, a parallelogram  $a b e d$ , and a triangle  $e c d$ , and let a point  $c$  be placed between the points  $a$  and  $b$ , and connect the right line  $a d$ . Because, therefore, the parallelogram is double of the triangle  $e c d$ , but the



triangle  $a d c$ , is equal to the triangle  $e d c$ , hence, the parallelogram is double of the triangle  $e c d$ . And hence it is evident that a parallelogram is double of a triangle on the same base. But if the bases are equal, we can shew the same by drawing the diameters of the parallelograms : for if the triangles are equal, the parallelogram which is double of the one, will also be double of the other. But triangles are equal, on account of the equality of bases, and the identity of altitude. The geometer, therefore, very properly omits this, for the demonstration is the same : since they will either have the same part, or they will be conjoined in one point only, or they will be separate from each other. But in whatever manner they may receive this variety, there is one demonstration according to all the cases.

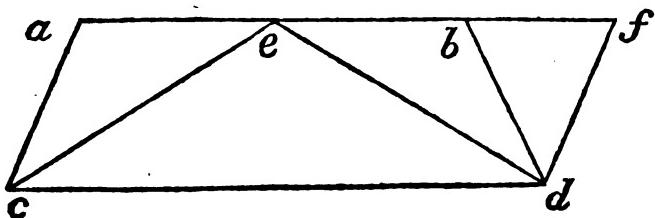
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We can likewise demonstrate the converse propositions to this theorem, after the same manner. One of which is : *If a parallelogram is double of a triangle, and they have the same or equal bases, and are at the same parts, they shall be between the same parallels.* For if they are not the whole shall be equal to the part, and the same proportion shall prevail : since it is necessary that the vertex of the triangle should either fall within, or external to the parallels. But in either case, the same impossibility will be the result, by drawing a parallel to the base, through the vertex of the triangle. But the second converse theorem is : *If a parallelogram is double of a triangle, and both are between the same parallels, they will either be situated upon one base, or upon equal bases.* For if they are upon unequal bases, since we have assumed the figures to be equal, we may shew that the whole will be equal to its part. Hence, all these theorems end in this common impossible : and on this account, the institutor of the Elements leaves us to investigate the variety they contain, as he himself, has contracted his speculation to such as are more simple, and of a more primary nature. However, as we have recognized these observations, let us see for the sake of exercise, by not assuming a parallelogram, but a trapezium, two of whose sides only are parallel (because it has the same base with the triangle, while it is situated between the same parallels), let us, I say, consider what proportion it possesses to the triangle. That it has not, therefore, a duple proportion is evident : for if it had a duple ratio, it would be a parallelogram, since it is a quadrilateral figure. But I say that it is either greater than double or less ; for since the two sides are parallel, one is greater, but the other less ; because if equal, the sides conjoining them will be parallel. If, therefore, the triangle has its greater side for the base, the quadrilateral figure will be less than double of the triangle : but if the lesser side, it will be more than double. For let  $a b c d$ , be a quadrilateral figure, and let the side  $a b$ , be less than the side  $c d$ , and produce the side  $a b$ , in infinitum, and let the triangle  $e c d$  have the same base with the quadrilateral figure, that is  $c d$  ; and lastly, through  $d$ , draw

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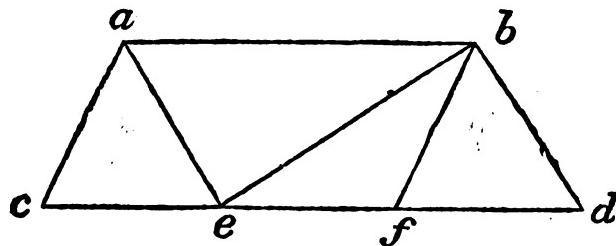
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df,

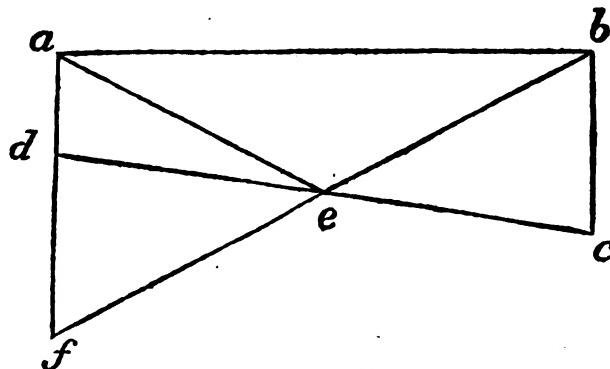


$df$ , parallel to  $ac$ . Hence, the parallelogram  $acdf$ , is double of the triangle  $ecd$ ; and so the quadrilateral figure  $abcd$ , is less than double of the triangle.

Again, let the triangle have the base  $ab$ , and draw  $bf$ , parallel to  $ac$ . The parallelogram, therefore,  $abfc$ , is double of the triangle.



And hence, the quadrilateral figure,  $abcd$ , is more than double of the triangle. This being shewn; we affirm, that when there is a quadrilateral figure, whose two opposite sides only, are parallel, if one of the parallel sides is bisected, and right lines are drawn from it to the other side, the quadrilateral figure, is either more or less than double of the triangle resulting from such a construction. But if one of the sides by which the parallel lines are conjoined, is bisected, and certain right lines are drawn from it to the remaining side, the quadrilateral figure, will be perfectly double of the triangle which is produced. And this may be shewn as follows. Let there be a quadrilateral figure  $abcd$ , and let the side  $ad$ , be parallel to the side  $bc$ , and bisect  $dc$ , in the point  $e$ , and connect the right lines  $ae$ ,  $eb$ , and produce  $be$ , till it coincides with  $ad$ , in some point, as  $f$ . Because, therefore, the angles at the point  $e$ , are equal, for they are vertical; likewise, because the angle  $fde$ , is equal to the angle  $bce$ , the side also  $fe$ , will be equal to the side  $eb$ , and the triangle  $def$ , will be equal to the triangle  $bce$ . Let the common triangle  $ade$ , be added. The whole triangle, therefore,



fore,  $aef$ , is equal to the two triangles  $ade$ ,  $bce$ . But the triangle  $aef$ , is equal to the triangle  $aeb$ : for they are upon equal bases,  $be$ ,  $ef$ , and between the same parallels, if a line parallel to  $bf$ , is drawn \*. Hence, the triangle  $aeb$ , is equal to the triangles  $ade$ ,  $bce$ , and the quadrilateral figure  $abcd$ , is double of the triangle  $aeb$ , which was to be shewn. After the same manner, we may shew, that if the side  $ab$ , is bisected, and certain right lines are drawn from it, to the side  $ed$ , the quadrilateral figure will be double of the triangle formed by such a construction. If, therefore, one of the sides by which the parallel lines are conjoined is bisected, and from it certain right lines are drawn to the remaining side, the quadrilateral figure shall be double of the triangle. And these things are demonstrated for the sake of geometrical exercise. Let us now proceed to the subsequent propositions.

### PROPOSITION XLII. PROBLEM XI.

To construct a parallelogram equal to a given triangle, in a given rectilineal angle †.

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### P R O-

\* Barocius is of opinion, that this Commentary was originally mutilated; and that the part which follows the word *drawn*, was added by some skilful geometrician, as necessary to the perfection of the demonstration. See his *Scholium to this Commentary*.

† The Commentary of Proclus, on this proposition is wanting in the Greek, and, as we are informed by Barocius in all the MS. copies which he had an opportunity of consulting.

Cc<sup>2</sup> Barocius

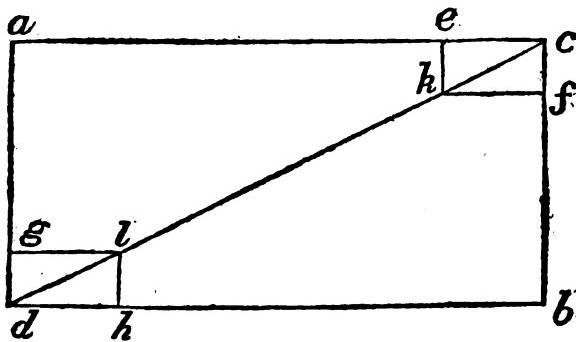
## PROPOSITION XLIII. THEOREM XXXII.

The complements of parallelograms, situated about the diameter of every parallelogram, are equal to each other.

The beginning of this commentary is wanting.

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that parallelograms are not mutually conjoined according to one-point, and that the complements are not quadrilateral ; it is requisite that placing this also as a case, we should regard the same accident. For let there be a parallelogram  $a b$ , having the parallelograms  $c k$ ,  $d l$ , about the same diameter, and let a certain right line  $k l$ , which is

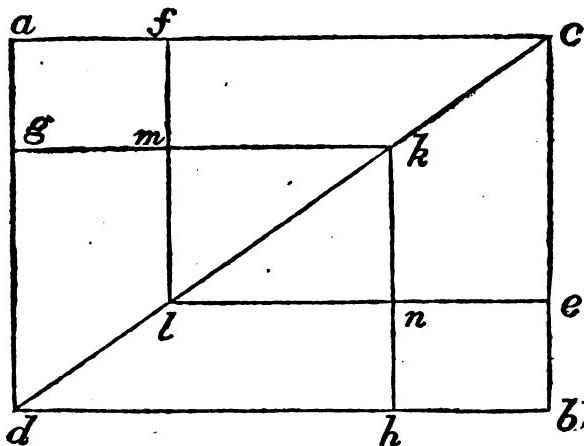


a part of the diameter intervene between them. Again, therefore, you may say the same, viz. that the triangle  $a c d$ , is equal to the triangle  $b c d$ ; and the triangle  $e c k$ , to the triangle  $k c f$ ; and likewise the triangle  $d g l$ , to the triangle  $d h l$ . The remaining figure, therefore,  $a g l k e$ , of five sides, is equal to the remaining five-sided figure  $b f k l b$ . But these were the complements. Again, if the parallelograms are neither conjoined according to a point, nor distant from

Barocius has endeavoured to supply this deficiency, after the manner of Proclus ; but he appears to have fallen into prolixity, by a too minute division of the problem.

each

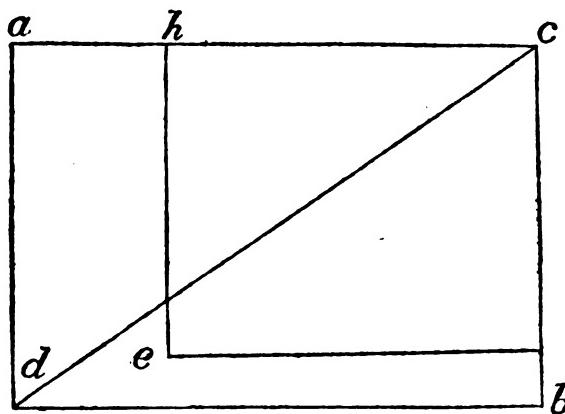
each other, but mutually cut each other, on this hypothesis also, the demonstration will be the same. For let there be a parallelogram  $a b$ , and a diameter  $c d$ , and let parallelograms be constructed about it,



one of which is  $e c f l$ , but the other, by which this also is intersected,  $d g k b$ . I say that the complements  $f g, e b$ , are equal: For since the whole triangle  $d g k$ , is equal to the whole triangle  $d b k$ , but a part of it also, the triangle  $k l m$ , is equal to the triangle  $k h n$ ; (since  $l k$  is a parallelogram); hence the remaining trapezium  $d l n b$ , is equal to the remaining trapezium  $d l m g$ . But the triangle  $a d c$ , is equal to the triangle  $b c d$ , and the triangle  $f c l$ , in the parallelogram  $e f$ , to the triangle  $e c l$ , and the trapezium  $d g m l$ , to the trapezium  $d b n l$ . The remaining quadrilateral figure, therefore  $g f$ , is not unequal to the remaining quadrilateral figure  $e b$ . And hence, the theorem is exhibited according to all its cases. But there are three only, and neither more nor less. For the parallelograms consisting about the same diameter, either cut each other or touch each other, according to a point, or are distant from each other by a certain part of the diameter.

But the institutor of the Elements assumes the appellation of *complements*, from the thing itself, so far as these also, besides two parallelograms, fill up the whole: and on this account, it was not of itself thought worthy of being remembered in the definitions. For, indeed, variety is requisite to its declaration, such as the knowledge of a pa-

rallelogram, and what those parallelograms are, which are about the diameter of the whole parallelogram ; since, when these are explained, this likewise becomes known. But those parallelograms are about the same diameter, which have a part of the whole diameter for their own : and those which have not this condition, are by no means about the same diameter. For when the diameter of the whole parallelogram is cut by the sides of an internal parallelogram, then this parallelogram is not about the same diameter with the whole parallelogram. As for example, in the parallelogram  $a\,b$ , let the diameter  $c\,d$ , cut the side  $e\,b$ , of the parallelogram  $c\,e$ . The parallelogram,



therefore,  $e\,c$ , is not about the same diameter with the parallelogram  $c\,d$ .

#### PROPOSITION XLIV. PROBLEM XII.

To a given right line, to apply a parallelogram equal to a given triangle, in an angle which is equal to a given right lined angle.

According to the Familiars of Eudemus, the inventions respecting the *application*, *excess*, and *defect* of spaces, is ancient, and belongs to the Pythagoric muse. But junior mathematicians receiving names from these, transferred them to the lines which are called conic, because one of these they denominate a *parabola*, but the other an *hyperbola*,

perbola, and the third an ellipsis \* ; since, indeed these ancient and divine men, in the plane description of spaces on a terminated right line, regarded the things indicated by these appellations. For when a right line being proposed, you adapt a given space to the whole right line, then that space is said to be *applied* ; but when you make the longitude of the space greater than that of the right line, then the space is said to *exceed* ; but when less, so that some part of the right line is external to the described space, then the space is said to be *deficient*. And after this manner, Euclid, in the sixth book, mentions both excess and defect. But in the present problem he requires application, wishing to apply to a given right line a parallelogram equal to a given triangle ; that we may not only have the construction of a parallelogram equal to a given triangle, but also an application to a determinate right line. As for example, a triangle being given, having an area of twelve feet, but a right line being proposed, whose length is four feet, we may apply to the right line a parallelogram equal to the triangle, if when we assume the whole length of four feet, we find how many feet the breadth ought to contain, that the parallelogram may become equal to the triangle. When, therefore, we have discovered that the breadth is three feet, and have multiplied the length with the breadth, the proposed angle being right, we shall obtain the desired space. And such is the verb *to apply*, formerly delivered by the Pythagoreans. But there are three things given in the present problem ; one, the right line to which it is to be so applied, that it may become the whole side of that space ; but the other is the triangle to which that which is applied ought to be equal ; and the third is the angle to which it is requisite that the angle of the space should be equal. And here it is again perspicuous, that when the angle is right, the space which is applied, is either a quadrangle, or an oblong ; but when it is either acute or obtuse, the space is either a rhombus, or rhomboides. Besides, this too is manifest, that the right line ought to be finite ; since this cannot be accomplished on an infinite

\* See Simf. Seq. Con Lib. I. Prop. 13. and Lib. II. Prop. 23, and Lib. III. Prop. 48.  
line.

line. At the same time, therefore, as he says, *to apply to a given right line*, he indicates that the right line must be necessarily finite. But he uses in the construction of the present problem, the construction of a parallelogram equal to a given triangle; since, as we have observed, application is not the same with construction. For the latter, indeed, constructs both the whole space, and all the sides; but the former, when it has one side given, constitutes on this the space, because it is neither deficient, nor exceeds according to this extension, but uses this one side which comprehends the area. But you may perhaps say, why does he use theorems, when he shews triangles equal to triangles; but problems, when he shews triangles equal to parallelograms? We reply, it is because equality spontaneously arises in things of the same species; but requires origin, and fabrication, in things of a dissimilar species, on account of the mutation subsisting according to species, since it is by itself difficult of invention.

### PROPOSITION XLV. PROBLEM XIII.

To construct a parallelogram equal to a given right-lined figure, in a given rectilineal angle.

The present is more universal than the two problems, in which he invented as well the construction, as the application of parallelograms equal to a given triangle. For whether a triangle, or a quadrangle, or any other quadrilateral figure is given, we may construct a parallelogram equal to it, by the present theorem; since every right-lined figure, as we have previously observed \*, may be essentially resolved into triangles, and we have delivered a method of discovering the multitude of triangles. When, therefore, we have resolved a given rectangle into triangles, and have constructed a parallelogram equal to one of them, and have applied to a given right line, parallelograms equal to the rest; then, by assuming that to which we have made

\* In the sixth Commentary of this book.

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the first application, we shall have a parallelogram composed from these parallelograms, equal to the right-lined figure composed from those triangles, and the thing desired, will be accomplished. Hence, though such a rectangle should be a figure of ten sides, yet, by resolving it into eight triangles, and constructing a parallelogram equal to one of them, and seven times applying parallelograms equal to the rest, we shall obtain the object of investigation. But, as it appears to me, the ancients being incited by this problem, sought how to describe a quadrangle equal to a circle. For if a parallelogram can be found equal to any right-lined figure, it deserves to be enquired whether right-lined figures also, can be shewn equal to such as are curve-lined. And Archimedes shews that every circle is equal to a right-angled triangle, one of whose radii is equal to one of the sides which are about the right angle of the triangle; but whose ambit is equal to the base. However, of this elsewhere: let us now proceed to the consequent propositions.

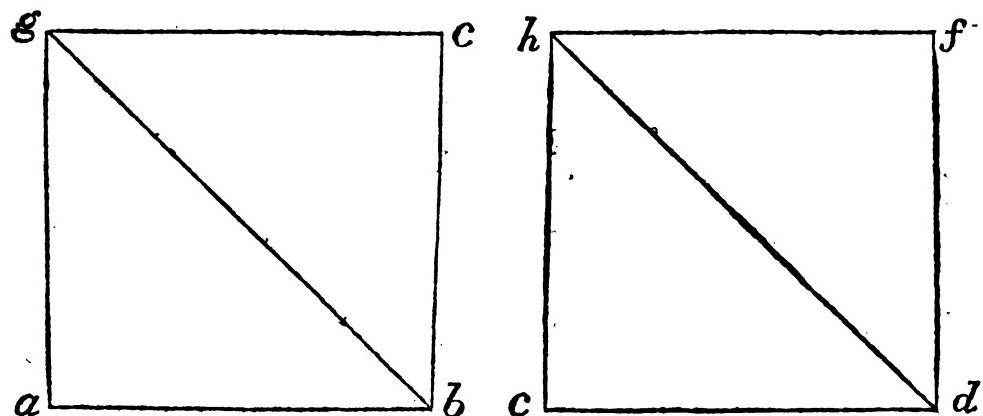
## PROPOSITION XLVI. PROBLEM XIV.

To describe a quadrangle from a given right line.

Euclid requires this problem, most particularly, in the construction of the following theorem. But he appears to have been desirous to deliver the origin of the two best rectilineal figures, viz. the equilateral triangle, and the quadrangle; because these right lined figures are required in the constitution of the mundane figures, and particularly of those four, to which origin and dissolution belong. For the icosaedron, and the octaedron, and the pyramid, are composed from equilateral triangles; but the cube from quadrangles. And on this account, as it appears to me, he has principally *constructed* the former, but *described* the latter. For he has discovered appellations adapted

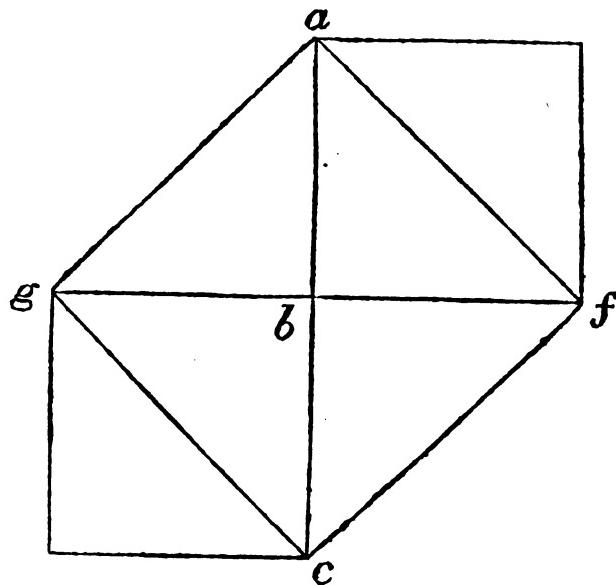
to these figures : since the equilateral triangle, so far as its composition is various, requires *construction*; but the quadrangle, so far as it originates from one side, requires *description*. For we cannot produce a triangle in the same manner as a quadrangle, by multiplying the number of a given right line into itself; but when we have conjoined right lines produced by other means, with the extremities of the given right line, we construct from these one equilateral triangle: and the description of circles, profits in discovering that point from which it is requisite to connect right lines, to the extremes of the proposed right line. But these observations are indeed perspicuous.

It may, however, be shewn, that the right lines, from which quadrangles are described, being equal, the quadrangles also shall be equal. For let the right lines  $a b$ ,  $c d$ , be equal, and from  $a b$ , describe the quadrangle  $a b c g$ , but from  $c d$ , the quadrangle  $c d b f$ .



and connect the right lines  $g b$ ,  $b d$ . Because, therefore, the right lines  $a b$ ,  $c d$ , are equal,  $a g$ ,  $b c$ , are also equal; and they comprehend equal angles, and the base  $g b$ , is equal to the base  $b d$ , and the triangle  $a b g$ , to the triangle  $c d b$ ; and the doubles of these are equal. Hence the quadrangle  $a c$ , is not unequal to the quadrangle  $a f$ . But the converse of this also is true. For if the quadrangles are equal, the right lines, also, from which they are described, will

be equal. Thus let the quadrangles  $af, cg$ , be equal, and let them be so placed, that the side  $ab$ , may be in a right line with the side  $bc$ . Since therefore, the angles are right, the right line also  $fb$ , will be in a



direct position, with the right line  $bg$ . Let the right lines  $fc, ag$ ,  $af, cg$ , be connected: Because, therefore, the quadrangle  $af$ , is equal to the quadrangle  $cg$ ; the triangle, also,  $afb$ , is equal to the triangle  $cgb$ . Let the common triangle  $bcf$ , be added. The whole triangle, therefore,  $acf$ , is equal to the whole triangle  $cfg$ .

Hence,  $ag$  is parallel to  $fc$ . Again, because, as well  $afg$ , as the angle  $cgb$ , is the half of a right angle,  $af$ , is parallel to  $c g$ . The right line, therefore,  $af$ , is equal to the right line  $c g$ , since they are the opposite sides of a parallelogram. Because, therefore, there are two triangles,  $afb$ ,  $b c g$ , which have the alternate angles equal, since  $af$ ,  $c g$ , are parallel; likewise one side  $af$ , equal to the side  $c g$ , the side, also,  $ab$ , shall be equal to the side  $bc$ , and the side  $bf$ , to the side  $bg$ . And thus it is shewn, that the quadrangles  $af, cg$ , being equal, the sides, also, from which they are described are equal.

## PROPOSITION XLVII. THEOREM XXXIII.

In right angled triangles, the quadrangle, which is described from the side subtending the right angle, is equal to the quadrangles which are described from the sides comprehending the right angle.

If we attend to the historians of antiquity, we shall find them referring the present theorem to Pythagoras, and asserting that he sacrificed an ox for its invention. For my own part, I admire those who first investigated the truth of this theorem : but I possess a greater admiration for the elementary institutor, not only because he establishes its truth by evident demonstration, but likewise, because he persuades us by scientific reasons, which cannot be confuted of a theorem more universal than this in his sixth book\*. For in that he shews universally, *that in right-angled triangles, the figure described from the side subtending the right angle, is equal to the figures described from the sides comprehending the right angle, when they are similar to the former figure, and are similarly described.* For every quadrangle is similar to every quadrangle ; but all right-lined figures similar to each other, are not quadrangles : since in triangles, and other multangles, similitude is inherent. Hence, the reason which demonstrates that the figure described from the side subtending the right angle, whether it is quadrangular, or of some other form, is equal to the figures subsisting about the right angle, similar to the former, and similarly described ; exhibits something more universal, and which possesses a greater power of producing science, than the reason exhibits, affirming a quadrangle alone, equal to quadrangles. For in the former case, it becomes manifest by an universal ostension, that the rectitude of the angle affords to the figure described from its subtending side, equality, to all the figures, subsisting about its comprehending sides, similar to the former, and similarly described : just as obtuseness is the cause of

\* In the 3<sup>rd</sup> proposition.

excess;

excess; but acuteness of diminution. But how this theorem is evinced, will be perspicuous, when we comment on it in the sixth book.

But let us now consider the truth of the present theorem, only adding this, that *universal* ought not to be shewn here, by him who has taught nothing concerning the similitude of right lined figures, and the doctrine of proportion: for many things which are here exhibited more particularly, are in that theorem shewn more universally by the same method. The institutor of the Elements, therefore, shews the thing proposed in the present, from the common contemplation of parallelograms. But since right-angled triangles are two-fold, i.e. either isosceles, or scalene; in isosceles triangles, we shall never find numbers corresponding with the sides: for there is no quadrangular number, exactly double of another quadrangular number; since the square from the septenary is double of the square from the quinary, by a deficiency of unity. But in scalene triangles it is possible, that numbers may be assumed, so as evidently to evince, that the square from the side subtending the right angle, is equal to the squares from the sides subsisting about the right angle. And of this kind is the triangle in the republic, whose right angle is contained by the ternary, and quaternary, but is subtended by the quinary. The quadrangle, therefore, from the quinary, is equal to the quadrangles from the other numbers: for this is twenty-five; but the quadrangle from the ternary is nine, and from the quaternary sixteen. And thus what we have asserted is perspicuous in numbers.

But there are delivered certain methods of inventing triangles of this kind, one of which they refer to Plato, but the other to Pythagoras, as originating from odd numbers. For Pythagoras places a given odd number, as the least of the sides about the right angle, and when he has received the quadrangle produced from this number, and diminished it by unity, he places the half of the remainder, as the greatest of the sides about the right angle; and when he has added unity to this, he produces the remaining side which subtends the right angle. Thus for example, when he has assumed the ternary,

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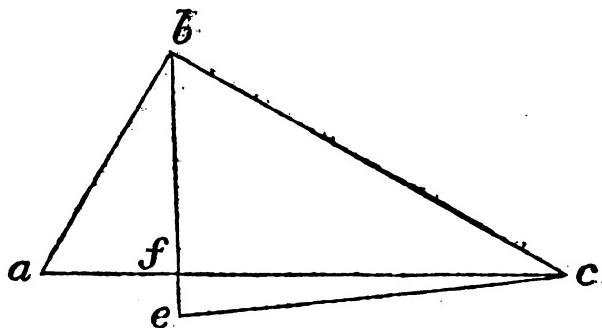
nary, and has produced from it a quadrangular number, and from this number nine, has taken unity, he assumes the half of eight, that is four, and to this again he adds unity, and makes five ; and thus discovers a right-angled triangle, having one of its sides of three, but the other of four, and the other of five units. But the Platonic method originates from even numbers. For when he has assumed a given even number, he places it as one of the sides about the right angle, and when he has divided this into half, and has produced a quadrangular number from the half, when he has added unity to this quadrangle, he forms the subtending side, but when he has taken unity from the quadrangle, he forms the remaining side about the right angle. Thus for example, when he has assumed the number four, and has multiplied the half of this into itself, and produced four, when he takes away unity he forms the number three, but when he adds unity, he produces the number five ; and thus he has the same triangle effected, as by the Pythagoric method. For the square from the number five, is equal to the squares from the numbers three, and four. And thus much for the digression of the present narration. But as the demonstration of the elementary instructor is perspicuous, I do not think, that any thing should be added, because it would be superfluous ; but we should be content with what is written. For those who have added any thing more, as the familiars of Hero and Pappus, have been obliged to assume in an affair of no difficulty, some of the propositions of the sixth book ; and the cause which regards this affair. We shall therefore pass on to the following theorem.

### PROPOSITION LXVIII. THEOREM XXXIV.

If the quadrangle described from one side of a triangle, is equal to the quadrangles described from the other two sides of the triangle : then the angle comprehended by the remaining two sides of the triangle, is right.

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This theorem is the converse of the preceding, and the whole is converted to the whole. For if the triangle is rectangular, the quadrangle which is described, from the side subtending the right angle, is equal to the quadrangles described from the other sides : and if the square from this, is equal to the squares from the other sides, the triangle is rectangular, because it has the angle right, which is comprehended by the remaining sides. And the demonstration of the Elementary institutor is indeed conspicuous. But when there is a tri-

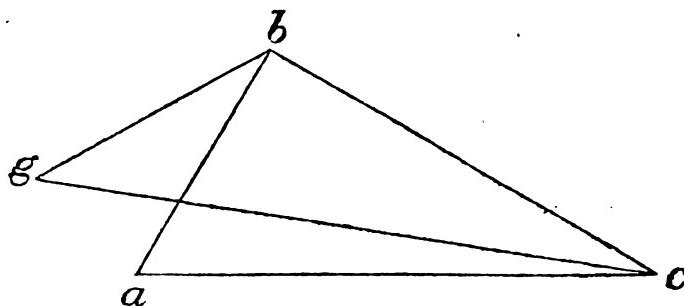


angle  $a b c$ , having the quadrangle, which is described from the side  $a c$ , equal to the quadrangles from the sides  $a b$ ,  $b c$ , since in the triangle, a right line from the point  $b$ , is raised at right angles to the side  $b c$ , if it should be said, that the right line must be raised at right angles, to other parts, and not at those to which the elementary institutor raises it, we assert that this is an impossibility. For it can neither fall within, nor without the triangle ; and can be no other than  $a b$ . For if possible, let it fall as  $b e$ . Because, therefore, the angle  $e b c$ , is right, the angle  $c f b$ , is doubtless acute ; and hence, the remaining angle  $a f b$ , will be obtuse. The side, therefore,  $a b$ , is greater than the side  $b f$ . Let a line  $b e$ , be placed equal to  $a b$ , and connect  $e c$ . Because, therefore, the angle  $e b c$ , is right, the quadrangle described from the side  $e c$ , is equal to the quadrangles from the sides  $e b$ ,  $b c$ . But  $e b$  is equal to  $b a$ . The quadrangle, therefore, from the side  $e c$ , is equal to the quadrangles from the sides  $a b$ ,  $b c$ . But the quadrangle from the side  $a c$ , was also equal to the same. Hence, the quadrangle from the side  $e c$ , is equal to that which is described from

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the side  $a c$ ; and so  $e c$  is equal to  $a c$ . Two right lines, therefore,  $b e$ ,  $e c$ , are equal to the two  $b a$ ,  $a c$ , each to each, and are constructed upon the right line  $b c$ , which is impossible. And hence, the line raised at right angles, does not fall within the right line  $a b$ .

But neither can it fall without, towards other parts of the right line  $a b$ . For if possible, let it fall as  $b g$ , and let  $b g$  be equal to  $a b$ ,



and connect  $c g$ . Because, therefore, the angle  $g b c$ , is right, the quadrangle described from the side  $g c$ , is equal to the quadrangles from the sides  $b g$ ,  $b c$ . But the quadrangle also, from the side  $a c$ , was equal to the quadrangles from the sides  $a b$ ,  $b c$ , but  $a b$  is equal to  $g b$ ; and so  $g c$  is equal to  $a c$ . But the right line  $g b$ , also, is equal to the right line  $b a$ , upon one right line  $b c$ , which is impossible. Hence, the right line which is raised from the point  $b$ , at right angles to  $b c$ , neither falls within, nor without the side  $a b$ ; and therefore falls upon it. And so the objection is dissolved. But the instructor of the Elements, thus far completes his first book, in which he has delivered many species of conversions; (for he often converts the whole of theorems to the whole, and wholes to parts, and parts to parts) and has invented a great variety of problems; (for he has delivered the sections, positions, constructions, and applications of lines and angles. He likewise touches upon that mathematical place which is called admirable; and sufficiently brings local theorems into our remembrance. Besides, he unfolds the elementary institution of universal and particular theorems, and indicates the difference of indeterminate, and determinate problems; all which, attending him in his progress, we have orderly explained. Lastly, he refers the whole

## COMMENTARIES OF PROCLUS. 209

book to one purpose, I mean the elementary institution, of the contemplation respecting the more simple rectilineal figures ; and finally, he investigates their constructions, and considers their essential properties. But we, indeed, shall give thanks to the gods, should we be able to comment on the other books, in a similar manner. In the mean time, if other cares should prevent the execution of our design, it is my opinion, that such as are studious of these contemplations, ought to expound the other books, after the same mode ; by investigating that which is every where difficult, and pertinent to the subject, and capable of an easy division. For, indeed, the commentaries which are circulated at the present period, are replete with great and various confusion, because, at the same time, they neither infer any assignation of cause, nor dialectic judgment, nor philosophic contemplation.

END OF THE COMMENTARIES.

VGL. II.

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OF THE  
R E S T O R A T I O N  
OF THE  
P L A T O N I C T H E O L O G Y,

By the latter PLATONISTS.

Ex 2



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THE  
H I S T O R Y  
OF THE  
RESTORATION OF THE PLATONIC THEOLOGY.

By the latter PLATONISTS.

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S E C T . I.

THE Grecian theology, the history of whose restoration by the latter Platonists is the design of the present dissertation, did not originate among the Greeks, but was the progeny of barbarian propagation. This will be evident by considering that Orpheus was a Thracian ; Thales, a Phoenician ; Hermes Trismegistus, an Egyptian ; Zoroaster, a Persian ; Anacharsis, a Scythian ; and Pherecydes, a Syrian. Yet though Greece was not the parent of theology, she was notwithstanding her benevolent nurse, by whom she was kindly educated, and received the full perfection of her nature. Indeed, though illustrious men flourished in the East, and theology was there particularly cultivated, yet her education was limited and rough, entangled with inexplicable ceremonies, and guarded by the sanctity of inviolable oaths. But when she was removed into the Grecian soil, and experienced the happy temperature of its climate, her genius became both elegant and profound ; her person magnificent and graceful ; and her ceremonies rational and sublime. Particular nations, indeed, seem to have been distinguished for particular pursuits. Thus the Egyptians appear to have excelled in the powers of invention ; and the East, in general, has been remarkable for its attachment to the most recondite and mystic philosophy. Thus the Romans were famous for the arts of eloquence and war ; and the Greeks have ever been celebrated as a people by whom every branch of knowledge received its ultimate perfection. They were a nation equally favoured by the graces, the muses, and philosophy ; whose celestial union formed the divine genius of Homer, and inspired that elegance and depth with which

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the works of Plato are replete. They were, in short, the standards of excellence to the ancient, and are the objects of imitation to the enlightened part of the present world ; and their theology, as well as their arts, will be admired when modern systems are no more.

It appears at first view strange that this sublime theology should rise to its pristine perfection during the decline of the Roman empire ; and at a period when a new religion (I mean the Christian) was continually increasing in reputation, and advancing with rapid steps to a despotic establishment. But if we attentively consider, we shall find that the very causes which apparently threatened its destruction were the natural and proper sources of its renovation. As every part of the universe subsists by perpetual change, it is necessary that philosophy and the sciences, with respect to their appearance or the contrary, should share in the general mutability of things : but at the same time, it is necessary to their preservation to after-ages, that the order of their revolution should be retrograde to that of sensible particulars. Hence we shall often find, that while kingdoms descend in the circle of vicissitude, philosophy ascends, and perhaps attains to her ultimate perfection, at the very period when the most powerful nations become extinct. Thus the falling empire of the Romans was naturally connected with the rising greatness of philosophy ; and the foreign ceremonies of a new religion, were the proper means of bringing to light the secret mysteries of the old. We may add too, that the same circumstances produced the great difference between the first and last appearance of this sublime theology. While Greece maintained her independence unconscious of the Roman yoke, and undisturbed by religious invasions, she disdained to expose her genuine wisdom to vulgar inspection, but involved it in the intricate folds of allegory ; and concealed it from the profane under the dark veil of impenetrable mystery. But when she lost her liberty and submitted to foreign dominion, when her most ancient rites were threatened with invasion, and her sacred mysteries were treated with contempt, she found it necessary to change the dress of theology and to substitute a simple and elegant garb, instead of one highly marvellous and mystic.

Yet we must not imagine that theology, now stript of her ancient concealments, became the object of open inspection to the profane and vulgar eye. She had not lost her resplendence, though she had changed her appearance : for the rays of celestial majesty yet beamed from her countenance, with a light awful and terrific to the multitude, but lovely and alluring to the wise. Hence the splendors of divinity no less secured her person from impious curiosity than the dark symbols in which she was formerly involved. The enchanting imagery of a celestial phantasy, and the pure light of an exalted intellect, while they captivated and converted the philosophical part of mankind, were inaccessible to the vulgar, whose mental eye, yet lost in the night of oblivion, was darkened by the splendid vision. However, though the real person of theology was not the ob-

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ject of vulgar inspection, her shadow at least was beheld by the benighted multitude, and became the subject of ridiculous opinions, and idle investigation. Hence some of these astonished with the majesty of her image, fondly fancied she was the progeny of the Jewish religion ; and that her sacred mysteries were nothing but corrupt imitations of Mosaic divinity : while others, measuring the obscurity of her real person by the darkness of her shadow, considered her doctrines as delusions, and her sublimest truths as the reveries of a distempered imagination. Thus was true theology perverted and vilified by the multitude, when she appeared in her natural dress to mankind ; till, in a few centuries after, indignant of the daring profanation, she ascended to her native heaven, and left the sons of folly involved in the shades of midnight error, and the gross delusions of fancied inspiration.

But let us contemplate her history more minutely, and mark the several particulars which distinguished her appearance on the earth. Let us survey the lives of the great geniuses who so largely participated her celestial light ; and who so admirably transfused it in their writings for the benefit of hitherto ungrateful posterity. Let us view with wonder how she rose in majesty, as Rome declined in power, and appeared in full perfection invested with celestial honours, and surrounded with a godlike band of philosophic heroes, while that mighty empire was rapidly diminishing in bulk, and on every side nodding to its dissolution..

We are informed by Proclus \*, that all the Grecian theology is the progeny of the mystic discipline of Orpheus ; and that Pythagoras was the first who learned the orgies of the gods from Aglaophemus the disciple of Orpheus. This sacred theology was fully displayed by Orpheus, with all the graces of poetical diction, accompanied with the fury of the muses and divine illumination, in a great work entitled, *The Sacred Discourse*, which was divided into twenty-four rhapsodies, and which has unhappily perished in the ruins of time. In this inestimable work, if we may be allowed to conjecture from a treatise of the same name composed by Pythagoras, and often mentioned by Syrianus, all the orders of the gods were celebrated from the highest principle of things, to the last processions of the mundane divinities. But Pythagoras was no doubt deeply indebted for a part of this knowledge to the doctrine of Zoroaster, whose dogmata, according to Apuleius †, he embraced, and whose profound mysteries involved in oracular darkness, we may presume he communicated to his initiated disciples. The whole of this recondite theology was afterwards received by Plato from the writings of Archytas, Philolaus, and other Pythagoreans, but was so concealed by poetical embellishments, and mystical traditions, that, like the numbers of Pythagoras, it was alone adapted to the comprehension of a penetrating and sagacious few.

\* Λαζαρος γεγ. ἡ μάρτιον Ἑλληνος Θεολογία τῆς Ὀρφεῖας; ιερί μυσταγωγίας ἵκυμος. τοιούτου μὲν πιθαγόρεων περὶ Λυγλασφύριν  
ταῦτα επὶ Οὐρανῷ ἐγένετο διδάχθειν. Procl. in Plat. Theol. p. 13. (b) 20

† In Lib. ii. Florid.

It is, however, a remarkable historical fact that this theology was lost for many centuries among the disciples of Plato, on the death of their divine master. But we are informed by Numenius \* the Pythagorean that Plato's successors, Speusippus, Zeno-crates, and Polemo, perverted his dogmata, and almost entirely changed the whole of his philosophy. And Aristotle, it is well known, however he might retain some essential doctrines of his master, altered others of the highest importance ; and confining himself chiefly to natural disquisitions, ascended but rarely and feebly to theological contemplations. However it was not irrecoverably lost ; and it disappeared for a time, only to shine with brighter splendors on its return. Truth, like the light of the sun, may suffer concealment, but cannot be destroyed ; for it would rather have its rays broken by resistance than bound to obscurity. About two hundred and fifty two years, therefore, after the Christian religion had made its appearance, this sublime theology was restored by one Ammonius Saccas an Alexandrian. This extraordinary person, was, it seems, at first nothing more than a porter : though by what methods he rose from this servile employment to the summit of philosophy, and what happy circumstances first affected this wonderful change, are enquiries which can never be answered, but whose loss will always be regretted by the liberal few. But though he was not  $\Deltaιογ\u03b9\u03b9$ , nobly born, his doctrines, as transmitted to us by his disciples, eminently evince his possessing in high perfection all the other endowments of a true philosopher : such as a penetrating genius, a docile sagacity, a tenacious memory, and every other ornament of the soul, requisite, according to Plato, to form the philosophic character. Indeed he must have possessed these qualifications in a most remarkable degree ; or he could never have emerged from the obscurity and servility of a porter, to the splendor and liberty of an exalted and divine philosopher. The truth of this observation is confirmed by the appellation of  $\Theta\deltaιδα\xi\lambda\epsilon$ , or *divinely-taught*, which was unanimously conferred on him, by his contemporary philosophers.

This great man opened a philosophical school at Alexandria, but with a determination not to commit the more abstruse and theological dogmata of his philosophy to writing. Indeed he was so fearful of profaning these sublime mysteries, by exposing them to vulgar inspection, that he revealed them to his disciples Erennius, Origen, and Plotinus, on the conditions of inviolable secrecy, and under the guard of irrevocable oaths. However, fortunately for posterity, Erennius dissolved the compact, and Origen (different from the Christian father of that name), imitating Erennius, disclosed a part of his master's secrets, in a curious treatise on daemons, which, among many other valuable productions, is lost in the ruins of time. But the publications of these two great men were but trifling efforts to unveil the mystic wisdom of antiquity : since a perfect revelation was reserved for the divine genius of Plotinus, who consider-

\* Euseb. *Prepar. Evang.* lib. xiv.

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ing himself now freed from his engagements, by the examples of his fellow disciples, resolved to bring theology from her dark concealments and to present her to the astonished world, in all the celestial graces and irresistible majesty of her natural appearance. This wonderful man (if he was not something more, since his writings discover a genius superior to the human), who was born to astonish and enlighten mankind, was the first who committed to writing the secrets of theology, free from the obscure enigmas in which she had been enveloped by the fages of antiquity. The celestial vigor and profundity of his genius, render his conceptions indeed, unavoidably abstruse : but he who has once fathomed his depth, will find himself amply compensated for the labour of investigation, by the rewards of uncommon knowledge and inexpressible delight. There is a long and curious life of this high priest of theology, and *dæmon of wisdom*, extant by his disciple Porphyry, the substance of which, as it will not I presume be unacceptable to the reader, and as it will throw great light on the history of theology, I have selected from that invaluable work.

Plotinus, was an Egyptian by birth, and was a native of Lycopolis, as we are informed by Eunapius, for Porphyry is wholly silent as to this particular. Indeed this is not wonderful, if we consider what Porphyry asserts in the beginning of his life, *that he was ashamed, that his soul was in body*. Hence says he, he would neither tell the race, nor the parents from which he originated, nor would he patiently relate in what country he was born. This I know will be considered by a genuine modern, as either rank enthusiasm, or gross affectation ; but he who has perused and fathomed his writings will immediately subscribe to its truth. The same vehement love for intellectual pursuits, and contempt for body, made him disdain to sit for his picture ; so that when one of his disciples Amelius, begged that he would permit his likeness to be represented, his answer expressed the true greatness of his mind : as if (says he) it was not sufficient to bear this image, with which nature has surrounded us from the first, you think that a more lasting image of this image should be left as a work worthy to be inspected. However the desire of Amelius was at length accomplished; by the ingenious contrivance of one Carterius a painter, who by frequenting the school of Plotinus, and viewing his countenance with fixed attention, produced at length from his memory a happy likeness of the philosopher. Though he was often afflicted with the colic, he always refused the assistance of clysters, asserting that cures of this kind were not proper to a man advanced in years. Nor would he ever receive the assistance of theriacal antidotes, since he said, his nourishment was not derived from the bodies of even tamer animals. He likewise abstained from baths : but daily used frictions at home. But when a grievous pestilence raged † at Rome, and the servants who were accustomed to rub him,

† This pestilence was in the time of the emperor Gallienus, and raged so vehemently, according to Trebelius Pollio, that five thousand men perished with an equal disease in one day. This happened in the year of Christ 262, and of Galienus 9, 10 ; and not long after Porphyry applied himself to Plotinus. Fabricius.

fell victims to the disease, from neglecting cures of this kind, he gradually became a prey to the pestilence. So great was the violence of this distemper, and its effects so dreadful on Plotinus, as Eustochius informed Porphyry who was absent, that through a very great hoarseness, all the clear, and sonorous vigour of his musical voice was lost; and what was still worse, his eyes were darkened, and his hands and feet were covered with ulcers. Hence, becoming incapable of receiving the salutations of his friends, he left the city; and went to Campania, to the estate of one Zethus, an ancient departed friend. Necessaries were here administered to him from the hereditary possessions of Zethus, and were likewise brought from Minturnus, from the fields of Castricius \*. But when this divine man drew near to his dissolution, that period which is no less the dread of the vulgar than the transport of the philosopher, and which to Plotinus must be the moment of *extatic rapture*, Eustochius who dwelt at Puteolus, was not very hasty in his approaches; doubtless not imagining he was on the point of making his triumphant exit from a corporeal life. However when he came into the presence of this departing hero, he was just in time to receive his dying words, and to preserve the sacred sentence to posterity. Listen ye profane with reverence, and treasure in your memories ye wise, the weighty truth it contains! *As yet (says he) I have expected you; and now I consent that my divine part, may return to that divine nature, which flourishes throughout the universe.* Such were the last words of this mighty man, which like those contained in his writings are great and uncommon, wonderful and sublime. He died at the conclusion of the second year of the emperor Claudius' reign; and was at the time of his death in the sixty-sixth year of his age, according to the information given by Eustochius to Porphyry. The most trifling particulars relative to the life and death of so extraordinary a man merit our attention; and indeed we may presume without being guilty of either superstition or enthusiasm, that scarcely any thing trifling could mark the existence of such a powerful and celestial genius. There is nothing, properly speaking, can be little which has any relation to a character truly great: for such is the power of uncommon genius, that it confers consequence on every thing within the sphere of its attraction, and renders every surrounding circumstance significant and important. Thus immediately on the death of Plotinus, we are informed by Porphyry that a dragon ‡ which had been concealed under his bed, wandered through a hole in the wall, and disappeared. But how great must the grief of Porphyry have been, to be separated from his beloved master, at the time of his death: from a master by whom he had been esteemed beyond the rest of his fellow-disciples; and whose loss no succeeding period was ever likely to repair. Indeed his disciples seem to have been unaccountably dispersed, at this important crisis: for Porphyry was at Lilybæum, Amelius at Apamea in

\* This is the Firmus Castricius to whom Porphyry inscribes his books on Abstinence.

‡ This was probably nothing more than a small serpent resembling the form of a dragon.

Syria,

Syria, Castricius at Rome, and Eustochius was alone present at his departure. Porphyry afterwards informs us, in perfect agreement with the genius of Plotinus, that he never would tell to any one, the month, or day in which he was born: because he by no means thought it proper that his nativity should be celebrated with sacrifices and banquets. Indeed we cannot suppose that he who had such a vehement contempt for a corporeal life, would be anxious that his entrance into mortality should be solemnized with festivity; but rather considering himself with Empedocles, as

“Heaven’s exile straying from the orb of light,”

he would be disposed to lament his captivity, and mourn the dégradation of his nature. However he was not averse to celebrate the nativities of Socrates and Plato; for he assisted at the sacred rites, and invited his friends to a philosophic banquet, were it was required that every guest should recite a written oration, adapted to the occasion of their amicable association.

But the few particulars which this great man condescended to relate of himself, in familiar discourse, are the following: When he was eight years of age, and was even under the tuition of a literary preceptor, he used to frequent his nurse, and to uncover her breasts, through an avidity of sucking her milk. And this custom he continued, till being accused of troublesomeness, and covered with shame through the reproof, he neglected this extraordinary custom. This story however trifling it may appear, indicates in my opinion the native innocence, and genuine simplicity of manners which marked the character of Plotinus. It is a circumstance, which does not merely point to something uncommon; but it was the harbinger as it were of that purity and sanctity of life, which so eminently formed the conduct, and adorned the writings of our philosopher. But when he was in the twenty-eighth year of his age, being vehemently inflamed with the love of philosophy, he was recommended to the most excellent masters of Alexandria: but he left their schools with sorrow and disappointment. By a fortunate event, however, he told a certain friend, who was well acquainted with the disposition of his mind, the cause of his affliction, and he brought him to the celebrated Ammonius, whose school Plotinus had probably overlooked among the great multitude with which that illustrious city abounded. But when he had entered the school of Ammonius, and had heard him philosophize, he exclaimed in transport to his friend, *this is the man I have been seeking.* From that day he gave himself up to Ammonius with sedulous attention for eleven years; and made such rapid advances in his philosophy, that he determined to study the philosophy of the Persians, and the wisdom particularly cultivated by the Indian sages. For this exalted purpose, when the emperor Gordian marched into Persia, in order to war upon that nation, Plotinus joined himself to the army, being at that time in the nine and thirtieth year of his age. But after Gor-

## RESTORATION OF THE

dian was destroyed about Mesopotamia, Plotinus fled to Antioch, where he received a fortunate shelter from the dangers and devastations of war; and in the reign of the emperor Philip came to Rome, in the fortieth year of his age. It seems therefore that Plotinus was disappointed in his purpose at that time of procuring the Persian and Indian wisdom: it is however certain that he afterwards obtained his desire; and most probably without the inconvenience of a long and dangerous journey. This will be evident from perusing his works; and attending to the latent dogmata they contain.

It was a long time before Plotinus committed his thoughts to writing; and gave the world a copy of his imitable mind. That light which was shortly to illuminate mankind, as yet shone with solitary splendour; or at best beamed only on a beloved few. It was now destined to emerge from its awful sanctuary, and to display its radiance with unbounded diffusion. But a disciple like Porphyry, was requisite to the full perfection of its appearance. Amelius was indeed laborious, but he was at the same time verbose: he neither appears to have possessed the inquisitive spirit, nor the elegant genius of Porphyry; and his commentaries were too voluminous to be exquisitely good. Porphyry gives a singular specimen of his endurance of labour, when he informs us, that he committed to writing almost all the dogmata of Numenius, and retained a very considerable part in his memory. He was not however, though an excellent philosopher, calculated to urge Plotinus to write, or to assist him in its prosecution: but this important task was reserved for Porphyry, who in the words of Eunapius, "like a mercusial chain, let down for the benefit of mortals, by the assistance of universal erudition, explained every thing with clearness and precision." Plotinus indeed began to write in the first year of the emperor Galienus; and he continued just to note such questions as occurred to him, for the ten following years, in the last of which he became acquainted with Porphyry, who was at that time in the thirtieth year of his age. He had then composed one and twenty books, which were in the hands but of a few: for the edition was difficult to be procured, and was not universally known. Besides Plotinus, was neither hasty nor rash in his publications: but he gave those only to the light, which had been approved, by a mature and deliberate judgment. The one and twenty books we have previously mentioned, after various inscriptions, at length obtained the following titles:

- On the beautiful.
- On the immortality of the soul.
- On fate.
- On the essence of the soul.
- On intellect, and ideas, and being.
- On the descent of the soul into body.

How

How that which is posterior to the first, proceeds from the first; and concerning  
the one.

Whether all souls are one:

Concerning the good itself, or the one.

On the three principal hypostases.

On the generation and order of things posterior to the first.

On the two matters, intelligible and sensible.

Various considerations.

On the circular motion of the heavens.

On every ones peculiar Daemon.

On the rational exit, from the present life.

On quality.

Whether there are ideas of particulars.

On virtues.

On dialectic.

How the soul is said to be a medium between an impartible and partible essence.

These one and twenty books were finished when Pophyry first became acquainted with Plotinus; and when this great man was fifty-nine years old. During the six years in which Porphyry was his companion as well as disciple, many questions of a very abstruse nature, were discussed in their philosophical conversations, which at the joint request of Porphyry and Amelius, Plotinus committed to writing, and produced from their investigation, two elaborate and admirable books, proving that true being is totally present in every part of the universe. He wrote besides two others; one of which asserts, that the nature superior to being, is without intellection; and the other distinguishes primary from secondary intelligence. He likewise composed at the same period, the following books:

Concerning that which exists in capacity, and energy.

\* That incorporeal natures are free from passivity.

Two books concerning the soul.

A third concerning the soul, or the manner in which we see.

On contemplation.

On intelligible beauty.

That intelligibles are not external to intellect; and concerning intellect, and the good,

Against the Gnostics.

\* It is strange that Fabricius should think it ought to be entitled *περὶ τῆς ἀναγνώσιας τῶν σωμάτων*: for he who reads this book must see that such a title would be ridiculous. Vide Bibl. Grec. tom. iv. p. 143.

On

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On numbers.

Why things seen at a distance appear small.

Whether felicity consists in length of time.

Concerning total mixture.

How the multitude of ideas subsists, and concerning the good.

On that which is voluntary.

On the world.

On sense and memory.

Three books on the genera of beings.

On eternity and time.

But while Porphyry resided in Sicily, Plotinus composed the five following books, which he sent to him for his revision :

On felicity.

Two books on providence.

On gnostic essences, and that which is superior to their nature.

On love.

These books were transmitted to Porphyry in the first year of the emperor Claudius' reign. And about the beginning of the second year, a little before his death, he sent him the following, and the last :

An enquiry into evil.

Whether the stars operate on sublunary natures.

What the nature is of man, and animal.

On the first good, and other goods.

The whole amount therefore of the books written by Plotinus, connecting the preceding with the present, is fifty-four, which Porphyry has divided into six enneads, assigning agreeable to the meaning of the word, nine books to every ennead. But they bear evident marks (says Porphyry) of the different periods, at which they were composed. For the first one and twenty, which were written in the former part of his life, if compared with the next in order seem to possess an inferior power, and to be deficient in strength. But those composed in the middle of his life exhibit the vigour of power, and the summit of perfection. And such with a few exceptions are the four and twenty we have already enumerated. But the last nine, composed in the decline of life, carry the marks of remitted energy, and drooping vigor. And this the four last declare, more evidently than the preceding five. It must however be observed that this difference is only visible, when they are contrasted with one another. To an impartial observer, zealous of truth, and not deeply read in Plotinus, each of his books will appear to

to be what it really is, uncommonly profound, and inimitably sublime. Each is an oracle of wisdom, and a treasury of invaluable knowledge; and the gradations of excellence consist in the power of composition, and not in the matter from which they are composed.

Plotinus had many auditors, and likewise a multitude of zealous partizans, and philosophic familiars. This indeed must necessarily be the case, if we consider the reputation of philosophy at that golden period, and the extraordinary abilities and celestial genius of its godlike restorer. Among the latter of these, Amelius the Tuscan, and Paulinus the Scythopolitan, a physician, held a distinguished rank. To which may be added Eustochius of Alexandria, a physician, who enjoyed the familiarity of Plotinus to the last, was present at his death, and giving himself entirely to the institutes of Plotinus, assumed the habit of a genuine philosopher. Besides these, Zothicus, a critic and poet, was conversant with Plotinus, who amended the works of Antimachus, and rendered the Atlantic history very poetically in verse: but after this he became blind, and died a short space of time prior to Plotinus. Zethus too, was very familiar with our philosopher, who derived his origin from Arabia, and married the wife of one Theodosius, the familiar of Ammonius. He was deeply skilled in medicine, and very much beloved by Plotinus, who endeavoured to dissuade him from engaging in the administration of public affairs. Such indeed was his familiarity with our philosopher, that, as we have already observed, Plotinus spent the last hours of his life at his rural retreat. Porphyry likewise informs us, that not a few senators were the sedulous auditors of Plotinus. Philosophy indeed, as it is the most noble and liberal of all pursuits, ought never to be separated from noble birth and exalted rank. It is naturally allied with every thing great, and is calculated to confer dignity, even on greatness itself. It exalts the majesty of the monarch, stamps nobility with true grandeur, and raises the plebeian to immortality. In the age of philosophy, therefore, we cannot wonder that she was reverenced by the senators of Rome. That illustrious body, even at this declining period, retained a portion of its ancient independence; and the generous ardor of unbounded liberty was not yet extinguished by the frozen hand of despotic usurpation. The Roman manners and religion were not yet destroyed; and nobility was not contaminated by the sordid occupations of traffic. *Meekness* was not esteemed a virtue, nor *merchandise* an honour!!! Among this illustrious body of men, Marcellus Orontius diligently applied himself to philosophy, and made rapid advances in its attainment. This too, was the case with Sabinillus, and above all with the senator Rogatianus\*. So deeply enamoured was this last nobleman with the charms of wis-

\* This Rogatianus is the person Porphyry alludes to in his Treatise on Abstinence, lib. i. p. 106, when he speaks as follows. "There was once an instance, where a negligence of terrene concerns, and a contemplation and intuition of such as are divine, expelled an articular disease, which had infested a certain person for the space of eight years. So that at the very same time, that his soul was divested of a solicitous concern for riches, and material affairs, his body was freed from a troublesome disease."

dgm,

dom, and the discourses of Plotinus, and so attentive to the care of separating his soul from his corporeal life, that he neglected his wealth, and secular affairs, dismissed his servants, and rejected the dignities of the state. Hence, when he was chosen prætor, and the lictors waited for his appearance, he neither came into public, nor regarded the duties of his office, nor dwelt in the house allotted for his reception : but he supped and slept with certain of his friends and familiars, and gave himself to absolute retirement in the day. By this negligence and carelessness of life (says Porphyry), from being so vehemently afflicted with the gout, that he was obliged to be carried in a chair, he resumed his pristine strength and vigour. And from being so diseased in his hands, that he could not extend them when necessary, he so recovered their use by philosophic endurance, as to employ them with greater expedition than the manual mechanic. This great man, as we may suppose, possessed the principal place in the esteem of Plotinus, who was not sparing in his praise of so uncommon a character, and proposed him as an illustrious example to the pupils of philosophy. Happy Rogatianus ! who could relinquish power for knowledge, and prefer the perpetual inheritance of wisdom to the gaudy splendors of title, and the fleeting honours of command. Alexandrinus Serapion too, was one of his associates, who was once a rhetorician, but afterwards, gave himself to philosophical disputations ; though, shameful to relate, he was at the same time a slave to usury, and avarice. Besides all these (says Porphyry), he reckoned me a native of Tyre, among his most friendly adherents, whom he appointed to correct his writings.

The following particulars relative to composition are related by Porphyry of this extraordinary man. He could by no means endure to review twice what he had written, nor even to read his composition, through the badness of his sight. But while he was writing, he neither formed the letters with accuracy, nor exactly distinguished the syllables, nor bestowed any diligent attention on the orthography : but neglecting all these as trifles, he was alone intent to the intelligence of his wonderful mind ; and, to the admiration of all his disciples, persevered in this custom to the end of his life. To a man of mere words, Plotinus will doubtless appear inexcusable for such *important* omissions : but to the sublime and contemplative genius, his negligence will be considered as the result of vehement conception, and profound cogitation. Such, indeed, was the power of his intellect, that when he had once conceived the whole disposition of his thoughts from the beginning to the end, and had afterwards committed them to writing, his composition was so connected, that he appeared to be merely transcribing from a book. Hence he would discuss his domestic affairs without departing from the actual intention of his mind ; and at the same time transact the necessary negociations of friendship, and preserve a perpetual intelligence of his thoughts. In consequence of this uncommon power of intellec<sup>t</sup>ion, when he returned to writing, after the departure of the person with whom he had been conversing, he did not review what he had written, owing,

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as we have observed, to the defect of his sight ; and yet he so connected the preceding with the subsequent conceptions, as if his composition had never been interrupted. Hence he was, at the same time present with others, and with himself, so that, as Porphyry observes, the self-converted energy of his intellect was never remitted, except perhaps in sleep, which he very moderately indulged. And so vigorous and frequent was the conversion of his soul to intellect, that he would often abstain from bread, swallowed up, as it were, in the depths of contemplation.

Several women too, enamoured with the love of wisdom, were the auditors of Plotinus. The Platonic philosophy, indeed, as it necessarily combines truth with elegance, is naturally adapted to captivate and allure the female mind, in which the love of symmetry and gracefulness is generally predominant. Hence, in every age, except the present, many illustrious females have adorned the Platonic schools, by the brilliancy of their genius, and an uncommon vigour and profundity of thought. This too, would doubtless be the case in our own country, if all the works of Plato and his disciples were but once faithfully and elegantly translated into English : but till the obstacle of Greek is removed, we may in vain expect thinking females \*, and I had almost said Platonic philosophers among men. Porphyry adds, that many men and women of noble birth, when at the point of death, delivered up, and commended their children and all their substance to Plotinus as to a sacred and divine guardian. Hence, says he, you might see the house of Plotinus full, both of young men and virgins, among the number of which was one Potamon, whom he educated with diligence and care. Nor was he wearied in hearing the procurators of his pupils, often rendering an account of their administration ; nor did he disdain to pay attention to their expences, affirming, that as they did not yet philosophize, they ought to possess their own goods, and to receive, without detriment, an increase of their estate. Yet though he procured for so many pupils the chief necessaries of life, the intellectual energy of his soul while he was awake, never suffered any interruption from externals, nor any remission of vigour. He was extremely mild, though not *meek* †, in his manners, and was easy of access to all his adherents and friends. Hence, so great was his philosophic urbanity, that though he resided at Rome twenty-six years, and had been the arbiter of many litigious causes, which he amicably dissolved, yet he had no enemy throughout that great and illustrious city. This last circumstance, indeed, reflects the highest honour on the philosophic character of Plotinus ; but, at the same time, some merit is due to the age in which he fortunately lived. Had he been destined to make his appearance in the present times, unsupported by fortune, and with no other recommendation than an uncommon great-

\* I have, however, the happiness of being intimate with a lady, who is a noble exception to this remark ; and is both an excellent Greek scholar, and skilled in the Platonic philosophy.

† For this is alone the virtue of traffic ; and is the chief support of its professors.

ness of mind, and an unequalled depth of thought; from being despised, insulted, and distressed, he must surely have been indignant, though not morose, and severe though not agitated with wrath. He would have been scornful without pride, contemptuous without weakness, patient without servility, and solitary without affectation. He would have lived without notice, wrote with success, and died without regret. But born to a happier fate, his genius was not doomed to languish in the shades of obscurity, but attained to the blossom of perfection in the sun-shine of philosophy, and through the liberal pains of Grecian and Roman cultivation.

But though Plotinus was thus universally esteemed at Rome, and in general by all who had the happiness of his acquaintance, yet he had one vehement enemy in the person of Alexandrinus Olympius, who had been for a short time the disciple of Ammonius, who desired to arrogate to himself the chief place in philosophy, and endeavoured to render Plotinus the object of general contempt. So deadly, indeed, was his hatred of our philosopher, that he attempted to invade him, by drawing down the baneful influences of the stars. The attempt was, however, vain, and its effects noxious to their author. For the sidereal defluxions, instead of being hurtful to Plotinus, were reflected on Olympius. Hence he exclaimed to his companions, "that the soul of Plotinus possessed such a mighty power, that it immediately repelled malignant influences directed upon his person, on the authors of the evil." But Plotinus, when Olympius first machinated his sidereal enchantments, was conscious of his design, and said to his friends; "Now, the body of Olympius is contracted like a purse, and all his members are bruised together." After Olympius, therefore, had often found to his own detriment, that the baneful influences intended for Plotinus were repelled on himself, he desisted from such base and fruitless undertakings. Indeed, says Porphyry, Plotinus naturally possessed something greater than the rest of mankind, which the following extraordinary relation abundantly evinces. A certain Egyptian priest, who at that time visited Rome, and who became suddenly known to Plotinus by one of his friends (perhaps Porphyry himself), desirous to exhibit his wisdom in that illustrious city, persuaded our philosopher to attend him, for the purpose of beholding, through his invocations, his familiar dæmon; to which request he readily consented. But the invocation was performed in the temple of Isis; this being the only pure place in Rome the Egyptian priest was able to find. However, instead of a dæmon, as was expected, a god approached, who was not (says Porphyry) in the genus of dæmons. The Egyptian astonished at the unexpected event, exclaimed, "Happy Plotinus, who hast a god for a dæmon, and whose familiar attendant does not rank among the inferior kind!" But this extraordinary and delightful vision was of short duration: for the priest affirmed it was not then lawful to interrogate any thing, nor any longer to enjoy the vision, because a certain common contemplative friend, who was present at the spectacle, suffocated

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some birds which he held in his hands for the sake of safety, either impelled by envy, or terrified with fear. As Plotinus, therefore, was allotted a dæmon belonging to the diviner orders, the divine eye of his soul was perpetually elevated to this guardian deity. On this account he composed a book *concerning every man's familiar dæmon*, in which he diligently endeavours to assign the causes of the diversity subsisting among these attendants on mankind. As a still farther proof of his uncommon greatness of mind, Porphyry adds, that when Amelius who was an observer of sacred rites, in which he officiated, according to the Roman calends, once requested Plotinus to attend him in the discharge of these religious ceremonies, he replied, "It becomes them to approach to me, and not me to them." But from what conceptions (says Porphyry), he spoke in such an exalted manner of himself, we were unable to conceive, and afraid to ask. We may, however, presume, that Plotinus meant to insinuate the high degree of purity and perfection of his intellectual part, which rendered him so superior to the use of corporeal sacrifices, and the cultivation of material deities, and dæmons, that he ought rather to be propitiated by others, than to propitiate himself. For a soul like his, was, indeed, to use his own expression, *ὕριστος θεός, a posterior god,* ready winged for flight, and scarcely detained by the fetters of body. This I know will pass for great arrogance and presumption among the philosophers of the present day, who consider *meekness* and *humility* as the highest ornaments of their nature, and the truest characteristics of genuine worth. But surely a sublime and godlike soul can never think *meanly* of its nature, or be willing to suppress and extinguish the inevitable consciousness of its own dignity and elevation. Humiliating conceptions flourish no where but in the breasts of the servile, or the base; and are the ornaments of no characters, but those of the impotent and the mean. Their influence is baneful to the advancement of science, and destructive of all genuine excellence and worth. They damp the glowing ardour of true theology, curb the celestial flight of philosophy, and blast the vigorous blossoms of genius. Let it, however, be remembered, that while we banish meekness, we are by no means the advocates of arrogance and conceit; but are alone desirous of vindicating the proper dignity of the worthy soul, and of rescuing its generous and ardent confidence from the frigid embraces of humiliating opinion. It is one thing to be *modest*, and another to be *meek*: for the former is the shadow attendant on genius, inseparable from its progress, and the symbol of its reality; but the latter is the dæmon of traffic, the inspirer of its projects, the support of its credit, and the harbinger of its appearance. It flies from the face of genius like the shadows of night before the beams of the morning, and, terrified at the approach of the elevated mind, hides itself in the dark retreats of trembling pusillanimity. But to return from this digression: Plotinus appears to have possessed an unequalled skill in physiognomy, as the following circumstance eminently evinces. A lady, named Chion, who, together with her daughters resided in his house, and there

happily passed a chaste widowhood, was fraudulently deprived of a very valuable necklace. In consequence of this, all the servants and domestics were summoned into the presence of Plotinus, who regarded their several countenances, selected one, and accused him of the theft. The man was immediately chastised, and for a long time denied the fact, but at length confessed his guilt, and restored the necklace. In a similar manner (says Porphyry) he wonderfully predicted the destiny of the young men of his acquaintance ; as of one Polemo, he foretold, that he would be very much addicted to love, and not arrive to the maturity of his age, which happened according to his prediction. But the last instance of his sagacity, related by Porphyry, excels all the rest, both in the singular skill displayed in its execution, and the happy consequences it produced. Porphyry, as we are informed by Eunapius, in his life, on his first acquaintance with Plotinus, bade a final farewell to all his preceptors, and totally applied himself to the friendship and confidence of this wonderful man. Here he filled his mind with science, and drew abundantly, without satiety, from the perennial fountain, seated in the sanctuary of the soul of Plotinus. But afterwards being conquered, as it were, by the magnitude of his doctrines, he conceived a hatred of body, and human nature, and could no longer endure the fetters of mortality. "Hence (says Porphyry) I formed an intention of destroying myself, which Plotinus wonderfully perceived, and as I was walking home, stood before me, and said, *Your present design, O Porphyry, is not the dictate of a sound intellect, but rather of a soul raging with an atrabilious fury.* In consequence of this, he ordered me to depart from Rome, and accordingly I went into Sicily, particularly when I heard that a certain worthy and elegant man dwelt at that time about Lilybæum. And by this means, indeed, I was liberated from this perturbation of soul, but was in the mean time hindered from being with Plotinus till his death."

But the great reputation of this divine man was not confined to the senate and people of Rome, for the emperor Galienus, and his wife Salonina, honoured his person, and reverenced his doctrine. Indeed, so highly was he esteemed by the emperor, that, relying on his benevolence, he requested that a city in Campania, which had been formerly destroyed, might be restored, and rendered a fit habitation for philosophers ; and besides this, that it might be governed by the laws of Plato, and called Platonopolis. Had this design succeeded, Plotinus intended to have dwelt there with all his disciples, and to have realized the beautiful republic, conceived by the godlike genius of Plato. The emperor, indeed, assented to his wishes, and the philosopher would have easily accomplished his intentions, if some of the emperor's familiars, impelled by envy or indignation, or some other unjust and selfish cause, had not warmly opposed its execution.

This extraordinary man (as we are informed by Porphyry) was strenuous in discourse, sagacious in invention, and prompt in the most opportune perceptions : but he was frequently incorrect in his speech, as well as in writing ; and this most probably owing to the

the vehemence and vigour of his conceptions. Besides this, while he was engaged in discourse, his intellect beamed through his corporeal frame, and diffused over his countenance its intimate light. He was, indeed, of a most beautiful aspect, but when he disputed (says Porphyry) he seemed far more lovely to the view. Then a placid gentleness appeared in receiving questions ; and a vigour uncommonly robust was demonstrated in their dissolution. When Porphyry once had interrogated him for three days, *by what means principally the soul was united with the body*, he persevered in demonstrating the manner of its conjunction. And when a certain person, named Thaumasius, entered his school, for the purpose of discussing common questions in philosophy, and premised that he wished to hear the explanatory sentences of Plotinus, but that the questions and answers of Porphyry were by no means adapted to a disputation of this kind, Plotinus replied : *unless we dissolve the doubts arising from the interrogations of Porphyry, we shall not be able to comment any thing, in an uninterrupted series of discourse.* But he wrote with a most intense acuteness of thought, and an abundant intellect. His writings are remarkably sententious, and he abounds, every where, more with profundity of sense than copiousness of words. " He poured forth (says Porphyry many things agitated by the impulse of inspiring deity ; and was often wonderfully affected with the object of his investigation." With me indeed every page of his works is a *volume*, and every sentence an *oracle*. The latent dogmata of the Stoics, and Peripatetics, are inserted in his books ; and more particularly the Sentences of Aristotle posterior to his Physics. He was ignorant of nothing pertaining to geometry, arithmetic, optics, and music, though he had never reduced these sciences to practice. The commentaries of the Platonic philosophers, Cronius, Numenius, Gaius, Atticus; as also of the Peripatetics, Aspasius, Alexander, Adrastus, &c. were read in his schools : but nothing was repeated from these in an uninterrupted series. For his conceptions were entirely his own ; and his contemplations were different from theirs. In interpretation, and the discussion of questions, he bore the intellect of Ammonius. As soon as he was sufficiently imbued with reading, and had given, in a short discourse, sentences full of profound contemplation he arose, and left the school. Having once read the book of Longinus concerning principles, he said, *that Longinus was indeed a philologist, but by no means a philosopher* ; and this indeed, as it appears to me, by a necessary consequence : for the knowledge of words is entirely foreign from the study of things. When Origen (not the Christian father of that name) once came into his school, Plotinus whose cheeks were covered with blushes, wished to rise, and being solicited by Origen to continue his discourse, he replied " that discourse ought to cease, when he who speaks perceives he addresses himself to those who are well acquainted with his doctrine." And thus after a short dissertation he arose from thence.

When in the celebration of Plato's nativity, Porphyry recited a poem *on the sacred marriage*\*, and a certain person who was present objected that Porphyry was mad, because many things were said in the poem mystically, and inspired by a divine fury, Plotinus openly exclaimed, " You have shewn yourself at the same time both a philosopher and a priest." On a certain time too an orator, named Diophanes, read an apology for the intoxicated Alcibiades in the banquet of Plato, endeavouring to prove that it was proper for the sake of learning virtue that the lover should expose himself to the object of his attachment, and not even refuse venereal congress. But while he was reading this licentious defence, Plotinus often rose from his seat, as if he would suddenly leave the assembly : but he restrained himself till it was finished. However, when he left the company, he commanded Porphyry to confute the oration. But when Porphyry desired the orator to lend him his discourse, for this purpose, and was refused, he answered him from recollection, and delivered his answer in the presence of the same auditors as had attended Diophanes. On this occasion Plotinus was so much rejoiced, that he often repeated in the assembly

" Thus write, and you'll illuminate mankind †."

Our philosopher too, applied himself to the rules of astronomy, though (says Porphyry) not according to a very mathematical mode. That is, as we may presume, he very little regarded the calculation of eclipses, or measuring the distance of the sun and moon from the earth, or determining the magnitudes and velocities of the planets : for he doubtless considered employments of this kind as more the province of the mathematician than of the profound and intellectual philosopher. The mathematical sciences are indeed the proper *means* of acquiring wisdom, but they ought never to be considered as its *end*. They are the bridge as it were between sense and intellect, by which we may safely pass through the night of oblivion over the dark and stormy ocean of matter, to the lucid regions of the intelligible world : and he who is desirous of returning to his true country will speedily pass over this bridge, without making any needless delays in his passage. Plotinus also diligently applied himself to the judgments of the astrologers ; but when he found that their predictions were not worthy of belief, he often confuted their presages in his writings.

At that time there were many Christians, and likewise some heathens, who forsaking the ancient philosophers, became the followers of Adelphius and Aquilinus. These men circulated a variety of books of Alexander, Libycus, Philocomus, Demostratus,

\* Concerning the ἱεροὶ γάμοι, or ho'ly marriages, which were first celebrated by the Orphic Theologers, see the note to the hymn to Proserpine, in my translation of the Orphic Initiations.

† A line somewhat altered from Homer. The original is θάλλ' εὐως μίκτη φίως Δαναῶσι γίνεται. Iliad. 1.1. 282.

Lydus;

Lydus; and openly exhibited certain revelations of Zoroaster, Zostrianus, Nichoteus, Allogenes, Mefus, with others of a similar kind. By this means they deceived many, and were themselves deceived, asserting that Plato had by no means penetrated the depth of an intelligible essence. On this occasion, Plotinus urged many arguments in his disputation against these impostors, and composed a book in confutation of their tenets, inscribed, *against the Gnostics*, leaving a farther discussion of their errors to the labours of his disciples. Hence Amelius composed forty books against the book of Zostrianus; and Porphyry shewed by a variety of arguments, that the writings which they attributed to Zoroaster, were adulterated and recent, and were composed by the propagators of the heresy, that their institutions might pass for the genuine doctrines of the ancient Zoroaster.

Many of the Greeks (says Porphyry) falsely accused Plotinus of privately usurping the doctrines of Numenius, which calumny Tryphon, a Stoic and Platonist, told to Amelius. On this occasion Amelius composed a book, inscribed by Porphyry; *Concerning the difference between the dogmata of Plotinus and Numenius*, which he dedicated to Porphyry. Every one of the books, indeed, of this great man bear such evident marks of original thought, and singular depth, the execution in each is so similar, and the conceptions so uncommonly abstruse, that no one can understand his meaning, and believe him indebted to the labours of others. Porphyry adds, that he was likewise considered by as many as a mere trifler, and treated with contempt, because says he they could by no means comprehend his sayings. Besides the manners of Plotinus contributed to produce and increase this disdain, for he was foreign (says Porphyry) from all sophistical ostentation, and pride; and conducted himself in the company of disputants, with the same freedom and ease as in his familiar discourses. With the superficial and the vain, a haughty carriage and severe aspect are considered as the badges of wisdom: but nothing in reality is more foreign from its possession. For true wisdom when it is deeply possessed gives affability and modesty to the manners, illuminates the countenance with a divine serenity, and diffuses over the whole external form an air of dignity and ease. Add to this, that Plotinus did not hastily disclose to every one, the syllogistic necessities, which were latent in his discourse. "The same thing (says Porphyry) happened to me, when I first heard Plotinus. On which account I endeavoured to provoke him, by writing against him, endeavouring to shew that *intellections are not external to intellect*." But after the writings of Porphyry on this subject were read to Plotinus, he said, smiling; "It must be your employment Amelius, to dissolve these doubts, occasioned by his ignorance of our opinion." After Amelius, therefore, had composed no small book against the objections of Porphyry, and Porphyry had again contradicted his writings, and was once more answered by Amelius: "At length (says Porphyry) having scarcely after all these attempts fathomed the depth of Plotinus, I changed my opinion, wrote a recantation of my error, which I recited in a general assembly, considered the books of Plotinus ever after as most worthy of belief, and provoked my master by every possible

possible means to disclose his opinion in a more particular and copious manner." This relation is a most egregious instance of the unequalled profundity, as well as excellence of Plotinus's writings: for who will presume to question the merit of composition, which was at first so difficult to be comprehended, and afterwards so greatly admired by such a genius as Porphyry? By a genius equally accurate, elegant and profound, who knew how to combine the Graces with Philosophy and Science, and to adorn the majestic brows of truth with the flowers of enchanting elocution.

But the testimony of the celebrated Longinus concerning our philosopher, sufficiently evinces his uncommon excellence and worth; and in the present age will probably be more esteemed than the eulogium of Porphyry. In a letter, therefore, which he wrote to Porphyry desiring him to come from Sicily into Phoenicia where he resided, and to bring with him the books of Plotinus, he writes among other things as follows: These books (meaning those of Plotinus) are not moderately faulty, so that I have no means of using them, though I desire above measure to consider what Plotinus has written concerning the soul, and on being." And again: "Do not send these books but bring them with you, and not these alone, but any others which may have escaped the notice of Amelius. *For why should I not enquire, with the greatest diligence, after the writings of this man, which deserve the highest honour and veneration?* This indeed I have always signified to you, both when present and absent, and when you resided at Tyre, that I could not understand many of the hypotheses of Plotinus's books, *but that I immoderately loved and reverenced the manner of his writing, the density of his conceptions, and the very philosophic disposition of his questions.* And indeed I judge that the investigators of truth ought only to compare the books of Plotinus with the most excellent works."

This testimony of Longinus is the more remarkable, as, prior to this, he had for a long time despised our philosopher, through the ignorant aspersions of others. The wonderful genius of Plotinus, was indeed so concealed under the garb of modesty, that before fame had announced this worth, it was only visible to a penetrating and sagacious few. But Longinus (says Porphyry) thought the works of Plotinus which he had received from Amelius incorrect, through the fault of the transcribers, because he was unacquainted with his usual elocution: for if any, the books in the possession of Amelius were correct, because they were transcribed from the manuscripts of Plotinus. Porphyry has likewise preserved the preface of a book composed by Longinus, inscribed, concerning the end, and dedicated to Plotinus and Amelius, in the course of which he thus speaks of our philosopher." "*Plotinus and Gentilianus Amelius are replete with a copiousness of propositions, which they studiously discuss, and have seriously chosen the employment of writing, using a mode of contemplation peculiar, and their own. And Plotinus indeed, as it seems, has more certainly explained the Pythagoric and Platonic principles than his predecessors. For the writings of Numenius, Cronius, Moderatus, and Thrasyllos, are not to be compared, for accuracy in any part, with the books of Plotinus on the same subjects.*"

If

If such then is the decision of Longinus concerning the abilities and writings of this extraordinary man : of Longinus who is celebrated by one of our first poets, *as inspired by all the Nine*; and whose literary reputation is universal; what judgment must we form of the philosophic taste of the present age, when we find that the very name of Plotinus is known but to a few, and his works scarcely to any? The inference is obvious: let the reader draw it, and lament. But (says Porphyry), if it be requisite to employ the testimony of the wife, *who is wiser than a god?* Than a god, who truly said of himself:

- \* The number of the sands is known to me,  
And the broad measure of the mighty sea.  
I know the thoughts within the dumb conceal'd,  
And words I hear, by language unreveal'd.

And this is no other than Apollo, who when Amelius enquired, where the soul of Plotinus had emigrated, answered in divine numbers, as follows:

To strains immortal full of heav'nly fire,  
My harp I tune well strung with vocal wire;  
Dear to divinity a friend I praise,  
Who claims those notes a god alone can raise.  
For him a god in verse mellifluous sings,  
And beats with golden rod the trembling strings.  
Be present Muses and with general voice.  
And all the powers of harmony rejoice;  
Let all the measures of your art be try'd,  
In rapt'rous sounds, as when Achilles dy'd:  
When Homer's melody the band inspir'd,  
And god like furies every bosom fir'd.  
And lo! the sacred choir of Muses join,  
And in one general hymn their notes combine.  
I Phœbus in the midst to whom belong  
The sacred pow'r's of verse, begin the song.  
Genius sublime! once bound in mortal ties,  
A dæmon now and more than mortals wife;  
Freed from those members that with deadly weight  
And stormy whirl inchain'd they soul of late:  
O'er lifes rough ocean thou hast gain'd that shore,  
Where storms molest, and change impairs no more;

\* In the original. Οὐδὲ δὲ ἵνα τάξιμον, οὐ μετρα βαλασσονε,  
Καὶ παρθὲν ἔνιημι, καὶ δὲ λαλεστος, εἴπαν.

## RESTORATION OF THE

And struggling through its deeps with vig'rous mind,  
 Pass'd the dark stream, and left base souls behind.  
 Plac'd where no darkness ever can obscure ;  
 Where nothing enters sensual and impure ;  
 Where shines eternal minds unclouded ray,  
 And gilds the realms of intellectual day.  
 Oft merg'd in matter by strong leaps you try'd,  
 To bound aloft, and cast its folds aside ;  
 To shun the bitter stream of sanguine life,  
 Its whirls of sorrow, and its storms of strife.  
 While in the middle of its boist'rous waves,  
 Thy foul robust, the deeps deaf tumult braves ;  
 Oft beaming from the gods thy piercing sight,  
 Beheld in paths oblique a heav'ly light :  
 Whence rapt from sense with energy divine,  
 Before thy eyes immortal splendors shine ;  
 Whose plenteous rays in darkness most profound,  
 Thysteps dire sted and illumin'd round.  
 Nor was the vision like the dreams of sleep,  
 But seen while vigilant you brave the deep ;  
 While from your eyes you shake the gloom of night,  
 The glorious prospects burst upon your sight :  
 Prospects, beheld but rarely by the wise,  
 Tho' men divine, and fav'rites of the skies..  
 But now set free from the lethargic folds,  
 By which th' indignant soul dark matter holds ;  
 The natal bonds deserted, now you soar,  
 And rank with Dæmon forms a man no more.  
 In that blest realm where love and friendship reign,  
 And pleasures ever dwell unmixed with pain ;  
 Where streams ambrosial in immortal course  
 Irriguoous flow, from Deity their source.  
 No dark'ning clouds those happy skies assail,  
 And the calm æther knows no stormy gale.  
 Supremely blest thy lofty soul abides,  
 Where Minos and his brother judge presides ;  
 Just Æacus, and Plato the divine,  
 And fair Pythag'ras there exalted shine ;

With

With other souls who form the general choir,  
 Of love immortal, and of pure desire ;  
 And who one common station are assign'd,  
 With Genii of the most exalted kind.  
 Thrice happy thou ! who life's long labours past,  
 With holy Dæmons dost reside at last :  
 From body loosen'd, and from cares at rest,  
 Thy life perpetual, and divine thy feast.  
 Now ev'ry Muse who for Plotinus sings,  
 Here cease with me to tune the warbling strings ;  
 For thus my golden harp with art divine,  
 Has told, Plotinus ! endles blis is thine.

" According to this oracle then (says Porphyry) Plotinus was worthy, and mild, gentle and endearing, and such as we found him to be by invariable experience. And again, it asserts that he was vigilant, endued with a purified soul, and always elevated to divinity, which he ardently loved. Likewise that he endeavoured by exerting all his powers to emerge from the bitter waters of this sanguine life. Hence when by the assistance of this blessed light, he had often raised himself by intellectual conceptions, to that first god who is superior to intellect, and had ascended according to all the gradations in the banquet of Plato to an union with his ineffable nature, this supreme principle suddenly appeared to him, neither possessing any form, nor any idea, but established above intellect, and every intelligible essence. And to this supreme god I Porphyry once approached, and was united with his nature, when I was sixty-eight years of age. The end of life therefore appeared to Plotinus : for the end and scope of existence to him, was a conjunction with that deity who is every where present. But he four times obtained this end, while I resided with him, not in capacity, but by an ineffable energy. Besides the oracle adds, that the gods often surrounding Plotinus with divine splendors, directed him in the right path, while they benignantly extended to his eyes abundant rays of celestial light : so that he may be said to have composed his books from the contemplation, and intuition of divinity. But from internal and external vigilance, he is said by the oracle, to have seen many and most beautiful spectacles, which no other philosopher has easily beheld. For human contemplation may indeed have various degrees of excellence, but when compared with divine knowledge, cannot fathom a depth, such as is penetrated by the gods. Hitherto the oracle shews, what were the energies of Plotinus, and what he obtained, while surrounded with body. But after his solution from body, it declares that he arrived at the blessed society, where friendship,

sweet desire, joy, and love united with the deity, perpetually reign. Besides this, how the sons of the divinity, Minos, Rhadamanthus, and Æacus, are appointed the judges of souls ; and that Plotinus departed to these, not for the purpose of receiving their decisions of his conduct, but to enjoy their conversation, with whom also other gods of the most exalted order converse. Where Plato and Pythagoras reside, and other sublime souls, who compose the choir of immortal love ; and where the most blessed dæmons have fixed their abode. And lastly, that the life of the inhabitants, in these celestial regions is ever flourishing and full of joy, and perseveres in perpetuity of bliss, from the benignant communications of divinity." And thus much for the life of Plotinus, who was a philosopher unequalled for the strength and profundity of his intellect, and the purity and elevation of his life. He was a being, wise without the usual mixture of human darkness, and great without the general combination of human weakness and imperfection. He seems to have left *the orb of light*, solely for the benefit of mankind ; that he might teach them how to repair the ruin contracted by their exile from good, and how to return to their true country, and their proper kindred and allies. I do not mean that he descended into mortality for the purpose of enlightening the vulgar part of mankind : for this would have been a vain and ridiculous attempt. The splendour of truth cannot be apprehended by eyes totally fixed in the dark night of oblivion ; but previous to this, punishment must be inflicted, and purgation employed ; the labours of Hercules must be accomplished, and the sufferings of Ulysses endured. But he came as a guide to the liberal few, who are struggling to gain the lost region of light, but know not how to break the fetters by which they are detained ; and who are impatient to leave the obscure cavern of sense, where all is delusion and shadow, and to ascend to the realms of intellect, where all is substance and reality.

Let us now consider what were the principal tenets of the Platonic theology, which this extraordinary man restored, and illustrated in his writings. And, in the first place, he every where profoundly and copiously proves the superefficient nature of *the one*, or the supreme principle of things, which is one of the principal doctrines in the Parmenides, and is more plainly asserted in the republic \*. However prior to Plotinus †, the interpreters of Plato ascended no higher than to intellect and being, and by this means placed a compound, and not a perfectly simple nature at the summit of the universe. This doctrine, which is called by Cudworth, *high-flown, phantastical, and unsafe*, will, I am sure, be deemed no better than *jargon and reverie*, by modern philosophers, who, so far from being able to conceive a cause superior to being, scarcely possess a thought which is not the progeny of body and sense. It is, however, sufficient for our

• Lib. 6-

† Vide Procl. in Plat. Theol. p. 89, et 90.

present

present purpose to prove that this is the opinion of Plato, and to shew how it is defended by Plotinus. In the sixth book of the Republic then, Plato most beautifully evinces the superefficient nature of the first cause, whom he calls the good, by his analogy to the sun, as follows. "Have you not observed, with respect to the author of the senses, in how perfect a manner he has formed the power of sight, and of being visible? I have not entirely perceived it, replied he. But consider it in this manner. Do hearing and sound require any other species, in order that the one may hear, and the other be heard, which third nature when absent, the one shall not hear, nor the other be heard? There is nothing, said he. It appears to me, indeed, that many others (that I may not say none), require no such third species. Or are you able to find any such power? By no means. Have you not perceived that sight, and the object of sight, require such a nature? How? When sight is present with the eyes, and they are directed to vision, when colour also is present, unless a third species is present, naturally formed for the purpose, the sight will be without vision, and the colours will be invisible. But what do you call this? What you call light. You speak the truth. Indeed, the sense of seeing, and the power of being seen, are joined together by a bond the most honourable of all other conjunctions, and by no trifling form, if light be not dishonourable. Whom then of the celestial gods can you assign as the cause of this, that light causes our sight to see in the best manner, and that objects are perceived by the eyes? The same as you and others assign; for you interrogate concerning the sun. But the sight is thus affected with reference to this god. How? Neither is the sight the sun, nor is the eye in which vision resides the sun. It is not. But of all the organs of sense, the eye participates most of the sun. Greatly so. Does it not preserve the power which it possesses, as infused from the sun? Entirely. Besides, the sun is not sight, but its cause, and is on this account beheld by sight. It is plainly so. This is what I called the son of the good, which the good generated analogous to itself: that as *this* in the intelligible place, is to intellect and the objects of intelligence, so is *that* in the visible place to sight. How is this? Explain yourself more largely. You know that the eyes as often as they are not turned towards objects whose colours the splendour of day irradiates and discloses, but to such as are faintly illuminated by nocturnal rays, grow dim by the vision, and appear almost blind, as if perfect sight was not resident in their nature. So it happens. But as often as they are turned to objects which the sun illustrates, they perspicuously perceive, and in the very same eyes, sight appears to be contained. It is so. Thus think also concerning the soul. For when it adheres to that in which truth, and being itself shines forth to view, then it understands and knows this, and appears to possess intelligence. But when it is carried to that which is mingled with darkness, which is generated and destroyed, the sharpness of its sight is blunted, it is conversant with various opinions, and it seems to be destitute of intellect. So it appears. Hence, that which affords truth to objects of intelligence, and extends the power of intellection to him who under-

understands, you may call the idea of the good, the cause of science, and truth perceived by intellect. But since these two are so beautiful, I mean knowledge and truth, if you esteem the good itself, as something different from these, and far more beautiful, you will think in a proper manner. And, as it is proper to believe, that light and sight possess a certain form of the sun, but are by no means the sun itself : so here it is proper to judge, that knowledge and truth possess a certain form of the good itself, but are by no means the good ; for its majesty is far more venerable and august." And a little after he adds : " You may say therefore, that the good, not only affords to objects of knowledge the power of being known, but likewise distributes their being and essence, while in the mean time *the good itself is not essence, but above essence*, excelling it both in dignity and power."

It is plain, therefore, from the words of Plato himself, that he considered the supreme principle of things superior to being ; and consequently this doctrine was not devised by the latter Platonists, contrary to the opinion of their divine master. But this is likewise evident from the testimony of Speusippus, the immediate successor of Plato, who, as we are informed by Proclus\* confirmed this doctrines from the most ancient authority, and asserted, " that the ancients considered *the one*, as better than being, and that the principle of being was free from all proportion to the subsequent order of things, as the good itself is separated from every condition of any particular good." To this most respectable evidence we may also add, that of the philosopher Moderatus, who, as we are informed by Simplicius†, declares, " that, according to the Pythagoreans (from whom Plato, it must be observed, received the greater part of his philosophy), the first one is above all essence."

This sublime theory was supported by Plotinus, with all that truly philosophic accuracy and depth, for which his writings are every where so remarkable. Indeed, it appears to have been his favourite topic ; for he has employed considerable parts of many of his books in its illustration and defence. Nor can we wonder at his partiality for this exalted speculation, if we consider that a union with this ineffable nature, was the great aim of all his desires, and the only end of all his studies and pursuits. This was the divinely solitary light to which his intellectual eye was ever directed, and which so abundantly illuminated the most secret recesses of his soul. Here he discovered the true fountain of good, and drank deep of its perennial streams. And lastly, here he derived those inestimable stores of knowledge, which he so fortunately transmitted to future generations. That the English reader, therefore, may have a specimen of his imitable writings on this abstruse subject, and may see some of the deepest mysteries of

\* This passage is cited by Ficinus, in the second volume of his works, p. 1186. and is, I doubt not, taken from the manuscript Commentary of Proclus on the Parmenides.

† ἡτοι γέ (i. e. Moderatus) κατὰ τὸν Πυθαγόρεως τὸ μὲν ἀφεῖται ἐπί τὸν καὶ πᾶσαν φύσιν αὐτοπεποίησα. Simp. in Ar. Phys. fol. 50.

the

the Greek theology disclosed, I shall present him with a paraphrased translation of two books of Plotinus : the first of which is inscribed, *That Intelligibles are not external to Intellect, and concerning the Good*, and the other, *Concerning intelligible Beauty*. I have particularly chosen these, not only because they admirably unfold the depths of the Platonic philosophy and theology ; but because the first relates to the vision of the *supreme*, explaining the wonderful manner in which it is accomplished ; and the second describes the method of becoming united with the intelligible world. The Platonic reader will find in these books (if I have done justice to their divine author), instances of sublimity beyond all comparison with any other writings ; and specimens of a profundity of thought unequalled by any other philosopher.. I am sensible that the great labour I have employed in the translation, will be, most probably, lost on the present generation : but though I write with no views nor desires of popular renown, yet I flatter myself with the approbation of more equitable posterity. The fifth book, therefore, of the fifth Ennead of Plotinus is as follows :

“ Is it possible any one can think that true intellect, possessing true being, can at any time be deceived, and believe in things which have no real existence ? Certainly no one. For how could it be intellect, if it is ever liable to deception ? It is requisite, therefore, that it should always understand, and that nothing should ever be concealed from it, like those natures that are subject to oblivion.. But it is likewise necessary that knowledge should reside in its essence, not like one imagining or doubting, or deriving information from another. Nor yet again, like knowledge collected from demonstration.. For though it is granted that some things are collected by demonstration, it cannot likewise be denied that something is of itself known to intellect, at the same time that reason dictates, that all knowledge is essential to its nature. But it is now necessary to enquire after what manner we must distinguish the essential knowledge of intellect, and that which it obtains by investigation.. Likewise from whence the certainty is derived to intellect, of its essential knowledge ? From whence its faith is derived, that it is in such a condition ? Because about things offered to the senses, the belief of which appears more certain, it is usual to doubt whether they possess their apparent nature in the subject things, or in certain passions only ; where certainly the judgment of intellect, or, at least, of thought, is required.. For though it should, perhaps, be granted, that the natures of sensible objects are contained in their subject bodies, yet what is known by sense, is nothing more than an image of the object ; for sense cannot apprehend the thing itself, since it abides external to its perception. But intellect when it understands and apprehends intelligibles, if it knows these as something different from itself, after what manner is it connected with them ? For it may happen that it shall not meet with them, and consequently that it may not understand : or perhaps then at last when it meets with them it will immediately understand, and thus it will not always possess intellect. And if it should be said, that intelligibles are conjoined with

with

with intellect, it remains to enquire what such a conjunction means. Besides the intelllections themselves will be certain figures : and, if this is the case, they will be adventitious, and nothing more than certain pulsations. But after what manner will intellect be figured ; and what will be the form of intelligibles ? Lastly, from this hypothesis intelligence will be like sense, a perception of externals. After what manner then do these disagree among themselves ? Shall we say in this, that one of them comprehends lesser concerns ? Also, how can intellect know that it perceives something in reality ? Or how will it be able to judge that this is good, or beautiful, or just ? For every one of these will be different from intellect, nor will it contain the principles of judging by which it believes, but these also will be external to its essence ; and in the same manner truth. Again, intelligibles themselves, are either destitute of sense, life and intellect, or they possess intellect. If they possess intellect, they will equally contain both, and this will be the true and first intellect. But of this also we enquire how it contains truth, intelligible itself, and intellect. Whether subsisting in the same and together, or in some other manner ? But if intelligibles themselves are destitute of intellect and life, we must enquire what they are. For they are neither certain propositions, nor axioms, nor dictions. For if this were the case they would affirm something of other things, but would not be things themselves : as if they should say, that what is just is beautiful, when at the same time justice itself is different from the beautiful itself. But if they should consider as simple essences, the just itself, and the beautiful itself, apart from each other : in the first place, intelligible itself will not be a certain one ; but every intelligible will be separate from others. In which case we must enquire where they are, and in what places they are separately disposed : afterwards in what manner intellect every where running round in a discursive procession, is able to find these : also how it abides : and again, how it abides or perseveres in the same ; and what form or figure it is endued with. Unless, perhaps, intelligibles are situated like certain images formed from gold, or from some other matter by a statuary or painter. But if this be the case, intellect in its perceptions will be the same as sense. Besides in what respect among these, is *this* intelligible, justice, but *that*, something else. Lastly, this is the most powerful objection of all : viz. if any one should entirely admit, that these are extrinsical, and that intellect speculates them as having an external position, it necessarily follows that intellect does not possess the truth of these, but is deceived in the contemplation of each. For the object of its contemplation will be truly external : it will therefore behold them deprived of their intimate possession, and containing only their images in a knowledge of this kind. Since, therefore, it does not possess truth itself, but only contains certain images of truth, it will possess what is false, and have nothing of truth. If then it knows that it contains only what is false, it must undoubtedly confess itself to be destitute of truth : but if it is ignorant of this, and thinks that it participates of truth, when at the same time it is destitute of its possession, it is deceived by a twofold fallacy, and is very far distant

from

from truth. For it is on this account, as I think, that truth is not to be found in sensible objects but opinion alone : because opinion is conversant in receiving, from whence its name is derived. On this account it receives something different from itself, since that also is different from which it possesses what it receives. If then truth is not resident in intellect, such an intellect cannot be truth, nor a true intellect, nor intellect at all : nor indeed will truth be resident in any other place.

Hence it is not proper either to investigate intelligibles separate from intellect, or to confess that the figures of things only are contained in intellect, or to deprive it of truth, while we admit it is ignorant of intelligibles, and that the objects of its intellection have no existence in the order of things. But it is necessary to attribute all things to true intellect, if it is requisite to induce knowledge and truth ; to preserve beings themselves ; and that knowledge by which the essence of every thing is known ; and no longer to acquiesce in the resemblances and images of things, as when we alone understand the particular mode of existence, and not the real essence of a thing ; in this case neither possessing the object itself, nor dwelling with it, nor conspiring into one with its nature. For intellect indeed truly knows, nor is any thing concealed from its essential intelligence, nor is it liable to oblivion, nor does it wander by investigation, but it contains truth, and the seat of things in its essence, and is ever vital and intelligent. All which properties, indeed, ought to reside in the most blessed nature, or where can any thing honourable and venerable be found ?

Hence it neither requires demonstration, nor the faith of persuasion, that intellect is thus essentially intelligent : for it is entirely manifest to itself, and there is nothing more worthy of faith than its own essence. So that it contains real truth, not consonant to any other but to itself, nor does it pronounce and exist any thing besides itself : and that which it is, it pronounces. Who then can confute it ? And from whence can he bring his confutation ? For the argument which is adduced must revolve into the same with the former. And although it is employed as different, it is nevertheless referred to the thing proposed by the first argumentator, and is with it entirely one and the same. For nothing can be found more true than truth.

This one nature intellect therefore is all beings : it is truth : it is a great deity : or rather it is not any particular god : but is deservedly every deity. And such is the nature of this second divinity, appearing to beholders, before they survey that superior God, who is seated in sublimer majesty on the illustrious throne of intellect, depending from his ineffable nature. For it is highly proper that he should not subsist in an inanimate seat, nor again immediately occur to us moving in the circular chariot of soul, but that an inestimable beauty should wonderfully shine before his appearance, as before the presence of a mighty king. For to such as advance to his intuition it is obtained

that lesser things should first occur and afterwards that such as are greater should gradually present themselves to the view; and that such as surround the king should be more royal, and the rest in a degree proportionate to their distance from his ineffable glory. But after all these, the mighty king himself, suddenly shines forth to the view: while the rest venerate the king, in a suppliant manner; such I mean as do not depart from thence till they have proceeded to the last spectacle of all, like those who are satisfied with the splendor of the attendants on majesty. Another king, therefore, reigns in this intelligible world, and his attendants are different from his nature. But this supernal king does not rule over foreign subjects, but he possesses a just and natural government, and a true kingdom: since he is himself the king of truth, and is naturally the lord of his offspring the universe, and of the divine company of immortal gods. Hence he is the king of a king, and of kings, and is called by a juster name, the father of the gods. Whom indeed Jupiter in this respect imitates, since he does not acquiesce in the contemplation of his Father, but proceeds beyond this to his grand-father, as to an energy in the very subsistence (*ὑπόστασις*) of his essence.

But let us now ascend to the one itself, which is indeed truly one, not like other things which at the same time that they are many, are also one through the participation of unity. For we must now receive one itself, which is not one by participation, like such things as are not more truly one than many. We must likewise assert that the intelligible world is more one than other things, and that nothing is nearer than this to unity itself: at the same time that it is not purely one.

But for the present we desire to contemplate, if possible, the nature of the pure and true one, which is not one from another, but from itself alone. It is therefore here requisite, to transfer ourselves on all sides to one itself, without adding any thing to its nature, and to acquiesce entirely in its contemplation; being careful lest we should wander from him in the least, and fall from one into two. But if we are less cautious we shall contemplate two, nor in the two possess the one itself; for they are both posterior to unity. And one will not suffer itself to be numerated with another, nor indeed to be numbered at all: for it is a measure free from all mensuration. Nor is it equal to any others, so as to agree with them in any particular, or it would inherit something in common with its connumerated natures; and thus this common something, would be superior to one though this is utterly impossible. Hence neither essential number, nor number posterior to this, which properly pertains to quantity can be predicated of one: not essential number whose essence always consists in intellection; nor that which regards quantity, since it embraces unity, together with other things different from one. For the nature pertaining to number which is inherent in quantity, imitating the nature essential to prior numbers, and looking back upon true unity, procures its own essence,

essence, neither dispersing nor dividing unity, but while it becomes the duad, the one remains prior to the duad, and is different from both the unities comprehended by the duad, and from each apart. For why should the duad be unity itself? Or one unity of the duad rather than another, be one itself? If then neither both together, nor each apart is unity itself, certainly unity which is the origin of all number, is different from all these; and while it truly abides, seems after a manner not to abide. But how are those unities different from the one? And how is the duad in a certain respect one? And again, is it the same one, which is preserved in the comprehension of each unity? Perhaps it must be said that both unities, participate of the first unity, but are different from that which they participate: and that the duad so far as it is a certain one participates of one itself, yet not every where after the same manner: for an army, and a house are not similarly one; since these when compared with continued quantity, are not one, either with respect to essence, or quantity. Are then the unities in the pentad, differently related to one, from those in the decad? But is the one contained in the pentad, the same with the one in the decad? Perhaps also if the whole of a small ship, is compared with the whole of a large one, a city to a city, and an army to an army, there will be in these the same one. But if not in the first instance, neither in these. However, if any farther doubts remain, we must leave them to a subsequent discussion.

But let us return to unity itself, asserting that it always remains the same, though all things flow from it as their inexhaustible fountain. In numbers, indeed, while unity abides in the simplicity of its essence, number producing another is generated according to this abiding one. But the one which is above beings, much more abides in ineffable station. But while it abides, another does not produce beings, according to the nature of one: for it is sufficient of itself to the generation of beings. But as in numbers the form of the first monad is preserved in all numbers, in the first and second degree while each of the following numbers do not equally participate of unity; so in the order of things, every nature subordinate to the first, contains something of the first, as it were his vestige or form in its essence. And in numbers, indeed, the participation of unity produces their quantity. But here the vestige of one gives essence to all the series of divine numbers, so that being itself, is as were the footstep of ineffable unity. Hence he who asserts that *το ενα*, which is a denomination declarative of essence, is derived from *το ον*, that is, *one*, will not perhaps deviate from the truth. But that which is called *το ον*, that is, *being*, first of all shining forth from the depths of unity, and as it were not far proceeding from thence, is unwilling to advance beyond its original, but abides converted to its most interior retreats, where it becomes essence, and the essence of all things, and that which pronounces these; containing itself as it were in its labouring with sound; and declaring by its speech that it flows from one: and indeed *το ον* thus pronounced, signifies its origin as much as possible. So that what becomes *ενα*, that is *essence*, and *ενα* or *το ον*,

imitate to the utmost their author, from whose unwearied power they perpetually flow. But intellect perceiving this, and being moved by the spectacle, and imitating what it knew, suddenly produces with an energetic voice the words *I, thou, now, &c.* For these sounds endeavour to express the substance of that which is generated, (the pronouncing nature labouring with the expression;) and imitate as much as possible the origin of being itself.

But this must be left to every one's particular determination. But since generated essence is form, (and that which is produced from thence can have no other appellation,) it is not a particular form, but universal, so that nothing else than this general form remains to species; and therefore it is necessary that one itself should be destitute of form. But since it is foreign from species neither can it be essence: since it is requisite that essence should be something determinate. But it is not lawful to consider unity itself as any thing particular and bounded, otherwise it would not be the principle, but that alone which you denominate something singular. If then all things are contained in that nature which is generated from the first, we must truly say that the author of all things, is not any one of these, and that he can alone be called that which is above all. But the natures produced from thence are beings, and being itself; and hence the one itself is superior to being. And that which is above being, does not say I am this, nor does it determine any thing concerning its nature, nor does it tell its name, but it alone pronounces, *I am not this*, i. e. *I am nothing comprehensible and definite.* But it is impossible by this means, to comprehend its nature: since it is ridiculous to attempt to comprehend immensity itself. So that whoever attempts it, removes himself far from the least vestige of this nature. For as he who desires to know intelligible essence, then only perceives what is above sense, when he possesses no image of a sensible object: so he who desires to contemplate a nature superior to intelligible essence, will enjoy the ineffable vision, if he neglects every thing intelligible, while merged in the most profound and delightful of all contemplations; learning from hence, that he is, but neglecting the enquiry into what he is, as impossible to investigate. For this which is called *such*, signifies when applied to him, not such: since the appellation of such cannot belong to a nature, to whom the predication *what*, is not applied. But we labouring as it were with our difficulty of conception, are ignorant what denomination is proper to his nature, and desiring as much as possible to signify something to ourselves give a name to that which is ineffable. But perhaps this name which is called one derives its appellation from a certain negation of many. On which account the Pythagoreans denominated him Apollo, according to a more secret signification, which also implies a negation of many. But if any one establishes this name one, and affirms something according to its signification, both the name and the thing named will be more obscure than if its appellation had been entirely neglected. For perhaps the name was expressed that the investigator beginning from something signifying the greatest simplicity

city

city of all might arrive at that perfection of contemplation, as even to deny him the appellation of one ; convinced that the best name indeed had been assigned him, but that it was unworthy to express the superlative excellence of his nature. For this cannot be reached by the hearing, nor be understood by any bearer : but if it is manifest to any one, it must be to the profound beholder. But if he who perceives, endeavours to behold form; he will lose the intuition of this ineffable nature.

Again, the energy of vision is twofold, as it happens with respect to the eye. For one thing, indeed, is a spectacle to the eye, that is, the form of the sensible object, but another, that by which it perceives the form, and which though itself sensible, is different from the sensible form. Hence it is the cause by which form is beheld, is inherent in form, and is perceived connected with its nature : though on this account it is not clearly perceived, since the eye more intently directs itself to the illuminated object than to the illuminating cause. But when there is nothing besides itself, it is beheld with a sudden and universal vision, though it should then be perceived adhering to some other object : for if it was entirely separate and alone, it could not be subject to sensible inspection ; since the light of the sun flourishing in the sun itself, would perhaps escape our sense, unless its more solid orb was the subject of its splendor. But if it should be said that the whole sun is light, it is perhaps only asserted for the sake of explanation : for light is in no form of other visible objects, and is perhaps nothing else than that which is visible \*, while other things are visible, but not light alone ; since their natures are

various

\* In the note to page 14, of this volume, we have shewn from Proclus that light is place, and is an immaterial body. Indeed that light is something superior to sensible matter may, I think, be evinced by the following consideration. As the supreme principle of the universe, (who can be compared to nothing so properly as light,) is the light of the intelligible world, and is at the same time more exalted than every thing which it contains ; so in this sensible, which is the image of the intelligible world, it is necessary that corporeal light, should be more excellent than any material nature. But as Proclus prosecutes this speculation on light, in the above mentioned passage, in a most admirable and uncommon manner ; let us attend him in the abstruse investigation. In the first place speaking of that light which emanates from the fountain soul of the world, and which, according to the Zoroastrian oracle,

"Ἄρδιν ἡμέτερα φῶς, φῶς, αἰθέρα, κίστην,

abundantly animates, light, fire, ether, and the world ; he says that this light is one fire, above the three others, viz. the empyrean, ethereal, and material fire ; and that it first unfolds the eternal quiet of the gods, and illustrates essential spectacles to such as are worthy of their inspection. For by the assistance of this light, things destitute of figure and impression, are impressed according to reason and intelligence. And perhaps (says Simplicius) he calls light place, because it is a certain description and type of the universe, and gives distance to things, which without the presence of light would not appear to be distant. But here Proclus very philosophically doubts against his own position. In the first place, how body can be received and penetrated by body. In the second place, whether place, i. e. light, is inanimate, or participates of soul. But (says he) it cannot be inanimate, because it is better than the animated natures which it contains, and because rational beings are said to be animated. Hence this body, will be animated the first of all others. But if it is animated, how can it be immovable ? And he dissolves the first doubt, because there are some immaterial bodies void of affection and passion. For (says he) a body void of matter neither resists nor is resisted ; since that which is impelled and repelled, is eminently passive in such operations. But neither can such a body be divided, since it is void of passivity.

various and composite. In like manner the eye of intellect, sees from another light things illuminated by that first nature, and in them it truly sees their illuminating source. But when it too earnestly converts itself to the nature of the illuminated objects, it perceives less their splendid original. And if at any time it should dismiss the visible objects, and attentively survey the light by which it perceives, it will then view light itself, and the principle of light. But because it is requisite that intellect should behold a light of this kind, not as any thing external ; let us return again to the example of the corporeal eye, which on a time does not perceive external and foreign light, but previous to this beholds a light more peculiarly its own, and by far more lucid, shining in a certain inviolate and pure seat ; either when it perceives before itself a ray darting from its transparent receptacle, through the darkness of night ; or when not disposed to behold other objects, it confines itself under the covering of the eye-lids, and in the mean time produces from itself a purer light within ; or lastly, when some one by pressing the corners of his eye-lids, views the inward light of the eye. For then, indeed, by not seeing he sees, and then sees in the most exalted degree ; for he views light itself : while other things which were the objects of this vision before, were indeed lucid, without being

vity. Hence neither can the absurdity be adduced, that the universe is received by a minimum, or penetrates a minimum. For if it cannot be divided, neither can it be equally divided with the smallest ; and if this is impossible, neither can it be permeated by the universe.

But he removes the second objection, by asserting, that this body, i. e. light, is animated by that soul, which is the fountain of others ; and that it possesses a divine life, and an essence self-motive, but not in energy. For if we assert that soul is self-motive, in a two-fold manner, in one respect according to essence, but in the other according to energy ; we must likewise affirm that place, or light is partly moveable, and partly immoveable. For what should hinder our asserting, that place participates of such a life, and that it lives according to an immutable essence, and not according to a self-motive energy ? But if you are desirous (says he) of considering the motion of place, according to energy, you will plainly perceive, that it is, as it were, the mover of the bodies which are moved, and which turn their parts according to a certain distance : since these cannot be every where, and are incapable of being present to all the parts of place, according to each of their parts. And this is that intervention and affinity, which it possesses with soul moving without dimension. For, life so far as life, appears to produce motion : and since place is that which first participates of life, motion to place, will confer a true motion on every part ; and will produce a desire in every part of the moved body of arriving at its whole, and since it is not able to accomplish this, on account of the natural property of interval, of subsisting divisibly in the universe. For whatever desires any nature, which it is not able to reach, through its peculiar defect, is then more enflamed in the pursuit of that, which it cannot possess through the imbecillity of its nature. For it is requisite (says he) that between an incorporeal and immutable life, such as that of soul, the fountain of the rest ; and one corporeal and mutable, that one immutable and corporeal should intervene as a proper medium. And thus much from Proclus concerning place or light, whose reasonings appear to me equally elegant and philosophical, subtle and profound.

But the opinion of the Phoenicians respecting light, as preserved by the emperor Julian in his elegant oration to the sun, is no less admirable than that of Proclus ; and at the same time asserts its immaterial nature. " According to the Phoenicians (says Julian) who are skilled in divine science and wisdom, the universally-diffused splendor of light, is the sincere energy of an intellect perfectly pure : i. e. of the solar intellect, which as Julian expresses it, scattering its light from the middle region of the heavens, fills all the celestial orbs with powerful vigour, and illuminates the universe with divine and incorruptible light." With great reason, therefore, does Plotinus assert, that light is nothing else than that which is visible : for this must be the necessary property of a nature, possessing a triple dimension without sensible matter.

light.

light. In like manner intellect concealing and separating itself from all other concerns, and confining itself in its most inward retreats, and perceiving nothing, will immediately behold light, not subsisting in another, but by itself alone, perfectly pure, and suddenly shining from itself, with a splendor ineffably sacred and divine.

But in this case it will be doubtful from whence such a light shines ; whether from something external, or rather from an internal source : and again when it departs we may happen to say, this was something intimate, and again not intimate. But, indeed, it is not lawful to enquire from whence it originated, for it neither approached hither, nor again departs from hence to some other place, but it either appears to us, or does not appear. So that we ought not to pursue it, as if with a view of discovering its latent original, but to abide in quiet, till it suddenly shines upon us ; preparing ourselves for this blessed spectacle, like the eye waiting patiently for the rising of the sun, until appearing above the horizon, and emerging, as the poets say, from the bosom of the ocean, he presents himself to the sight. But from whence does this light which the sun imitates supernally shine ? And what is the nature which it transcends, when it perspicuously presents itself to our view ? Indeed it illuminates intellect, intently surveying its lustre. So that intellect steps itself in beholding, as having now arrived at the desired end of its vision, looking upon nothing else than the beautiful itself ; converting itself wholly to its contemplation, and dedicating itself entirely to its enjoyment. Hence abiding in this delightful state, and as it were replete with divine vigour, it beholds itself in the first place now become more beautiful and resplendent, as being nearer to that which is highest and best. But he will not approach in the manner some may expect ; since he will come as if not coming. For he will be present before and above all things, even before intellect approaches to the vision. But it is intellect which properly approaches and departs ; which departs indeed when it is ignorant where it should abide, and where this divine principle abides : because indeed it truly abides in no being. And if intellect could be no where (I do not mean with respect to place only since this also is free from the affections of place) but entirely no where, it would doubtless always behold his divinely solitary nature, although it would become united with him, not as perceiving, but as abiding in his nature ; and this not as if intellect and this highest principle were two. But now because it is intellect, it thus sees when it sees, by that which it contains different from intellect, and which is the very summit and flower of its essence. And, indeed, it is wonderful in what manner this first god is present without approaching, and how while he is no where, he is at the same time every where. This indeed is wonderful from its very condition, but to him who profoundly knows the thing itself; it would rather be admirable if the contrary should be affirmed. Or rather, indeed, it cannot exist otherwise than as the object of vehement admiration. For such is the nature of the supreme.

What--

Whatsoever is produced by another, is either contained in its author, or in some other nature, if any thing besides its author remains: for since it is produced by another, and requires something different from itself to its generation, it every where requires another nature for its support, and consequently repose in another, from the necessary indigence of its being. And thus it is appointed by nature, that such things as are last, should be established in such as are immediately above them: and again things prior to these, in such as are similarly prior, and always one thing in another up to the first principle of all. But the highest principle, because he has nothing prior to his nature, cannot subsist in any other. And hence because he is not in another, but others subsist in their superiors, on this account he comprehends all things in the immensity of his nature. But while he embraces them, he is not dissipated into their essence, since he contains them without being contained; yet in this case, there is nothing exists, with which he is not present: for unless he was present he could not contain: and again, if he did not contain, he could not be present. So that he is present, and yet not present: for, because he is not comprehended by any thing, he is by no means present; but because he is free from all circumscription, he is not hindered from being present every where: for if he were restrained, he would certainly be defined by some particular being, and subsequent natures, would be left destitute of his presence; and thus far the first deity would reign, nor would any thing farther subsist in his nature, nor would he abide in himself, but become subservient to others. Whatever, therefore, subsists in any thing different from itself, is properly there, where it subsists. But such as are not any where, are on this account present every where. For whatever is excluded from some particular place, is comprehended in some other, so that it is false to affirm of such a nature that it is not contained some where. If then it is true that the supreme principle is not in any particular place, and false, that he is somewhere (lest he should be contained in another), he is on this account absent from no being or place. But if he is no where absent, certainly because he is not somewhere, he will be every where present in himself: for one part of him will not be here, and another there, nor yet the whole of him in one particular place only, so that he will be every where totally present; since no one being contains him, nor yet in another sense does not contain him, since he is so contained, that he may rather be said to contain. But in order to illustrate the present subject, let us consider our visible universe, for if there were no other world superior to this, it would neither be contained in the world, nor yet in place. For what place could there be prior to the existence of the world? But the parts of the world are reduced to the universe, and are placed in its comprehensive bound. And soul is not in the world, but rather the world is in soul: for neither is body the place of soul, but soul is in intellect, and body in soul. Lastly, intellect abides in another, which is no longer dependent on any thing superior, and in which it is compelled to repose: so that the highest principle is properly contained in

no other, and is on this account said to be no where. Where then do other things subsist? Doubtless in that which is first. Hence he is neither absent from others, nor is contained in them, while at the same time he contains all things in the immensity of his nature. Hence too, on this account he is considered as the good of the universe; because all things subsist by him, and are referred to him as their divine original. But they are so referred to him, that some are more excellent than others, because some are more proximate than others to his ineffable nature.

But let me intreat you, not to endeavour to perceive him through the medium of other natures, for otherwise you will not discover the highest principle himself, but only a vestige of his divinity. But consider with yourself what that is, which can alone be perceived abiding in itself, perfectly pure and unmixed, and which is of such a kind that all things participate yet none contain its nature; so that nothing else can be such as he is, and yet it is necessary that such a nature should subsist. What being then can at once apprehend the whole of his power? For if any one apprehends the whole, in what respect does he differ from his nature? Must he be received then according to a part? But you who are intent on beholding him, should survey him with a universal vision, and at the same time be cautious not to tell yourself the whole of your perception, or you will become intellect, intelligent: but he will immediately fly from your intuition, or rather you will retire from him. But when you behold, behold him totally; and when you energize with intellect concerning him, whatever you retain in your memory of his nature, be careful to understand it as the good. For he is the cause of a wise and intellectual life; since he is that power itself, from which life and intellect is produced; and he is the author of essence and being, because he is the one itself. And he is perfectly simple, and the first, because he is the principle of all. For all things flow from him as their original source; motion first proceeded from him, yet is not contained in his nature; station likewise originates from him, because he is superior to want: for he is neither moved, nor at rest, since he contains nothing in which he can either repose or revolve. For about what, or to what, or in what can he either be moved, or repose, since he is the first? But neither can he be defined, for what can bound his nature? Nor yet again, is he infinite, like an immense bulk. And where can he be said to advance, as if he were indigent, who is in want of nothing? But his power contains infinity itself. Nor is he ever deficient, since beings who are superior to defect derive this perfection from the inexhaustible plenitude of his nature.

But this infinite is so called, because it is not more than one, and because it does not contain any thing, by which any part as it were of its nature can be bounded. Indeed, from its being one, it is neither measured, nor proceeds into number; and therefore is neither terminated by another, nor by itself: for if this were the case it would become two. Nor again has it any figure, because it has no parts, nor form. Do not, therefore,

seek after its ineffable vision with mortal eyes ; nor attempt to perceive by any corporeal means, that which reason proves to be so remote from the comprehension of sense. Do not, I say, think it can be known in the manner they imagine, who consider all things as sensibles ; and thus entirely subvert that which *Is*, in the most exalted degree. For those things, which some consider as having the most real being, have the most unreal. And that which is great in quantity is least in being : but that which is first is the principle of being, and something more excellent than essence ; so that our opinion must become the very opposite to this, or we shall be destitute of the union with this most exalted deity. Just as those who in solemn festivals, through a shameful gluttony, fill themselves with food which it is unlawful for those to touch who intend an entrance to the gods ; esteeming the aliment of the belly more certain than the contemplation of the god whose rites are to be celebrated, and on this account they depart destitute of the sacred visions. For in such holy rites when the god is not beheld, his existence is denied by those who consider as alone certain that which is tasted and perceived by the flesh. Just as if any one should be lost in sleep through the whole of life, and should therefore believe in the visiages of sleep, as alone certain and real. But if any one happens to rouse him, as one who does not believe in objects beheld with open eyes, should suddenly return again to sleep, and the delusions of dreams.

Again, it is requisite for the purpose of perceiving, to assume that organ by which each particular ought to be beheld. The eyes for some, the ears for others, and so of the rest. And it is necessary to believe, that other things are the peculiar objects of intellect, and that to understand is not the same as to hear and to see ; for this would be as absurd as if any one should command the ears to perceive, and should on this account deny the existence of voices, because they are not the objects of sight. Hence we must consider such as these ignorant of that which from the beginning to the present day they desire and affect : for all things desire that which is first from a necessity of nature, prophesying, as it were, that they cannot subsist without the incomprehensible energies of his nature. Besides the knowledge of beauty, happens to such souls as are roused and knowing ; and is attended with a stupor, and the excitation of love. But good, because present from the beginning to our innate appetite, abides with us even when asleep, and never seizes its spectators with astonishment, because it is always present, and requires no peculiar reminiscence to convince us of its presence. But the love of beauty, when it first offers itself to the view, produces molestation, because it is requisite to seek after beauty by knowledge : but a love of this kind since it is the second, and belonging to those who are intelligent, plainly indicates that beauty is itself the second ; and the desire of good, since it is more ancient, and does not require the assistance of the senses, testifies that good itself is more ancient than beauty, and is superior to its nature. Add to this, that all beings think they shall be sufficient to themselves, if they obtain good ; as if secretly convinced they shall then at length arrive at the desired end : but all do not think the pos-

session

session of beauty, will be sufficient to the completion of their wishes. Besides some judge that what is beautiful, is beautiful to itself, but not to them, as is the case with this our apparent beauty. For they judge that its possessor is beautiful; and consider it sufficient to appear beautiful though deprived of its real possession: but they do not desire to possess good in opinion, but in reality. For all things especially strive to procure for themselves that which is first; and contend with beauty, as it were with a desire of victory, as if conscious it was generated, as well as themselves. Just as if some one posterior to a king, should study to equal in dignity another who immediately follows the king, and is the next to him in royal pre-eminence; because he depends on one and the same principle as his rival, being ignorant, indeed, that he himself depends on the king, but that the other precedes him in priority and perfection of nature. But the cause of the error is their both participating of the same; and one itself being prior to both. Besides it appears that good itself is by no means indigent of the beautiful, but the beautiful cannot subsist without the good. Hence good is gentle, mild, placid, delicate, and such as every one wishes it to occur. But beauty either renders the soul stupid, or mingles the excited pleasure with grief. Lastly, it often causes incautious souls to deviate from good, as the beloved object often separates the lover from his parent. For beauty is of a junior nature, but good is more ancient, not indeed in time, but in truth, because it possesses a prior power: for it possesses universal power. But that which is subordinate to the good, does not receive all power, but such only as it is requisite for a nature posterior to the first, and originating from him to receive. So that he is the lord of this posterior power, and is in no respect indigent of his offspring, the beautiful, since he existed such as he is prior to its generation; and would have suffered no loss in the perfection of his nature, if this had not been generated. And if some other could be produced from his nature, he would not envy it the possession of being. But now nothing farther can be generated: for nothing remains, which has not been already produced, since the universe is complete. But this highest principle is not all things, for in this case he would be indigent of all: but surpassing all things, he is able to produce and permit all things to themselves; while, at the same time, he is eminently exalted above all by the incomprehensible dignity of his nature.

But since the supreme principle is good itself and not merely good, it is requisite he should contain nothing in himself, since he does not even contain good. For if he possessed any thing, he would either possess good, or that which is not good: but in that which is properly the first good, non-good, can have no subsistence; nor yet can good itself contain good. If then it neither possesses non-good, nor good, it contains nothing; and if it contains nothing it is alone, dwelling in solitary unity, retired from the universality of things. If then other natures are either good (yet not good itself); or, perhaps, such as are non-good, but he contains neither of these, certainly by possessing nothing he is good itself. If then any one adds to his nature either essence, or intel-

left, or beauty, by such an addition he deprives him of being the good itself. As on the other hand by taking away all things, and affirming nothing concerning his nature, nor deceiving in any respect, as if something was present with his nature, we shall permit him to be what he is ; testifying concerning him none of these properties of being, which are not present with a cause so sublimely remote from essence itself.. In which respect those for the most part err, who, when they are ignorant how any one ought to be praised, detract from the glory of the subject of their praise, while they add such things to his nature, as are beneath its dignity ; not knowing how to accommodate true praise to its proper object.. On this account we ought also, in the first place, to beware, lest we add any thing posterior, and unworthy the divine object of our praise ; and to observe that he who surpasses all these; is, indeed, their proper cause without possessing any of their properties and affections. For the nature of good does not consist in being either all things, or some one particular of all. Since, if he was some one particular of all, he would be contained under one and the same nature together with all. But if he is under one and the same nature together with others, he will vary from others, only by a certain proper difference and addition. Hence, in this case, he will be two and not one ; one part of which two, I mean that which is common to it with the rest, will be non-good : but the other will be good. He will, therefore, be mixed from good and non-good, and consequently will not be the pure and first good. But that will be the first good, of which this participating becomes good beyond the common condition. This then will be good by a certain participation : but that of which this participates, will be none of the universality of things ; and such, therefore, must be the condition of the good itself. But if this too contains good as a part, for it is difference by which this is a composite good ; it is necessary that this should depend on another, which is entirely simple, and alone good. And hence this which is various depends upon that which is good alone. So that it appears, that what is first, and the good itself, is above all beings, is good alone, and contains nothing in its nature, but is perfectly free from all mixture ; and that it is above all, and is the divinely solitary cause of all. For neither does beauty nor being originate from evil, nor yet from such things as are indifferent : for the efficient is better than the effect ; since it is more perfect and divine." And this much for the first book of Plotinus, which we proposed to insert ; the other on intelligible beauty is as follows :

" Since we must confess that the soul which contemplates the intelligible world, and beholds the beauty of true intellect, may also perceive the father of this divine world, who is superior to intellect : let us now endeavour to the utmost of our ability to behold, and to express to ourselves (as much as such things can be expressed) how we may in the best manner survey the beauty of intellect, and the world which it contains. Suppose then, two stony masses placed near each other, one of which is incomposite, and destitute of artificial form : but the other is fashioned by art into some divine, or human statue. And if divine, let it be the statue of some Grace or a Muse : but if human, not that of any particular man, but rather of some one which

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art has collected together from all beautiful forms. The stone then which is disposed by art into the beauty of form, will immediately appear beautiful, but not because it is a stone ; or the other mass would be similarly beautiful ; it is therefore beautiful because it possesses the form which art applies. Matter, therefore, had not this form, but it existed in the thinking artist before it came into the stone. But it was in the artificer, not on account of his possessing eyes and hands, but because he was endued with art. This beauty, therefore, existed in art in a much more excellent manner. For the form itself which abides in art does not proceed into the stone, but this abides in indivisible union, while an inferior form proceeds from this, which neither remains in itself pure, nor is such as the artist wishes, but such as the subject matter is capable of receiving. But if art operates according to what it is, and to what it possesses, but it fashions beautiful forms, according to the reason by which it acts : hence reason is a much greater and truer beauty, since it contains the beauty of art ; and is greater and more excellent than every thing which proceeds into external form. For so far as form proceeding into matter is extended, so far it becomes more debile than that which abides in one. Since whatever suffers distance in itself, departs from itself, and the integrity of its nature ; whether it is strength diffused into some participant ; or heat, or power, or beauty extended to some subject, and divided about the fluctuating receptacle of matter. Again, every efficient according to itself, ought to be more excellent than its effect : for that which is unharmonious does not form a musician, but this is the work of harmony ; and that music which is above sense, produces the harmony in sensible sound. But if any one despises the arts, because they operate imitating nature, in the first place, it must be confessed, that natures also imitate other things : and in the next place, that arts do not simply imitate that which is perceived by the eyes, but recur to those reasons from which the energy of nature consists. Besides this, they produce many things from themselves, and add something where any thing is wanting to the perfection of the whole ; because they contain beauty in themselves. Lastly, Phidias himself fashioned his Jupiter, not by imitating any spectacle proper to the senses ; but conceiving the god such as he would appear, if he should be willing to exhibit himself to our eyes.

But for the present let us neglect the arts, and consider those beautiful natural effects, which art is said to imitate, i. e. all rational and irrational animals ; but especially whatever amongst these are more exactly finished : I mean where the Demiurgus ruling over matter, invests it with the form he desires it should participate. What then is beauty in these ? For it is not blood and menstrua, but colour and figure different from these ; or it is nothing ; or something destitute of figure ; or it is that which, as it were, contains something simple like matter. From whence arose the beauty of Helen, for which so great a contest ensued ? From whence shines the beauty of other forms similar to Venus ? And from whence did the form of Venus herself arise ? Or that of any man entirely beautiful, or of some god, whether they are among the number of things subject to our sight, or among those which are not subject, and yet have in themselves a conspicuous

spicuous beauty. Is not this every where form, descending into that which is produced by the artificer, in the same manner as it was said that the beauty of artificial figures, proceeded from the arts. What then? Are works beautiful indeed, and reason existing in matter? But is reason separate from matter, which exists in the soul of the agent, and which is first in dignity and rank, not beautiful, but is reduced into one with its subject matter? But if bulk is beautiful, so far as bulk, it follows that active reason, because it is not bulk, is not beautiful: though if form, whether contained in a small or in a large mass, moves and affects in a similar manner the mind of the beholder, certainly beauty is not to be attributed to the magnitude of bulk. Hence, so long as form is external to the soul, we do not perceive, and are not moved by its power: but when it is well conceived in the soul then it affects us with delight. Again, the form of things alone, flows through the eyes, otherwise the most ample figures could not penetrate through such narrow receptacles. But magnitude is contracted, not from its being great in bulk, but rather because great in species or form. Besides it is necessary that the cause itself of a beautiful effect, should be either deformed, or indifferent, or beautiful. If it is deformed, it cannot produce the contrary to deformity. If it is indifferent, why should it rather produce any thing beautiful, than deformed. But, indeed, it is necessary that nature the artificer of things so beautiful, should possess a beauty more primary and exalted. But with regard to us, when we behold nothing inward, and are entirely ignorant of internal beauty, we follow what is external, unconscious in the mean time that the cause of motion is profoundly latent in the depths of the soul; just like one, who on perceiving his own image, and being ignorant from whence it came, should follow its shadowy and unreal progression. But that there is something else which allures followers to itself, and that beauty does not consist in magnitude is sufficiently testified, by the beauty inherent in disciplines, offices, and the soul: where certainly a more true beauty flourishes; which is then manifest, when we contemplate the wisdom in a worthy mind, and are delighted with the contemplation, and in love with its beauty; not then surveying the corporeal face, which perhaps is not beautiful, but neglecting the whole form of the body and pursuing inward beauty to its most sacred and profound retreats. But if such a soul does not yet incite you to denominate it beautiful, neither on surveying yourself inwardly, will you be delighted with yourself as with something beautiful. Hence while so affected, you will vainly investigate true and intimate beauty: for you will seek after the purity of beauty, not with something pure, but with that which is base; and hence too, a discourse on things of this kind is not to be addrefsed to all men. Because if you behold yourself beautiful, you may obtain a reminiscence of beauty itself.

The reason therefore of the beauty contained in nature is the exemplar of the beauty appearing in body: but the exemplar of natural beauty, is a more beautiful reason contained in soul, from which the beauty of nature flows. But this shines brighter in a worthy soul, already advanced in beauty, than in nature herself: since it adorns such a soul,

soul, and affords a light, derived from one much greater; and which is no other than the first beauty. Thus abiding in the soul, it leads it to consider, what that superior reason of beauty may be, which is no longer generated nor placed in another, but abides perpetually in itself. Hence it is not reason, but the author of that reason which is first: since indeed the first reason is a certain beauty subsisting in soul as in matter. But its author is intellect, which is always the same, and not sometimes intellect; because intelligence does not happen extrinsical to this true and original intellect. But what image are we able to receive of such an intellect? For whatever is enquired after externally, is doubtless sought for from something worse than intellect. An image therefore of intellect must be obtained from intellect itself: so that we must not speak of it through the medium of an image; but we must receive a certain portion of gold, as a representative of universal gold. And unless this received gold is pure, we must purify it either in reality, or at least in our discourse; demonstrating that this which is received by us, is not universal, but only a particular portion of gold. Thus then let us ascend higher from our intellect now purified, to intellect itself; and let us begin with the gods themselves, contemplating the intellect which they possess. For all the gods are venerable and beautiful, and endued with an inestimable gracefulness. But what is the cause of such beauty? It is intellect, energising in the most exalted manner, which produces their divinely beautiful appearance. For it is not because their bodies are beautiful that they are gods, but from the possession of intellect, since the participation of body, is not essential to divinity. For they are not at one time wise, and at another time the contrary; but they are perpetually wise, with a tranquil, stable, and pure intellect, understanding all things, and knowing not human concerns properly, but their own, that is such as are divine, and such as intellect itself perceives. But the gods who inhabit this visible heaven, for they abound in divine leisure, assiduously contemplate, as if it were above them, what the primary and intelligible heaven contains. But those who are stationed in this higher world, contemplate its inhabitants possessing the whole of this diviner heaven. For all things there are heaven. There the sea, animals, plants, and men are heaven. Lastly every portion of this heaven is celestial. But the gods who reside there, do not disdain men, nor any other of its inhabitants, because every thing there is divine; and they comprehend the whole of this intelligible region attended with the most perfect repose.

Hence the life of these divinities is easy, and truth is their generator and nurse, their essence and nutriment: hence they perceive all things, not such indeed as are subject to generation, but such as abide in essence: they likewise perceive themselves in others. For all things are there perfectly perspicuous. Nothing there is dark, nothing opposing, but every thing is conspicuous to all, intrinsically and universally. For light everywhere meets with light. Each thing contains in itself all, and all things are again be-

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held in another. So that all things are every where, and all is all. There every thing is all. There an immense splendour shines. There every thing is great, since even what is small is there great. There the sun is all the stars; and every star is a sun, and at the same time all the stars. But one thing excels in each, while in the mean time all things are beheld in each. There motion is perfectly pure: for the proceeding motion is not confounded by a mover foreign from the motion. Station also there is disturbed by no mutation: for it is not mingled with an unstable nature. Besides beauty there is beauty itself, because it does not subsist in beauty. But every thing abides there not as if placed in some foreign land; for the being of each is its own stable foundation: nor is its essence different from its seat; for its subject is intellect, and itself is intellect. Just as if any one should conceive this sensible heaven, which is manifest and lucid to the eyes, germinating into stars by its light. In corporeal natures indeed, one part is not every where produced from another, but each part is distinct from the rest. But there each thing is every where produced from the whole; and is at the same time particular, and the whole. It appears indeed as a part: but by him who acutely perceives, it will be beheld as a whole: by him I mean, who is endued with a sight similar to that of the lynx, the rays of whose eyes are reported to penetrate the depths of the earth. For it appears to me that this fable, occultly signifies the perspicuousness of supernal eyes. Besides the vision of these blessed inhabitants is never wearied, and never ceases through a satiety of perceiving. For there is no vacuity in any perceiver, which when afterwards filled up, can bring intuition to an end. Nor can pleasure ever fail through the variety of objects; or through any discord between the perceiver and the thing perceived. Besides every thing there is endued with an untamed and unwearied power. And that which can never be filled, is so called, because its plenitude never spurns at its replenishing object. For by intuition it more assiduously perceives. And beholding itself infinite, and the objects of its perception, it follows its own nature as its guide in unwearied contemplation. Again, no life there is laborious, since it is pure life: for why should that labour, which lives in the best manner? But the life there is wisdom, a wisdom not obtained by arguments like ours, because it is always total, nor is in any part deficient, from which it might require investigation. But it is the first wisdom, not depending on any other; and essence itself is there wisdom; yet not in such a manner that essence is first, and then wisdom succeeds as secondary and an adjunct. Hence, no wisdom is greater than this, but there science itself is the associate of intellect, because they both germinate, and beam with divine splendors together: in the same manner as by a certain imitation they report that justice resides with Jupiter. For every thing of this kind exists there like a lucid resemblance perspicuous from itself, so as to become the spectacle of transcendently happy spectators.

The magnitude and power therefore of wisdom itself, is sufficiently evident from its containing with itself, and producing beings: for all things which are true pursue wisdom,

dom, depend on it for their being, originate together with it, and have one and the same essence: and lastly essence there is no other than wisdom itself. But we do not yet approach to this exalted knowledge, because we consider sciences, as certain speculations, and rules, and a flux of propositions, which indeed ought not with propriety to be attributed to the sciences we possess. But if any one doubts concerning our sciences, we must neglect the discussion for the present, at the same time affuming an occasion from hence, let us dispute concerning that science, which Plato beholding in the intelligible world says, that science there is not one thing in another. And this investigation will be proper to us, if we profess ourselves worthy an appellation of this kind.

Whatsoever is made by nature or art, is produced by a certain wisdom, and every where wisdom is the leader of action. But wheresoever a certain wisdom fabricates, there are indeed arts of this kind. But the artificer himself is again referred into natural wisdom, according to which art produces every work; not by being collected from speculations, but as one certain whole; nor as composed from many into one, but rather as resolving itself from one into many. If any one therefore places this wisdom as the first in intelligible dignity, it will be sufficient, since it does not originate from another, and does not subsist in any other essence. But if he should say that reason is placed in nature, and that the principle of this is nature, we must enquire from whence nature possesses reason. Because if it is said to possess it from another, we again enquire of that other; and if it possesses it from itself, our investigation is finished. But if they fly to intellect, there again we must enquire, whether intellect generates wisdom. And if they confess it does, we ask from whence? But if it conceives wisdom from itself, it could not accomplish this, unless intellect is wisdom itself. True wisdom therefore is essence, and true essence is wisdom; and the dignity of essence is derived from wisdom. For it appears that true essence originates from wisdom. Hence whatever things are destitute of the wisdom of essence; so far indeed as they are made by a certain wisdom, they are essences; but because they do not contain in themselves any wisdom, they are not true essences. No one therefore ought to think that in the intelligible world, either the gods themselves or any of its transcendently happy inhabitants, contemplate certain rules of propositions; but that each of the objects there contained, offers itself to the beholders, like a beautiful spectacle, such as may be imagined to exist in the soul of a man divinely wise. Not indeed like painted resemblances, but true beings shining with intellectual splendors: on which account the antients called ideas, beings and essences.

But the wise men of the Egyptians whether from a certain accurate science, or from natural instinct, when they determined to signify to us the mysteries of wisdom, appear to me not to have used figures significant of letters, discourses, and propositions, nor things imitating voices and axioms; but rather by describing and painting the par-

ticular images of particular things in their sacred concerns, to have occultly signified the discursive energy of the thing itself. For indeed every image is a certain science and wisdom ; it is likewise a subject ; and is a spectacle collected into one ; and is neither cogitation, nor counsel. But afterwards from this image, or wisdom collected into one, an evolved resemblance is produced in something else, speaking in a discursive transition, and finding out the causes why things are thus instituted : while the thing thus beautifully disposed, excites admiration. Hence it is said that he will admire wisdom, who considers how without containing the causes of her essence, she affords to others which are fashioned according to her nature, their particular mode of existence. This beautiful disposition of things then, which is scarcely manifest from enquiry, if any one should discover, he must own it requisite that in the intelligible world, things should subsist previous to all argument and enquiry, as in one great nature which harmonizes the whole.

Can we think that this universe, which we confess to be derived and to exist in this manner, from another, was so composed by its artificer, that he thought within himself concerning the earth ; and considered that it ought to rest in the middle ? And that afterwards he reasoned concerning the connection of water with earth, and the orderly disposition of things as far as to the heavens ? But in the next place concerning all animals, and such, and so many forms of particular vital beings, as they are at present ; and the disposition as well of the inward as of the external parts and members ? And lastly that he began to produce things in energy, as they were disposed in himself ? But such a consideration could not subsist with the artificer of the universe. For how could it take place in him, who had not as yet seen such things in existence ? Nor is it possible that he could fabricate, by receiving external assistance, after the manner of human artificers, who operate with hands and instruments : for hands and feet were posterior to his energy. It remains therefore that all things must subsist in their divine cause, and since no medium intervenes, that by the propinquity of being itself, to another, its image and similitude should as it were on a sudden shine forth, whether from itself alone, or through the ministry of soul. For it is of no consequence at present whether or not the world was fabricated properly through a certain soul, if it is but admitted that all things emanated from thence, and subsist there in greater beauty and perfection. For here they are mixt, but there they are pure. But this universe proceeding, from thence, is comprehended by forms from beginning to end. In the first place matter is the receptacle of the elementary forms, and of others in continual succession ; so that it is difficult to find matter, thus concealed under a multitude of forms. But since it possesses a certain ultimate form, it easily becomes the subject of every form. Hence since the exemplar of the universe is form, he produced all forms ; and this without any difficulty or violence, because the artificer there is a divine universe, and essence, and form. Hence too his fabrication.

fabrication was easy, and without labour: for there was no impediment; and on this account he now rules over his work with absolute dominion. And although some particulars are every where in opposition to others, yet they cannot now oppose the universal fabric, for it abides as the whole. Indeed I think if we were the first exemplars of things, and at the same time essences, and forms, and if the form which operates here was our essence, that our fabrication would rule without labour, though man as at present should fabricate a form different from himself. For becoming man he ceases to be the universe: but when he ceases to be man as Plato says, he raises himself on high, and governs the world. *For being made of the whole, he also makes the whole.* But that we may return to our design, you may indeed produce a reason, why the earth is placed in the middle, and why it is round; or why the zodiac is situated in a certain place: but in the intelligible world it was not deliberated so to be, because it was requisite; but rather because it is as it exists, on this account it is constituted as it ought: just as if previous to a syllogistic energy through causes, the conclusion itself should remain indubitably certain, without any propositions. For nothing there depends on consequences, nothing becomes certain from consideration: but it subsists prior to consequence, and all consideration. For all these are posterior, *reason, demonstration, faith.* Since on account of the principle all these exist, and are thus disposed. But it is rightly said that the causes of the principle are not to be sought after; especially of a perfect principle, which is the same with the end: for that which is both principle and end, is at the same time the whole, and perfect in every part.

Intellect itself, therefore, is the first beauty; it is total, and is every where total, without suffering a defect of beauty in any part. What then is the beautiful itself to be called? Certainly, not any thing which is not the whole itself, but either possesses a part only, or is entirely destitute of its participation. Indeed unless this is the beautiful itself, what else can merit this appellation? For that which is prior to intellect, does not will itself to be beautiful, but is something ineffably more excellent. Hence that which first presents itself to our view, because it is form, and a spectacle of intellect, is by this means lovely, and pleasant to the sight. On this account Plato wishing to intimate to us this truth, represents the demiurgus of the universe, approving his own perfect work; willing from hence to exhibit, by something more manifest to our apprehension, the beauty of the exemplar, and of his great idea, as perfectly lovely. For as often as any one admires a work, fabricated according to an exemplar, he must particularly admire the exemplar itself. Nor ought it to seem wonderful if in the mean time such a one, is ignorant of what he suffers: since terrene lovers, and those who admire corporeal beauty, are ignorant that they are thus affected, on account of supernal beauty. But that Plato refers the demiurgus of the universe loving his work, to the divine exemplar, is evident from hence: for he says, that he was delighted with his work, and wished to render it still more similar to its exemplar: evincing from this the

beauty of the exemplar, for says he its work is beautiful, because it is the image of its artificer. For indeed unless that was inestimably beautiful, what would be more beautiful than this universe, which is subject to our corporeal sight? On which account they do not perceive rightly, who detract from the beauty of this sensible world; unless in detracting they perceive that this universe is not the intelligible world.

Let us then receive by cogitation this our sensible world, so disposed that every part may remain indeed what it is, but that one thing may mutually reside in another. Let us suppose that all things are collected as much as possible into one, so that each particular object may first present itself to the eyes; as if a sphere should be the exterior boundary; the spectacle of the sun immediately succeeding, and an image of the other stars, and the earth, the sea and all animals should appear within, as in a diaphanous globe: and lastly let us conceive that it is possible to behold all things in each. Let there be then in the soul a lucid imagination of a sphere, containing all things in its transparent receptacle; whether they are agitated, or at rest; or partly mutable, and partly stable. Now preserving this sphere receive another in your soul, removing from this last the extension into bulk, take away likewise place, and banish far from yourself all imagination of matter: at the same time being careful not to conceive this second sphere, as something less than the first in bulk, for this must be void of all dimension. After this invoke that divinity who is the author of the universe, imaged in your phantasy, and earnestly intreat him to approach. Then will he suddenly come, bearing with him his own divine world, with all the gods it contains. Then will he come, being at the same time one and all, and bringing with him all things concurring in one. There indeed all the gods, are various amongst themselves in gradations of power, yet by that one abundant power they are all but one, or rather one is all: for the divinity never fails, by which they are all produced. But all the gods abide together, and each is again separate from the other in a certain state unattended with distance, and bearing no form subject to sensible inspection: or one would be situated differently from the other, nor each be in itself all. Nor again does any one of these possess parts different from others, and from itself: nor is every whole there a divided power, and of a magnitude equal to its measured parts; but it is indeed a universe, and a universal power, proceeding to infinity in a power, which is the parent of energy. But this divine world is so truly great, that its parts become infinite. For where can any thing be said to exist, with which it is not extended? This sensible world too is great, and all powers are contained in its ample bosom: but it would be much greater, and that in a manner perfectly ineffable, if it was free from the diminutive power of body. And if it should be said that the power of fire and of other bodies is great, it must be remembered that true powers are infinite, and that it is only from an ignorance of these, that corporeal natures appear to have being, and to operate by corrupting, separating, and ministering to the generation of animals. But these indeed corrupt, because they are

themselves

themselves corrupted, and they generate because they are generated. But the power which flourishes there, possesses being alone, and is alone beautiful, without any external and adventitious qualities, which only derogate from the dignity of essence. For where can there be any thing beautiful, deprived of being? And where again can essence abide, if it wants the presence of beauty? For while beauty is taken away, essence is destroyed. On this account being itself is desirable, because being, and beauty are the same: and the beautiful is lovely, because it is being. But it is not proper to enquire which is the cause of the other, since the nature of each is one and the same. The false essences indeed of bodies, require a certain image of beauty, extrinsically acceding, both that they may appear beautiful, and that they may inherit an obscure portion of being. For they so far partake of essence as they participate of beauty, consisting in form: and by how much the more they receive of this kind of beauty, so much the more of perfection do they inherit: for by this means a beautiful essence, and beauty itself is more peculiar to their nature.

On this account Jupiter himself, who is the most ancient of the other gods which he leads, proceeds first to the contemplation of the intelligible world. But afterwards the subordinate gods, daemons, and souls follow him, who are able to perceive such transcendently lucid objects. And this divine world shines upon them, from a certain occult place, which is no other than the abode of ineffable unity. But it illustrates all the divinities with its light: and excites to itself superior souls who are afterwards converted to its splendid vision, which before they were incapable of perceiving; and which like the sun dazzles the eye unaccustomed to intellectual light. And while some with elevated eyes, easily bear its intuition, others who are more distant from its nature are disturbed with the vision. But since each of these blessed inhabitants, perceives according to his ability, all of them indeed behold this intelligible world, with its various contents, yet they do not all retain the same spectacle, but while they are lost in attentive vision, one beholds the lucid fountain and nature of the just itself, while another abundantly perceives temperance itself, but not such as that which resides with men, when they enjoy its possession. For this our temperance imitates the supreme: but that diffusing itself in all things, as if about all the magnitude of its nature, is finally perceived by those, who have already beheld many perspicuous spectacles. On this account the gods behold every thing separate, and at the same time all things together: they perceive too divine souls there, whose vision is universal; and their nature becomes such from unbounded perception, that they contain all things from the beginning to the end.

These divine objects therefore, Jupiter himself and those of us who together with Jupiter love this intelligible world, happily contemplate, together with that universal beauty shining from all, and whatever participates of the beauty, which there abides. For every thing there glitters, and illuminates the spectators with its light, so that they

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become beautiful by its lustre : just as it happens to those who ascend the highest mountains, where the earth is yellow : for they are immediately infected with the colour, and become similar to the earth, to which they ascend. But the colour which flourishes in the divine world is beauty itself ; or rather every thing there is wholly colour, and profound beauty. For beauty there, is not like that which flourishes in the superficies of bodies : but among those who do not perceive the whole, that alone which is resplendent in the superficies is considered as beauty. But those who are totally filled with the intoxicating nectar of divine contemplation, since beauty diffuses itself through every part of their souls do not become spectators alone. For in this case the spectator is no longer external to the spectacle : but he who acutely perceives, contains the object of his perception in the depths of his own essence ; though while possessing, he is often ignorant that he possesses. For he who beholds any thing as external, beholds it as something visible, and because he wishes to perceive it attended with distance. But whatever is beheld as *perceptible*, is beheld externally : but it is requisite we should transfer the divine spectacle into ourselves, and behold it as one, and as the same with our essence : just as if any one hurried away by the vigorous impulse of some god, whether Apollo or one of the Muses, should procure in himself the intuition of the god ; since in the secret recesses of his own essence, he will behold the divinity himself. But if any one of us who is not able to perceive himself entirely comprehended by this divinity, should produce a spectacle into his view, for the purpose of assisting his vision, he should produce himself ; and he will then perceive an image of the intelligible world, now become more beautiful and divine. But afterwards neglecting the image although beautiful, and conspiring with himself into one, and no longer separating his essence, he will become *one all* together with that deity, who silently flows into his soul ; and he will be present with him as far as he is able, and as much as he desires. But if he should return from this divine union into two, and is in the mean time pure, he will nevertheless dwell proximate to its essence ; so that by conversion, he may again be present and become united with his divinity. But the gain of the soul will consist in this ineffable conversion. Indeed, when it first attempts this union, it perceives itself, as long as it is different from the god : but when it has penetrated into its most intimate recesses, it will then find itself in possession of the intelligible universe ; and casting sense behind, fearing lest it should become different, it will be one with this divine world. And if it desires to perceive as something different, it will place itself external to its object. But it is requisite that the soul which is about to perceive a divinity of this kind, should possess a certain figure of his nature, and assiduously persevere, while it endeavours perspicuously to know him ; and thus well understanding the importance of its pursuit, and trusting it is about to enter on the most blessed vision, should profoundly merge itself in contemplation, till instead of a spectator, it may become another specimen of the object of its intuition ; such as it came from thence, abundantly shining with intellectual conceptions.

But how can any one reside in the beautiful itself, unless he perceives it? Indeed, if he perceives it as something different, he will not as yet abide in beauty. But becoming beautiful, he will thus especially exist in beauty. If then vision is directed to something external, it is not proper that vision should be there, or if it is it should become one with the object of perception. But a doubt of this kind is like a certain consciousness of some one fearing, lest if he wished to perceive more vehemently, he should depart from himself. For thus disease more vehemently impels and excites our sensation: but health dwelling with us more quietly, exhibits a truer knowledge of itself, since it is present with silence and tranquillity, as something familiar and allied to us; and conspires into one with our composition. On the contrary disease possesses nothing domestic, but is entirely foreign from our nature; and hence its presence is more manifest on account of its diversity: but such things as are peculiarly our own, are present with us, without any manifest sensation. So that when we are in this condition, we are then most of all known to ourselves; since our science in this case is one and the same with our essence. Hence, in the divine world, when we are most knowing according to intellect, we appear to be ignorant, expecting the passion of sense, which says it does not perceive; nor indeed does it see; nor can it ever attain to the intuition of such exalted objects. That which distrusts its vision then is sense: but it is something else which perceives. And if this too should doubt, it is no longer its true self. For neither can this last when it places itself externally, behold that which is intelligible, as if it were sensible, and to be seen with corporeal eyes.

But it has been shewn how the soul may be able to accomplish this as different from its object, and how when the same. But what will the perceiver relate whether abiding as different, or the same? He will tell that he saw this god, who is the same with the intelligible world, generating a beautiful son, and producing all things in his essence without any labour and fatigue. For this deity being delighted with his work, and loving his progeny, continues and connects all things with himself, pleased both with himself, and with the splendors his offspring exhibit. But since all these are beautiful, and those which remain within are still more beautiful, Jupiter the son of intellect alone shines forth externally, proceeding from the splendid retreats of his father. From which last son, we may behold as in an image, the greatness of his sire, and of his brethren those divine ideas, who abide in occult union with their father. But this ultimate progeny does not affirm in vain, that he proceeds from his parent intellect: for he is another world, proceeding from this first, and becoming beautiful, like an image of beauty. For it is not lawful that the image of beauty and of essence, should not be beautiful. Hence, he in every respect imitates his exemplar. For he possesses life, and the gift of essence as a certain imitation of stable essence, and life ever vigilant: he possesses also beauty, so far as he proceeds from thence; and perpetual duration, as a moving image of the eternity of intellect abiding in one:

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for if this is not admitted, he would at one time exhibit his image and not at another. But he is not an image fabricated by art; and every image formed by nature, lasts as long as its exemplar endures. Hence they do not conceive rightly, who think this world may be destroyed, that which is divine remaining in the full perfection of its essence, and thus imagine the world generated, and that its author on a certain time consulted concerning its production. Such as these indeed neither wish to understand, nor are at all acquainted with the mode of its formation, and are ignorant that so long as the splendors of that divine world endure, so long will this visible universe beam from thence, and will never be destroyed, since the original of each is the same. But the intelligible world always was, and always will be: appellations of this kind being adopted from necessity, for the purpose of conveying the conceptions of our minds.

Saturn, therefore, who according to poetical fable is feigned bound, because he always perseveres in the same divine energies of his nature: who is also reported to have delivered the government of this universe to his son Jupiter (for it was not proper that he having dismissed his government, should follow a nature junior and posterior to himself, since he comprehends in himself the plentitude of all beauty.) Saturn, I say, omitting all subordinate natures established in himself his father Cælum, and raised himself on high as far as to this ineffable principle. He likewise established succeeding natures originated posterior to him, from his son. And thus he possesses a middle situation between both, through a diversity of section from that which is above him, and from his abstaining from inferior concerns, while he is fabled by a subordinate care to be bound in chains; thus obtaining a middle situation between his greater father, and his inferior son. But since his father Cælum, is something greater than beauty, hence Saturn or intellect is the first beauty, though soul is likewise beautiful: yet intellect is more beautiful than soul, because soul is only its vestige; and is naturally beautiful through this, though it is far more beautiful when it beholds the perfect nature of intellect. If then the soul of the universe (that we may use words more generally known), and Venus herself is beautiful, what must be the beauty of intellect? For if soul and Venus possess this from themselves, how great must be the splendor of intellect? But if from another, from whom does soul possess the beauty as well acceding, as natural to her essence? Indeed, whenever we are beautiful, we become so from the possession of our own nature alone: but we are base, when we are precipitated into an inferior nature. So that we are beautiful when we know, but base when we are ignorant of ourselves. Beauty, therefore, shines in Saturn or intellect, with primary splendors. But are these considerations sufficient to a knowledge of the divine world, the intelligible place? Or must we proceed another way in its investigation?"

And thus much for the doctrine of Plotinus, as delivered by him in the two preceding inestimable books. I shall only add the following observations concerning the Platonic

Platonic triad of principles, as conceived and illustrated by this extraordinary man, and some reflections concerning the Christian trinity, with which I shall take my leave of Plotinus.

According to Plotinus then, as the divinely solitary principle of things is perfectly simple, it necessarily follows that he must be perfectly sufficient, and perpetually exuberant. Hence, he must be a producing cause; and that which he first produces, must be the most similar of all things to himself. And this is no other than intellect, or the intelligible world, the nature of which as been so divinely explained by this philosopher, in the preceding book. Now this intelligible world on account of its perfect similitude to the one, contains all multitude in occult and indistinct union: for it is requisite that multitude should exist occultly, before it is scattered abroad and diffused into separate forms; and that it should be concealed in the profound recesses of intellect, — before it emerges into the diffused splendors of multitude perfectly divided and discrete. Just as the duad is posterior to unity, and contains number, without being perfect number itself. But as it is necessary that this occult multitude, should be perfectly diffused, in order to the actual diversity of things, and the existence of the sensible world, hence a third procession originates, in which multitude no longer subsists in indivisible union, but proceeds from the sanctuary of intellect into absolute diversity and separation. And this third principle is no other than soul, which expands the impenetrability of intellect, and unfolds all that was involved in the unity of intellectual perception. Now, besides these, there can be no other principles: for after the cause by which multitude is perfectly evolved, nothing but the gradation and diversities of multitude can subsist. Hence, as Plotinus justly observes \*, "we ought not to entertain any other principles, but having established the *simple good* as first, we should place the supreme intellect as the next, and then the universal soul as the third in descent. For this is the proper order according to nature, neither to make more, nor less intelligibles than these three. For he who contracts the number of these, must of necessity either suppose soul, and intellect to be the same, or else intellect and the first good. But that all these three are different from each other we have often asserted and proved."

It must here, however, be observed, as Dr. Cudworth justly remarks †, that this third hypostasis or principle, is not the immediate soul of the world (according to Plotinus, and the best of the Platonists) but *ψυχὴ υπερμονῆ*, a supermundane soul. For thus Proclus plainly asserts ‡, not only of Amelius, but also of Porphyry, who followed Plotinus in this particular. "After Amelius, Porphyry, thinking to agree with Plotinus, calls the supermundane soul, the demiurgus of the world, and that intellect to which it is converted not the demiurgus, but the paradigm of the world." Indeed, this supermundane soul must be too nearly allied to the supreme intellect, to become

\* En. 2. l. 9. c. i.

† Intellectual System, p. 562.

‡ In Timaeum. p. 93. 94.

the immediate animating principle of the world ; and as the gradation of things throughout the universe, subsists by the most gentle and easy declension, the descent would be precipitate, to make the highest soul connected with the mundane body. Besides as multitude subsists retired and concealed in the supreme intellect, such an intellect cannot be the artificer of the world, since all forms reside there in stable and indivisible union : but a procession and extension of these forms is requisite to the production of the visible universe. And as every cause is superior to its effect, and as the mundane soul must be connate with the world, hence the demiurgus of the world, must be superior to the mundane soul.

Such then is the Platonic triad, composed from three distinct, and different principles ; and having no similitude except in name, to the trinity of the Christian faith, as established by law. The Platonic philosophers indeed took a bold flight, for they soared to the principle of things, and drew abundantly from the ineffable and eternal fountain of good : but they never rose so high as to discover that the three persons of their triad were identically one. As men merely assisted by the illuminations of intellect, they saw the necessity of three principles, to the existence of the universe, but they had not yet penetrated the awful veil of the most mystic theology, and beheld the triple deity, seated on the tremendous throne of *unintelligible* faith. They were capable of demonstrating that the principle of all was perfectly *simple* and *one* : but their eyes were not acute enough to survey the *triplicity* of the *one*. Had they but been blest with the *light* of the moderns, what wonders would have opened to their view ! They would then have understood the trinity in unity, and the godhead in the manhood, absurdity involved in mystery, and mystery in absurdity. In short, they would have discovered, that the supreme so far from being separate from multitude, and superior to essence itself, as they fondly imagined, took upon himself the actual form of a man, that he might enlighten the vilest and most obscure of mankind, and that by suffering an ignominious death, he might appease his own wrath, and satisfy the vengeance of his injured deity. However, an impartial reader must confess, that considering their ignorance of these sublime particulars, their discoveries were admirable and profound ; and a sagacious modern will doubtless rejoice to find that they believed in a god, who was the principle of things, though at the same time they were so blind, as not to perceive that like *Cerberus* he was triple !

S E C T-

## S E C T I O N II.

**P**ORPHYRY the favourite disciple of Plotinus next demands our attention : but unfortunately we have scarcely any other particulars of his life, worthy our attention, than those we have already delivered in the history of Plotinus. I shall, therefore, only add, that he was born at Tyre, in the twelfth year of the emperor Alexander Severus, and of Christ 233 ; and that he died at Rome more than seventy years old, in the latter part of the emperor Dioclesian's reign. Of his great abilities we have already given ample testimony : it now remains that we shew how much he contributed to the restoration and perfection of the Platonic theology. We are informed by Proclus \*, that it was usual with Plato, and his most genuine associates, to call all beings, by the appellation of intellect. "Hence (says he), in many places they establish *the good intellect, and soul*, as the three principles of things, calling intellect *every being*." Now this was eminently the case with Plotinus, who in intellect or the intelligible world, comprehends all the intelligible gods, all true beings, and the multiform variety of ideas. Hence, he was more anxiously employed in profoundly investigating the nature of this divine world, than in scientifically unfolding the order of the beings it contains. Indeed, his genius on every subject was more adapted to an intimate perception of the occult essence of a thing, than to explaining its gradual evolution, and describing the mode of its participations. However, though he did not prosecute the more particular processions of divinity himself, yet he took care to insert the principles of this sublime investigation, in his writings ; and to lay the foundation of that admirable and beautiful system, which was gradually revealed by succeeding Platonists, and at last received its ultimate perfection, by the subtle and elegant genius of Proclus.

Porphyry, however, appears to have been the first who wrote any thing explicitly on this interesting subject. "For he composed (says Proclus †) a treatise concerning principles, in which he demonstrates by many and beautiful reasons, that intellect is indeed eternal, but that it contains in itself something more ancient than intellect, which is conjoined with the one." Now this something which is more ancient than intellect, but inferior to the one itself, can be nothing else than an Henad, or posterior monad ; and if so there must be an order of Henades prior to that of intellects, which is most beautifully and copiously proved by Proclus, in his books on Plato's Theology, and is demonstrated in the following theological institutions. But that this was likewise the doctrine of Plotinus is plain from his own words ‡. "It is necessary (says he) that the principle and cause of intellect, and the deity himself, should be present with the soul ;

• In Plat. Theol. p. 158.

† In Plat. Theol. p. 27.

‡ Ennead. 5. p. 492.

and that this supreme principle should be present, not in any divisible manner, but perfectly stable, not indeed in place, but in himself. And, in the mean time, he must be considered in each of the natures capable of receiving him, as something different from their essence, just as the centre also abides in itself. But every line in the circle terminates its point in the centre, to which it tends. And by something analogous to this, which our essence contains, we also may touch, coalesce, and be conjoined with the supreme." Hence it appears how egregiously Dr. Cudworth \* is mistaken, in asserting that the doctrine of the Henades, was a figment of the Platonists posterior to Plotinus : for if they are the summits of our souls, and intellects, they must likewise be the very flowers, as it were, of universal souls and universal intellects.

As the most valuable of Porphyry's works are unfortunately lost, and this most probably through the malice of sectaries, and the unjust interposition of regal authority, we can scarcely give any other proof of his great contributions to the advancement of the theology, than by enumerating his writings on this occasion. And in the first place, next to the two books on principles which we have already mentioned, his treatise, *Concerning the Philosophy from Oracles*, merits our attention. In this curious and valuable work (as we are informed by Eusebius †) he collects, a variety of the answers, as well of Apollo, as of other deities, and beneficent dæmons : proving by this means the virtue of theology, and promoting *theosophy*, or the study of divine wisdom.

2. In the next place, we may deservedly rank his *Explanation of many of Plotinus's Books* (according to the testimony of Eunapius.) For surely nothing could more contribute to the restoration of this theology, than a commentary on the writings of him who first uncovered its veil, and happily penetrated into the mysteries it contains.

3. To these succeed his *Commentaries on the Timeus of Plato*, which are every where cited by Proclus, with the praise they doubtless deserved. Indeed, if we consider the sanctity of Porphyry's genius, his great abilities, and universal attainments, we cannot sufficiently regret the loss of his writings on a dialogue which has ever been esteemed uncommonly abstruse, and replete with the most mystical theology.

4. *Concerning the divine Names.* This work is mentioned by Suidas, and promises by its title, to have been an inestimable treasury of recondite wisdom.

5. *Concerning the Allegories of the Grecian and Egyptian Theology.* How much Porphyry, excelled in works of this kind, is sufficiently evident from his explanation of the Cave of the Nymphs in Homer, which is fortunately preserved ; and the translation of which in the following pages, will both adorn the present history, and, I doubt not, be acceptable to the reader.

6. *On the Regress or Re-ascent of the Soul.* The design of this excellent work, as we are informed by St. Augustin, who often cites it, in his City of God ‡, was to admonish us, that we should fly from all body, that the soul may abide in felicity with the deity.

\* Intellectual System, p. 568.

† Præpar. Evan. lib. iv. c. 6.

‡ Lib. x. cap. 29.

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According to the same Augustin too, in this book there was a copious discourse concerning the purgation of the soul by the *theurgic* art, which we cannot wonder this reverend father should both ridicule and disclaim : for how can the faith of a *bishop* agree with the dogmata of a *philosopher*? But it is here necessary to inform the reader that the theurgical art, explained and recommended by Porphyry in this treatise, consists in purifying the imaginative spirit or vehicle of the soul, which according to its various condition either elevates the foul to a superior state, or draws it down into the darkest recesses of the earth. Lucas Holstenius in his account of the life and writings of Porphyry, is very diffuse in relating and defending the opinions of Augustin on this particular, but takes no notice of the Platonic Synesius \*, who in his admirable book on dreams, speaks so appositely and fully on this most interesting subject, that I cannot refrain from presenting the English reader with a translation of his doctrine concerning this phantaſtic spirit, or as he calls it spiritual soul. "In what the disease (says he) of this spirit consists, by what means it languishes and is dulled, and how it becomes purified and defecated, and restored to its natural simplicity and perfection, must be learned from the arcana of philosophy ; from which being purified by the lustrations of mysteries it passes into a divine condition of being. But it is requisite to banish all influxions externally, before the phantaſtic spirit can superinduce the divinity. And whoever preserves it pure by a

\* It is necessary to inform the genuine Platonic reader, that Synesius was not a bishop, and had not embraced Christianity, when he composed the present excellent book. This is not only evident from the recondite wisdom of antiquity, which it every where displays, but from his expressly using in one place, p. 151, the epithet, *the mundane god* (*Ωντες λύκειας*) in the same manner as it is always employed by heathen Platonists. There are three Latin versions of this admirable work : the first and best by Ficinus, the next both in the order of time and excellence, by Cornarius, and the last and worst by Petavius. This industrious Jesuit, indeed, though sufficiently learned on other subjects, appears by his version to be very ignorant of the Greek philosophy. This will be evident by comparing his translation with the Greek, and, I hope, I may add, with the above version, which contains the greatest, and most important part of this inestimable work. With respect to Synesius, he was a native of Cyrene, in Africa ; travelled into Egypt for improvement ; and perfec&ed his studies under the celebrated Hypatia : a woman no less eminent for her uncommon abilities, than remarkable for her tragical death, effected by the infernal revenge of the Alexandrian church, and the orthodox, patriarchal, Christian malice of St. Cyril. Though Synesius was reluctantly consecrated bishop of Ptolemais about the year 410, yet we may collect from his tenets, that he was not a perfect convert to Christianity ; and from his epistles, that he lamented his episcopal station. For with respect to the former, he denied the immediate creation of the world, its final destruction, and the resurrection of the dead : and with regard to the latter, he frequently and earnestly begs to be discharged from his office ; and declares that as he was by education a heathen, and by profession a philosopher, he had met with no success since he presumed to serve at the altar. Particularly in a letter to his friend Olympius (Epist. 95.) he declares that if his duty as a bishop should be any hinderance to his philosophy, he would relinquish his diocese, abjure his orders, and remove into Greece. For farther conviction in this particular consult Epist. 11, 13, 57, 67, 80, 105 : and the Clidophorus, and Hypatia of Toland, who largely and curiously descants on the life and writings of Synesius. I shall only add, that even Petavius, the editor of his works, affirms, that in some of the books composed after his profession of Christianity, Synesius appears as great a heathen as before : and this conduct may be reconciled, by what Synesius himself asserts in one of his letters : (Epist. 105.) "that it may be sometimes expedient to lie, in order to do good ; since philosophical truth is not necessary for the vul'gar, who may receive hurt from their knowledge. Wherefore (says he) if this method, (that is of concealing the truth), be consonant with the rights of episcopal dignity, I may be consecrated. I shall freely philosophize at home, and tell fables abroad ; neither teaching, nor unteaching, but suffering people to live in the prejudices they have imbibed." This must be allowed to be sound philosophy, excellent policy, and true orthodoxy !

life according to nature, will render it prompt for this most exalted employment. For this spirit understands the affection of the soul, and is not destitute of sympathy towards it, like its testaceous vestment the body, which has a condition opposite to the more excellent affections of the soul. But the primary and proper vehicle of this phantaſtic spirit, when the soul is in a flourishing condition, is attenuated and etherial: but when the soul is badly affected, then this vehicle is dulled, and becomes terrene. For this phantaſtic spirit is ſituated in the confines of the rational and brutal nature, is of an incorporeal and corporeal degree; and is the common boundary of both, and the medium which conjoins divine natures with the lowest of all. On this account it is difficult to comprehend its nature by philosophy: for it collects that which accords with itſelf, as it were from neighbouring natures, and from the extremes of each; and comprehends in one eſſence things ſeparated by ſo great an interval from its own. But nature extends the latitude of a phantaſtic eſſence, through many conditions of things: for it descends even to animals to whom intellect is not present. In this caſe, however, it is no longer the vehicle of a diuinier soul, but presides over its ſubject powers, becomes the reaſon of the animal with which it is connected, and is the occaſion of its acting with much wiſdom and propriety.

" But this phantaſtic spirit may be even purified in brutes, ſo that ſomething better may be induced; and all the genera of dæmons derive their eſſence from a life of this kind, for their whole eſſence is composed from the phantaſy, and from inward imaginations. But many of the energies of the human nature conſift from this alone, or if from ſomething else, yet this prevails the moſt: for we are not accustomed to cogitate without imagination, unless ſome one ſhould perhaps for a moment be able to paſs into contact with an immaterial form. But to tranſcend the phantaſy in rational energies, is not leſs difficult than bleſſed. Hence (says Plato) the poſſeſſion of intellect and wiſdom in old age is deſirabie above all things, ſignifying by this, intelligence ſhining without imagination; because intelligence when conveſtant with a common life, belongs to the phantaſy, or at leaſt to an intellect energizing through the medium of the phantaſy. Hence too, this animal ſpirit which diuinie men have denominated the ſpiritual ſoul, becomes a god, and an omniform dæmon, and an image, in which the ſoul ſuffers the punishment of its guilt. And in conformity with this the oracles also compare the life of the ſoul in this animal ſpirit to the imaginations of dreams. Philosophy too, agrees in aſſerting, that preceding lives are certain preparations to those in a ſubsequent order, while the poſſeſſion of the beſt habit in ſouls renders this ſpirit more adapted to elevation, and wipes away the profound stains of a baſer affection. Hence by natural allurements, this ſpirit is either elevated on high, on account of its heat and dryneſs, which Plato ſignifies by the wings of the ſoul, and Heraclitus when he ſays, *that a dry ſoul is the wifest*: or becoming bulky and humid, it merges itſelf in the recesses of the earth by a natural gravity; and is thus concealed in darkness, and hurled into a ſubterraneaſ

ranean region\*. For a place of this kind is peculiarly adapted to humid spirits; and the life there is unhappy, and obnoxious to punishment. It is however possible by labour and time, and a transition into other lives, for the imaginative soul when purified, to emerge from this dark abode: for it passes its course through lives of a twofold nature; and alternately approaches to superior and subordinate conditions of being.

" But the soul in its first descent, derives this spirit from the planetary spheres, and entering this as a boat associates itself with the corporeal world, earnestly contending that it may either at the same time draw this spirit after it, in its flight, or that they may not abide in conjunction. Indeed it is rarely though possible to be accomplished, that the one deserts the other in descending to the earth: for it is unlawful not to believe in mysteries of known credibility and truth. But the souls' regression will be base, if she neglects to restore, that which is foreign from her nature, and leaves about the earth, what she had received from on high. And this indeed one or two may obtain as a gift of divinity and initiation. For it is instituted by nature, that the soul, once seated in this phantastic spirit, should either follow, or draw, or be drawn, yet so as to remain copulated with this spirit, till it again ascends from whence it came. Hence when on account of its depravity this spirit grows heavy, at the same time, it draws down the soul, which had yielded to its gravitation. And the dread of this is what the oracles announce to our intellectual conceptions, when they advise: *Nor decline be-*

\* This is excellently amplified by Porphyry in Senten. 32. p. 232. " As the soul's residence on the earth, (says he) does not consist in being conversant in the earth, as bodies, but in presiding over bodies situated on the earth: so likewise the soul is said to be in Hades, when she presides over her image, which has a nature accommodated to place, but then obtains a subsistence in darkness. Hence if Hades is a subterranean dark place, the soul though not separated from being, dwells in this case in Hades, attracting to herself her image. For the spirit which she had collected from the spheres, attends her in her departure from a solid body. But from her affection towards body, reason having a partial object, according to which, she obtained a habitude towards a body of this kind, while she lived; from this propense affection, a form of the phantasm is impressed on the spirit, and so she attracts the image. But she is said to be in Hades, because she obtains a spirit of a formless and dark nature: and since a heavy and humid spirit, extends to subterranean places, on this account also the soul is said to dwell under the earth; not because her essence passes from place to place, and subsists in place, but because it receives the habits of bodies endued with a natural inclination to local transitions, and the possession of place. And bodies of this kind indeed, receive the soul, according to aptitudes, from a certain disposition towards her nature. For according to her particular disposition, she finds a body defined, correspondent to her order and properties. Hence when the soul is in a more pure condition of being, a body is natural to her, approximating very nearly to that which is immaterial; such as an ethereal body. But when she proceeds from reason to the object of imagination, she naturally obtains a solar-form body; and when effeminated and captivated with the love of forms, she is united with a lunar-form body. Lastly, when she falls into bodies composed from humid vapours, a perfect ignorance of being succeeds, and darkness and infancy. And indeed in her egress from body, when she retains a spirit disturbed by a humid evaporation, she attracts a shade, and is weighed down; a spirit of this kind, endeavouring to dwell naturally in the bottom of the earth, unless some other cause draws it into a contrary place. As therefore when surrounded with this terrene shell, it is necessary she should reside on the earth; so likewise when she attracts a humid spirit, it is necessary she should be surrounded with the *image*. But she attracts moisture, when she continually studies to be present with a nature, whose operation consists in moisture, and is mostly subterranean. But when she endeavours to depart from nature, she becomes a dry splendor, without a shadow, and without a cloud: for humidity constitutes a cloud in the air; but dryness produces from vapour, a dry splendor."

*neath*

*neath, into the obscure world, whose depth is always an unfaithful bottom, and an infernal darkness, squalid, rejoicing in shadows, and full of stupidity and folly.* For how can a stupid and foolish life be expedient to intellect? But the inferior region, accords with the image, or spiritual soul, on account of an affection of spirit corresponding with such a place: for like rejoices in like.

"On this account, if by conjunction, one is produced from the two, intellect also will be merged in pleasure. But the extremity of all evils consists in not perceiving the present evil: for this belongs to such as have no desire to emerge, but like those whose skin is hardened by disease, as they are no longer tormented with pain, so neither are they anxious to be cured. Hence penitence possesses a peculiar power of re-elevating the soul. For he who endures his present state with sorrow and remorse, will meditate his flight: *and the will is the greatest part of purgation.* Indeed through the means of this both our deeds and discourses extend their hands to assist us in our ascent: but this being taken away the soul is deprived of every purifying machine, because destitute of assent, which is the greatest pledge of reconciliation. Hence both here and elsewhere, punishments bring with them the greatest utility to the order of things, while they oppose molestation to delight, and banish stupid pleasure from the soul. Misfortunes too, which are said to happen contrary to our deserts, are of the greatest advantage in extirpating the affections by which we are captivated with externals: and thus the doctrine of a providence is confirmed to the intelligent, from the very circumstances which produce diffidence in the ignorant. For no place would be left for the soul to take her flight from the dominion of matter, if in the present state she lived free from the incursions of evil: and hence it is proper to believe, that the prefects of the infernal regions have invented vulgar prosperities, as the snares of the soul. It may therefore be said that souls emigrating from hence drink of oblivion: but the cup of oblivion is extended to souls entering into the present life, by pleasure and delight. For when the soul descends spontaneously to its former life, with mercenary views, it receives servitude as the reward of its mercenary labours. But this is the design of descent, that the soul may accomplish a certain servitude to the nature of the universe, prescribed by the laws of Adraستia, or inevitable fate. Hence when the soul is fascinated with material endowments, she is similarly affected to those, who though free born, are for a certain time hired by wages to employment, and in this condition captivated with the beauty of some female servant, determine to act in a menial capacity under the master of their beloved object. Thus in a similar manner, when we are profoundly delighted, with external, and corporeal goods, we confess that the nature of matter is beautiful, who marks our assent in her secret book: and if considering ourselves as free we at any time determine to depart, she proclaims us deserters, endeavours to brink us back, and openly presenting her mystic volume to the view, apprehends us as fugitives from our mistress. Then indeed the soul particularly requires fortitude, and divine assistance, as it

it is no trifling contest, to abrogate the confession and compact which she made. Besides, in this case, force will be employed : for the material inflictors of punishments will then be roused to revenge, by the decrees of fate, against the rebels to her laws. And this is what the *sacred discourses* (*λεγος λόγοι*) \* testify by the labours of Hercules, and the dangers which Hercules was required to endure ; and which every one must experience who bravely contends for liberty, till the phantastic spirit becomes superior to the dominion of nature, and is placed beyond the reach of her hands.

" But if this leap from matter should happen within the boundaries of nature, the soul will be depressed, and require more weighty contests : for matter now fully convinced that we are fugitives, will not be sparing of punishment ; and though we may despair of our ascent, she will chastize us for the endeavour, and no longer propose our choice of living from two urns, which Homer occultly intimates are two portions of matter. And Jupiter himself in this place, according to the same divine poet, is the moderator of matter, distributing a twofold condition of fate ; from which good is never found sincere, and without a mixture of evil, though it is possible that some unfortunate being may participate of the worse condition without any portion of good.

" In short all lives are conversant with the fluctuations of error, when the soul does not speedily return, after its first descent. But consider with how great an interval, this spirit energizes in our nature : for when the soul is inclined downwards, *the spirit also* (according to the *sacred discourse*) *grows heavy, and sinks, till it falls into a region profoundly dark : but when the soul rises from this obscurity, the phantastic spirit also attends it, as far as it is able to follow.* And it will attend, till the soul arrives at a condition of being the most opposite from its nature. Hear too, what the oracles declare on this occasion. *Nor should you leave the most abject part + in the precipice of matter : for there is a place for the image*

\* This *sacred discourse* composed by Pythagoras was twofold ; one in verse, mentioned by Heraclides, and the other ΠΕΡΙ ΘΕΩΝ, in Doric prose, both which are unfortunately lost. It appears, however, from the preface of this page, that a part of one of them consisted in explaining the labours of Hercules, who, like Ulysses, is an allegorical character, representing the progress of a man from the impurity of a sensible life, till he acquires the perfection and purity of a life intellectual and divine. Hence Proclus on Plato's Republic, p. 382. " Hercules being purified by sacred initiations, and having acquired undefiled advantages, deserved a perfect establishment among the gods." We may conceive, therefore, that by the club of Hercules is meant philosophy, and by his lion's skin, prudence ; through whose assistance he tamed the passions, those monsters of the soul, and destroyed vain cogitations : both which are occultly signified by the twelve labours he endured. I only add that Petavius appears to have been entirely ignorant, that any such writings as the *sacred discourses*, ever existed : for he translates them in one place *sacra historia*, and in another, where they are mentioned in the singular number, (*λόγος*) *natura bac oratio.* Such ignorance may be excusable in the modern priest ; but as a man of learning Petavius cannot be defended by any apology in such vile translations.

+ The Scholium of Nicephorus upon this passage is as follows : " Synesius (says he) calls the dross of matter, that which the phantasy, descending from the supernal spheres, derives from the natural elements of fire and air. And this he observes it is not lawful to leave in the precipice of matter, viz. in the world every way obscure and dark, but it is requisite to draw it upwards, by the assistance of temperance, continence, and the other virtues, and re-elevate it to the ethereal world : since there is a place allotted for *natura*, and the *image*,

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*image in the region every where resplendent with light.* But this place is opposed to the region totally dark ; and to him who acutely perceives, a still deeper meaning will be found in the words. For the oracle not only seems to recal into the spheres, the nature which had proceeded from thence, but also intimates that whatever of sublime fire or air, the soul descending from on high, had attracted into a phantastic essence, before she was invested with this terrene bark, must be elevated together with the more exalted part. For the dregs of matter, or the most abject part, cannot signify a divine body. But reason dictates that things which communicate, and conspire in unity, cannot be destitute of mutual relation, and connection with each other, particularly when the places of their residence have a kindred position : as fire is proximate to an orbicular body, and does not like the earth possess the extremity of things.

" Again, if better natures yielding to the subordinate, should at any time unite in conjunction with these, they would produce in matter an indissoluble body, from their superior dominion : and in this case perhaps, the baser nature, no longer opposing the energy of the soul, but becoming gentle, obedient, and obsequious, and exhibiting the middle nature of the phantastic spirit without dissipation, may become ethereal together with the dominion of the rational soul ; may be the attendant of its elevation ; and may ascend if not to the summit of all, at least to the extremity of the elements, and arrive at the region in every part lucid, and divine. For it possesses says the Oracle a certain place in this region, i. e. it is received into a certain order of an orbicular body. And thus much may suffice, concerning the parts and condition of the elements, which the reader may either believe, or reject as he pleases.

" But it cannot be denied that the corporeal essence of the phantastic spirit, when arrived at this place, is at the same time elevated with the returning soul, and adapted to the spheres ; or in other words it is brought back to its proper nature and pristine condition. These two regions, therefore, are situated in perfect opposition to each other : the one profoundly obscure, but the other every way lucid, obtaining the extremities \* of felicity and misery. But how many middle regions do you think are situated

in the region every where lucid. But he calls the *image*, the phantastic spirit, as being connate to the rational soul, though of a subordinate nature. For as *intellect* is the resemblance and image of the divinity, but the *rational soul* of intellect : so the *phantastic spirit*, or irrational soul, is the image of the rational soul. Hence, as *nature* or the *natural soul*, is the image of the phantastic spirit, and of this again *the body*, and of body, *matter* : on this account he observes that the phantasy obtains a certain familiar proportion to the natures which communicate with it on each side, by a certain union and affinity, and by a tendency to the same common end. Hence in its descent it not only acquires fire and air, from the rational soul, which has a supernal situation, but likewise from the natural elements, which are placed beneath." Nicophori Scholia, p. 390.

\* " Synefius says that there are two extreme seats, the summits of the universe, one beneath, every way dark, the other above, wholly luminous ; and that the former is miserable, but the latter blessed. But because opposites cannot be immediately copulated together, he asserts that in the middle of these two seats, there are many other regions, differing from each other, which he denominates in one part luminous, and in the other obscure, participating of the extremes, and beneath the region totally resplendent with light. Just in the same manner

Seated in the concave space of this mundane orb, partly lucid, and partly dark, in all which the soul lives, with this phantastic spirit, and alternately changes its forms, and manners, and life? When, therefore, it returns to its proper nobility, it becomes the storehouse of truth: for it is then pure and pellucid, and perfectly immaculate; and has power, if willing, to become a god and a prophet. But when it falls from this elevation, it becomes dark, erratic, and fictitious: for the obscurity of the spirit, cannot perceive the perspicuity of true beings. Lastly, when it possesses a middle situation, it partly wanders, and partly pursues the truth. You may also by this means, explore a demoniacal nature, and its order: for to pursue truth entirely, or to wander but a little from its contemplation, is divine, or nearly divine. But a condition of being, erroneous in predictions, necessarily belongs to such as are assiduously inclined to nature, who are obnoxious to passion, and perfectly ambitious: for by this means such a condition becomes subterranean\*, and forsakes divinity, and its more ancient daemon; though by a contrary mode of proceeding it may resume its pristine associations, and occupy the place prepared for a more excellent nature.

" And from hence we may apprehend the state of the soul while connected with the present body: for he whose phantastic spirit, is pure and composed, and who, both waking, and sleeping, receives true resemblances of things, be indeed, possesses a token that the figure of his soul will pass into a better condition of being. Nor is the judgment trifling which we may form respecting the affection of the animal spirit, from the imaginations which it principally produces, and in which it is employed, when free from external pulsation; philosophy supplying us with judgment and admonition, respecting its nutriment, and the diligent care we should employ to prevent its deviation from the right. But its best education consists in always energizing according to an intuitive and perceptive power, and in taking care that the principal energy of the soul is always intellectual; and that as much as possible we always pre-occupy the absurd and rash impetuositieis of the phantasy. But this is no other than a conversion of the soul to that which is best, and forsaking all communion with an inferior nature, except what the strongest necessity compels us to adopt. But an intellectual perception above all things separates, whatever is contrary to the true purity of the phantastic spirit: for it attenuates this spirit in an occult and insipid manner, and extends it to divinity. And when it becomes adapted to this exalted energy, it draws by a certain affinity of nature, a divine spirit, into conjunction with the soul: as on the contrary when it is so

as cogitation is the medium between intellect and soul: but opinion is seated between the phantasy and the soul; and sense between the phantasy and body. And these mediums mutually penetrate, and extend through each other, from body to intellect, and participate all their surrounding natures. As air possessest on each side two extremities and seats, by means of which, it is hot in its supreme situation, through its proximity to elemetary fire, but humid in its inferior extremity, through its vicinity to water." Necopori Schola, p. 392.

\* The Greek is very defective in this place, which I have endeavoured to supply, as the learned reader may observe.

contracted and diminished by condensation, that it cannot fill the ventricles of the brain, which are the seats assigned to it by providence, then, nature not enduring a vacuum, *an evil spirit* is insinuated in the place of one divine. And what will not the soul suffer, when assiduously pressed by such an execrable evil? For such is the constitution of things, that the regions of the phantastic spirit must either be filled with a superior or subordinate nature: but the latter is the punishment of the impious, who defile the divine part of their essence; and the former is either the end of piety, or proximate to the end." Thus far the excellent Synesius, who, I doubt not, was greatly indebted to Porphyry's book on the regress of the soul, for this admirable discourse; as it is evidently pregnant with the most recondite theology. But let us consider this interesting subject more minutely.

Though the theurgical art is unfortunately lost, by means of which we might obtain the best method of purifying the phantastic spirit; yet we must not suppose that it is utterly impossible to accomplish this desirable end, without its assistance. Synesius in the preceding beautiful quotation informs us, "that an intellectual perception attenuates this spirit, in an occult and ineffable manner, and extends it to divinity." Indeed, nothing can so effectually contribute to separate the phantasy from this terrene body, as a continual intellectual illumination. Now this can only be acquired by long habits of meditation, accompanied with a vehement thirst after truth, which gradually withdraw the soul from sensible perturbations, produce the contemplative virtues, and dispel the darkness of corporeal imaginations. Science, indeed, is the first requisite in acquiring this purification of the phantasy, I mean the mathematical science; by whose assistance, we first recognize the glimmerings of truth, and discover the dawning beams of intellect emerging, as it were, from the night of oblivion. When the liberal soul first discovers this light, though but feeble and transient, she rejoices at the happy event, and is anxious to procure its continuance and increase. She now despises outward corporeal form, and becomes deeply enamoured with those purer forms in the phantasy, which she has found to be the receptacles of truth. And this is the first degree of purification. But after this, if by a fortunate event, from contemplating universals in imaginative figures, she should rise to speculate their subsistence in cogitation, and in the rational soul, she will then discover a much brighter light; though even this will not be constant and serene: for it will be present only when she is deeply engaged in such *middle contemplations*. Indeed, as cogitation is the medium between sense and intellect, so the light attending its energies, has a middle subsistence between the obscurity of the former, and the invariable splendors of the latter. This light, however, will so purify the phantastic spirit, that all its images will possess a considerable degree of perspicuity and lustre. There now remains only the third step, in order to produce the perfection of purity, and to conjoin the phantasy with divinity: and this is no other than an intimate conversion of the soul to the energies of intellect. For by a long and vigorous exercise of this kind, a constant

constant and ineffable light will continually illuminate the phantasy, so as to render all its images pure and pellucid, and perfectly abolish the obscurity of sensible impressions. We may add too, as a symbol of this exalted purgation, that a perpetual serenity, unceasing delight, and occasional rapture will be produced in the soul. The *will*, now entirely free will be intimately converted to that which is best ; the desires will breathe nothing but the ardour of intellectual energy ; and the passions will no longer be at variance with reason. In this delightful state, the vehicle of the phantastic spirit will become so attenuated and etherial, that all sensible harmony will waken the soul to an immediate recollection of ideal harmony ; all external figure will recall to her memory ideal form ; and all lucid bodies will represent with advantage to her inward eye the brighter light reflected in the mirror of imagination. Indeed, sensible light, will be found to possess a remarkable sympathy with this purer light of the soul. For when this intellectual splendor is firmly introduced, and illuminates every part of the phantasy, the smallest spark, and the most glimmering ray of external light, will call forth into energy that sacred light, which is now perfectly seated in the sanctuary of the soul. Such too will be the temperament of the soul in this case, that she will spontaneously utter musical sounds, as indications of the harmony within ; and as echoes of the perpetual felicity she enjoys. And such are the methods of acquiring, and such the tokens of possessing purity of imagination, which he who obtains will understand ; but which will appear incomprehensible, and ridiculous to him, who is not advancing in its acquisition.

And here it may not be improper to observe, that the phantasy in this purified state, affords indubitable tokens of the possession of truth ; and serves as an instrument by which we may discover false opinions from such as are true. For the images attending the perceptions of reality, will always be lucid ; and this in proportion to the certainty they contain. Hence, whenever the soul is full, and as it were, pregnant with true conceptions, certain bright phantasms, as the progeny of her rational energies, will drop into the mirror of imagination, and appear like images clothed with light. For the phantasy will now no longer be similar to the dark and irriguous cavern of Calypso (which appears to be the emblem of imagination in an unpurified state), illuminated by sense as by an artificial fire ; but it will be totally diaphanous and full of light. It will, indeed, in every respect resemble the palace of Ithaca, when enlightened by the golden lamp of Minerva, during the removal of the arms by Ulysses and Telemachus. Of which we may say with the greatest propriety :

Not such the sickly beams which unsincere,  
Gild the gross vapour of this nether sphere \*.

And he who knows the truth of what I assert, may exclaim with rapture, like Telemachus :

\* Pope's *Odysscy*, book xix.

What

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What miracle thus dazzles with surprise !  
 Distinct in rows the radiant columns rise ;  
 The walls where-e'er my wond'ring sight I turn,  
 And roofs amidst a blaze of glory burn !  
 Some visitant of pure ethereal race,  
 With his bright presence deigns the dome to grace.

¶ In the last place, we may deservedly rank among the theological writings of Porphyry, his treatise *Concerning the Cave of the Nymphs*, in the 13th book of the Odyssey. This admirable work is fortunately preserved : and as it contains some deep arcana of the natural and symbolical theology of the ancients, together with some beautiful observations respecting the allegory of Ulysses, I persuade myself the following paraphrased translation of this work, will be acceptable to the lovers of ancient learning and philosophy.

" \* What are we to understand by the Cave, in the island of Ithaca, which Homer describes in the following verses ?

High at the head a branching olive grows,  
 And crowns the pointed cliffs with shady boughs.  
 A cavern pleasant, though involv'd in night,  
 Beneath it lies, the Naiades delight.  
 Where bowls and urns, of workmanship divine,  
 And massy beams in native marble shine ;  
 On which the Nymphs amazing webs display,  
 Of purple hue, and exquisite array.  
 The busy bees, within the urns secure  
 Honey delicious, and like nectar pure.  
 Perpetual waters thro' the grotto glide,  
 A lofty gate unfolds on either side ;  
 That to the north is pervious by mankind :  
 The sacred south t' immortals is confign'd.

i. e. "an olive with spreading branches stands at the head of the Ithacenian port ; and near it is a cave both pleasant and obscure, which is sacred to the nymphs who are called Naiades. Within the cavern, bowls and spacious amphora are formed from stone, in which the bees deposit their delicious honey. There are likewise within the cave long stony beams, on which the nymphs weave purple webs wonderful to the sight.

\* There are three editions of this excellent work. The first Greek and Latin by Holstenius, Cantab. 1655 ; the second by Barnes, prefixed to his Homer ; and the third by some German editor, which I have not seen.

Perpetual

Perpetual waters flow within the grotto. But there are two gates : one towards the north gives entrance to mortals descending : but the other towards the south which is more divine, is impervious to mankind ; and alone affords a passage to ascending immortals."

That the poet does not describe this cave according to truth, is evident from hence, says Cronius, that none of those who have handed down to us the situation of that island make any mention of such a cave. This likewise, says he, is manifest, that it would be very absurd for a mortal man, such as Homer, to expect, that in describing a cave fabricated merely by poetical licence, and thus arbitrarily opening by a new art a path to gods and men in the region of Ithaca, he should gain the belief of mankind. It is equally as absurd to suppose, that nature herself should point out in this place, one path for the descent of all mankind, and again another path for all the gods. For indeed the whole world is full of gods and men : but it is impossible to be persuaded that in the Ithacanian cave men descend, and gods ascend. Cronius, having promised thus much, affirms that it was evident not only to the wise, but also to the vulgar and unlearned, that the poet under the veil of allegory, concealed some mysterious signification. But the investigation of the particular meaning of these gates, and of the cave of the nymphs, he leaves to others more disposed to such curious enquiries, as likewise why it is both pleasant and obscure ; since darkness is by no means delightful, but is rather productive of aversion and horror. Also why it is not simply sacred to nymphs, but it is accurately added, which are called Naiads. Why likewise the cave is represented as containing bowls and urns, when no mention is made of their receiving any liquor, but the bees are said to deposit their honey in these vessels, as in hives ? Then again, why are oblong beams placed here for the nymphs ; and these not formed out of wood, or any other ductile matter, but from stone, as well as the bowls and urns ? Which last circumstance is indeed less obscure ; but that on these stony beams, the nymphs should weave purple garments, is wonderful not only to the sight, but to the auditory sense. For who would believe that goddesses weave garments in a cave involved in darkness, and on stony beams ; especially while he hears the poet affirming that the purple webs of the goddesses were exposed to human inspection. Besides this too is wonderful, that the cave should have a double entrance ; one prepared for the descent of men, the other for the ascent of gods. And again, that the gate previous by men should look to the north, but the portal of the gods to the south. Since the reason of this distribution affords just matter for surprise and enquiry : and why an eastern and western situation was not preferred. For almost all temples have their entrance and statues towards the east : but those who enter them look towards the west, when standing with their faces turned to the statues they honour and worship the gods. Hence since this narration abounds with obscurities, it follows that it is neither a fable, rashly devised, for the purpose of procuring delight, nor contains a true and certain description of a place : but that something is signified by the

poet, under its obscure disguise; who likewise places, with a mystic intent, an olive at the entrance of the cave. All which particulars the ancients thought very laborious to investigate and explain, and we who succeed them are of the same opinion, while endeavouring from our own inventions to unfold the concealed meaning of the allegory. Hence those men appear to have written very negligently concerning the situation of the place, who believe both the cave and its contents, to be a mere poetical figment. But the best and most accurate writers of geography, and among these Artemidorus the Ephesian, in the fifth book of his work, which consists of eleven books, thus writes: "The island of Ithaca, containing an extent of 85 stadii\*", is distant from Panormus, a port of Cephalenia, about 12 stadii †. It has a port named Phorcys; in which there is a shore, and on that shore a cave sacred to the nymphs, in which the Phaeacians are reported to have placed Ulysses."

By no means therefore is this cave a mere Homeric figment. But whether the poet describes it according to its real nature, or adds something of his own invention, yet the same questions remain to be solved; whether you are disposed to investigate the intention of the poet, or of those who consecrated the cave. Since neither did the ancients consecrate temples without fabulous symbols; nor is it usual with Homer to relate any thing rashly concerning their peculiarities. For indeed, by how much the more any one endeavours to shew, that this description of the cave is not an Homeric fiction, but was consecrated to the gods, before Homer's time; by so much the more he evinces, that this sacred cave is filled with ancient wisdom. On which account it is highly worthy our investigation, and necessary that its symbolical consecration and obscure mysteries should be rendered evident by the light of philosophical enquiry.

Antiquity then with great propriety consecrated caves and dens to the world, whether taken collectively as the universe, or separately according to its parts. Hence they considered earth as the symbol of that matter from which the world is composed; so that, according to the opinion of some, matter and earth are the same: by the symbol of a cave, signifying the formation of the world from matter. For indeed caves are most commonly spontaneous productions, congenial with the earth herself, and comprehended by one uniform stone; whose interior part is concave, and whose exterior parts are extended over an immense space of earth. But the world being self-born, (i. e. produced by no external cause but from a principle within,) and in perfect symphony with itself, is allied to matter which they call, according to a secret signification, a stone and a rock. For like these hard bodies it is sluggish and inert, and receives the impression of ornamenting form: at the same time they considered it as infinite on account of its formless nature. But since it is continually flowing, and of itself destitute of the supervening investments of species by which it is formed and becomes visible, the flowing waters, darkness, or, as the poet says, obscurity of the cavern exhibit apt symbols of what

\* I. e. about ten Italian miles.

† Viz. a mile and a half.

the world contains on account of that matter with which it is connected. Hence through the dark union of matter, the world is obscure and dark, but from the presence and supervening ornaments of form (from which its derives its name) it is beautiful and pleasant. The world therefore may with great propriety be called a cave; agreeable indeed, at its first entrance, on account of its participation of form, but involved in the deepest obscurity to the intellectual eye which endeavours to discern its dark foundation. So that its exterior and superficial parts are pleasant, but its interior and profound parts obscure: and its very bottom is darkness itself. After the same manner the Persians mystically signifying the descent of the soul into an inferior nature, and its ascent into the intelligible world, initiate the priest or mystic in a place which they denominate a cave. For according to Eubulus, Zoroaster first of all among the neighbouring mountains of Persia, consecrated a natural cave, florid and watered with fountains, in honour of Mithras the father of all things: a cave in the opinion of Zoroaster bearing a resemblance of the world fabricated by Mithras. But the things contained in the cavern, being disposed by certain intervals, according to symmetry and order, were symbols of the elements and climates of the world. We find too that after Zoroaster it was usual with others to perform initiatory rites in caves and dens, whether natural or artificial. For as they consecrated temples, groves, and altars to the celestial gods; but to the terrestrial gods and to heroes altars alone, and to the subterranean divinities vaults and cells; so to the world they dedicated caves and dens; as likewise to nymphs, on account of the waters trickling, and dispersed through caverns, in which the nymphs called Naiads, as we shall shortly observe, preside. But the ancients not only considered a cave as the symbol of this generated and sensible world, but as the representative of every invisible power: because as a cave is obscure and dark, so the essence of these powers is unknown. Hence Saturn fabricated a cave in the ocean itself, and concealed his children in its dark retreats. Thus Ceres educated Proserpine with her nymphs in a cave; and many other particulars of this kind may be found by any one who peruses the writings of Theologists. But that caves are attributed to nymphs, and especially to Naiads, who dwell near fountains, and are called Naiads from the waters over whose flowing streams they preside, the hymn to Apollo indicates in these words:

\* “ The nymphs residing in caves shall deduce fountains of intellectual waters to thee, (according to the divine voice of the Muses,) which are the progeny of a terrene spirit. Hence waters bursting through every river, shall exhibit to mankind perpetual effusions of sweet streams.” From hence as it appears to me the Pythagoreans, and after them Plato took occasion to call the world a cave and a den. For the powers which are the leaders and guides of souls thus speak in a verse of Empedocles.

“ We will enter into this cave covered with rocks.”

\* These verses are not to be found in any of the hymns now extant, ascribed to Homer.

And Plato in the seventh book of his Republic, speaking of the condition of mankind in this sensible world, says, "Behold men as if dwelling in a subterranean cavern, whose entrance opens through the whole cave to the admission of the light." But when the other person in the dialogue says, you relate an absurd similitude, he subjoins: "It is requisite, friend Glaucus, to apply this similitude to all that has been previously said: assimilating this terrene habitation which is the object of corporeal sight, to the dark residence of a prison: but accomodating the fire shining in the recesses of the cavern to the solar light." And thus it is sufficiently evident, that theologists have considered a cave as a symbol of the world, and of the powers it contains. But we observed that they likewise considered a cave as the symbol of an intelligible essence; led to this opinion by reasons very different from the former. For they placed it as a symbol of the sensible world, because caverns are dark, stony and humid; resembling in all these respects the world on account of the obscure nature of that matter from which it is composed, the continual impression of forms to which it is obnoxious, and the constant flowing of all its parts. But a cave resembles intelligible essence, both because invisible to the eyes and sense, and because its substance is solid, firm, and durable. And in the same manner particular virtues or powers are inconspicuous, especially such as are united with matter. For they did not consider a cave as the symbol of a material and immaterial nature on account of its figure as some have suspected: (since every cave is not circular as appears from this Homeotic cavern with a double entrance) but from surveying the natural condition of caves, involved in the depths of obscurity and night, and formed from the union of a hard and stony substance. Again, since a cave has a twofold similitude, it must agree in some particulars with sensible substance, but in others with an intelligible essence. Thus the present cave since it contains perpetual waters, in this respect resembles a substance united with matter, and not that which is immaterial and intelligible. On this account the cave is not sacred to mountain divinities, to those who dwell on hills, or to other deities of this kind, but to Naiads so called by the Greeks from νεῖλα, fountains; because they preside over waters: and this term is commonly applied to all souls passing into the humid and flowing condition of a generative nature. These souls they considered as incumbent on the water, which is nourished by a divine spirit as Numenius affirms: and hence a prophet said, *that the spirit of God moved on the waters.* The Egyptians likewise on this account place all daemons, not connected with any thing solid or stable, but raised on a sailing vessel; and it is known that humor invades the sun itself, and all animals descending into generation. Hence Heraclitus observes "that it appears delightful, and not mortal to souls, when they are born connected with humidity." And he says in another place, speaking of unembodied souls, "we live their death, and we die their life." Hence the poet calls men existing in generation διηγές, i. e. humid, because their souls are drenched in moisture. On this account too, such souls delight in blood and humid feed: but water administers nutriment.

ment to the souls of plants. Besides, according to the opinions of some men aerial and celestial bodies, are nourished by the vapours of fountains and rivers and other exhalations. Thus the Stoicks assert that the sun is nourished by the exhalation of the sea ; the moon from the effluvia of fountains and rivers ; but the stars from the exhalation of the earth. Hence according to them the sun is a certain intellectual composition formed from the sea ; the moon from river waters ; and the stars from terrene exhalations. It is necessary therefore that souls, whether they are corporeal or incorporeal, while they attract bodies, must verge to humidity, and be incorporated with humid natures ; especially such souls, as from their material inclinations ought to be united with blood, and confined in humid bodies as in a watery tegument. Hence the souls of the dead are evoked by the effusion of bile and blood : and souls insnared by corporeal love, and attracting to their nature a humid spirit, condense this watery vehicle like a cloud ; for a cloud is nothing more than humour condensed in the air. But the pneumatic part thus condensed, through too great an abundance of humour becomes the object of corporeal sight. And among the number of these we must reckon those apparitions of images, which from a spirit coloured by the influence of imagination, present themselves to mankind. But pure souls are averse from generation ; on which account the same Heraclitus observes "a dry soul is the wisest." But souls thus desiring to be mingled with body, and attracting a humid vapour, by their propensity to generation render their pneumatic part moist and wet, and by thus verging to the ever-flowing waters of generation, are deservedly called Naiads. Hence it is customary with the Greeks to call nymphs *νύμφας*, or married, as those who are copulated to generation ; and to wash in a bath whose waters are derived from fountains or perpetual rills. This world then is sacred and pleasant to nymphs, i. e. to souls proceeding into a material nature, and to genii participating of generation, although it is naturally dark and opaque ; on which account some are of opinion that souls are composed from a certain aerial opacity. Hence a cave is a habitation peculiarly adapted to such souls ; since it is both pleasant and obscure, like this material region, in which souls reside. A cave likewise through which perpetual waters flow is well adapted to nymphs, the divinities of waters. The present cave therefore must be allowed sacred to souls, and to those more particular powers denominated nymphs, who from their being praefects of rivers and fountains are called *νεράιδαι* and *νεράϊδαι*, i. e. fountain and river divinities. What then are the different symbols, some of which correspond to souls, and others to the divinities of waters, by which it may be manifest that this cave is at the same time dedicated and consecrated to both ? We reply that the stony bowls and urns are symbols of the aquatic nymphs. For vessels of the same form are symbols of Bacchus ; but their composition is testaceous, that is, from baked earth. And indeed such as these are correspondent to the gift of this god ; since the fruit of the vine is brought to a proper maturity by the celestial fire of the sun. But the stony bowls and urns, are most admirably accommo-

View  
Honey

dated to nymphs presiding over waters which flow from rocks. And what symbol is more proper to souls descending into generation, and the tenacious vestment of body, than as the poet says, "Nymphs weaving on stony beams purple garments wonderful to behold?" For the flesh is generated in and about the bones, which in the bodies of animals may be compared to stones. On which account these textorial instruments, are fabricated from stones alone. But the purple garments plainly appear to be the flesh with which we are invested; and which is woven as it were and grows by the connecting and vivifying power of the blood, diffused through every part. Besides, purple garments are tinged with the blood of animals; and flesh is produced and subsists from blood. Add too that the body is a garment with which the soul is invested; a circumstance indeed wonderful to the sight, whether we regard its composition, or consider the connecting band by which it is knit to the soul. Thus according to Orpheus, Proserpine who presides over every thing generated from seed, is represented weaving a web; and the ancients called heaven by the name of *τινάχος*, which is as it were the veil or tegument of the celestial gods. But why are the amphora represented filled with honey-combs, and not with water? For in these as he says the bees deposit their honey. But the word *τιθεσθαι μέλον*, signifies nothing more than *τιθεσθαι μέλον*, i. e. to deposit aliment. And honey is the nutriment of bees.

Indeed, theologists have made honey subservient to many and various symbols, because it is indued with a variety of powers: for it possesses a purging and preserving quality. Hence bodies are kept from putrefaction by its use, and ulcers of long standing are purified: besides it is sweet to the taste, and bees produced from putrid oxen, collect it by a wonderful art from flowers. On this account when in the sacred rites, called *μαζίας*, those who are initiated, pour honey instead of water on their hands, it is signified by this practice, that their hands should be pure from every sorrowful, noxious, and abominable concern. Thus, others purify the initiated by a purgatorial rite from fire, but are averse from water as the enemy of fire. Besides they purify the tongue from all the défilement of evil with honey. But when the Persians offer honey to the guardian of fruits, they regard its preserving power as a symbol of its similitude to a divine nature. In like manner when the poet pours nectar and ambrosia, into the nostrils of the slain, for the purpose of preserving the body from putrefaction, some have interpreted honey as the aliment of the gods. For Homer in a certain place calls nectar yellow; which is also the colour of honey. But whether or not honey is to be taken for nectar, we shall hereafter more accurately examine. Again, we find in Orpheus that Jupiter employs stratagems against Saturn from honey. For Saturn full of honey is intoxicated, his senses are darkened as if from the effects of wine, and he sleeps: just as Porus, according to Plato, is distended with nectar; for wine (says he) was not yet known. But night admonishes Jupiter to employ the stratagem of honey, according to Orpheus, in these words,

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"As soon as you behold him spread under the lofty oaks, intoxicated with the sweet honey, produced by the bees, bind him in chains."

Saturn, therefore, intoxicated with honey is bound by Jupiter; and castrated in the same manner as Cælum. But the theological poet intimates by this fable that the divine essences are, as it were, bound, and drawn down by delight into the fluctuating empire of generation; and that when resolved in pleasure, they produce certain powers by their seminal virtue. Thus Saturn castrates Cælum, who, by his desire of coition descends to earth. But the intoxication of honey, signifies among theologists nothing more than the desire of coition; by the ensnaring power of which Saturn is castrated. For Saturn and his orb is the first of the celestial spheres, which moves contrary to the course of Cælum or the heavens. But certain virtues descend as well from the heavens as from the wandering stars, and the influences of the heavens are received by Saturn, and those of Saturn by Jupiter. Hence, since honey is assumed in purgations, and as an antidote to putrefaction, and aptly represents the pleasure and delight of descending into the fascinating realms of generation, it is accounted a symbol well adapted to nymphs the divinities of waters; signifying the nature of the waters over which they preside free from putrefaction: intimating likewise the purgative quality of the waters and their co-operating in the business of generation. For water promotes generation. The poet, therefore, very properly represents the bees as depositing their honey in bowls and urns: since bowls signify fountains; and on this account a bowl or cup is placed next to Mythras instead of a fountain. But we draw the waters of fountains in Amphora; and fountains and rivers are proper to aquatic nymphs, and especially to the nymphs called by the ancients souls, which antiquity likewise peculiarly denominated μιλστας, i. e. artificers of sweetness or bees: for souls are, indeed, the authors of all the pleasure peculiar to our nature.

Hence Sophocles does not speak improperly, when he says,

"The swarm of the dead utters a buzzing noise."

But the priestesses of Ceres, as ministers to the terrene goddess were formerly called bees; and her daughter Proserpine μιλστας, or delicious, alluding to the sweetness of honey. Besides the moon who is the queen of generation was denominated by the ancients a bee, and likewise a bull: for the exaltation of the Moon is Taurus, and bees are generated from oxen; on which account they are called βεηνης, which name is likewise attributed to souls proceeding into generation. Also the god Mercury is esteemed a stealer of oxen, who is secretly conscious of generation. Besides honey is considered as a symbol of death, in the same manner as gall is of life; whether they indicated by such similitudes that the life of the soul dies by the noxious embraces of pleasure, but enjoys life from bitterness, which by its disgusting sensation prevents the soul from sinking into that drowsy oblivion produced by corporeal delight (on which account they sacrificed gall to the gods); or whether the symbol originated from considering that death is the end of evils, but that the present life is laborious and bitter. But it is here necessary

to

to observe that they did not promiscuously call all souls descending into the whirl of generation bees ; but only those who, while residing in this fluctuating region, acted justly; and who, after being in a manner acceptable to the divinities returned to their pristine felicity. For the bee is an animal, accustomed to return to its former place ; and is studious of justice and sobriety, on which account libations with honey are called *μελισσαί*, or sober. The ancients likewise refrained from sitting on beans, which they considered as a symbol of generation proceeding in a regular series without being interrupted ; because this leguminous vegetable is almost the only one, amongst other fruits, whose stalk is perforated throughout without any intervening knots. We must, therefore, admit that honey-combs and bees are symbols, as well peculiar as common to nymphs the divinities of waters ; and at the same time to souls wedded to the humid and fluctuating nature of generation.

*(and below)*

But let us now return to the cave and consider its double entrance. The most ancient of mankind then, before temples were raised to divinity, consecrated caves and dens to the gods. Hence the Curetes in Crete dedicated a cave to Jupiter ; in Arcadia a cave was sacred to the Moon, in Lyceum to Pan, and in the island Naxus to Bacchus. The worship of Mithras too, wherever this god was known was performed in caves. But with respect to this cave of the nymphs in Ithaca, Homer was not alone content with saying that it had two gates, but he adds that the one looks to the north, and the other, more divine, to the south ; concerning which he does not mention whether it is pervious to the descent of either immortals or mankind, as is the case with the northern entrance, but he only says,

“ The other of these tends to the south, which is not pervious to men, but is alone open to immortals”

It remains, therefore, to investigate either the secret meaning of those who first instituted this cave, according to the poets description ; or what occult signification Homer himself intended to convey, if it is nothing more than a fiction of his own inventing. Since then, the present cave in an eminent degree is a symbol and image of the world, as Numenius and his familiar Cronius affirm, it is necessary, in order to elucidate the reason of the position of the gates, to observe that there are two extremities in the heavens ; viz. the winter-solstice, than which no part of heaven is nearer to the south ; and the summer-solstice which is situated next to the north. But the summer tropic, that is, the solstitial circle is in Cancer, and the winter tropic in Capricorn. And since Cancer is the nearest to the earth, it is deservedly attributed to the moon, which is itself proximate to the earth. But since the southern pole by its great distance is inconspicuous to us, Capricorn is ascribed to Saturn, who is the highest and most remote of all the planets. Again, the signs from Cancer to Capricorn are situated in the following order ; the first is Leo called by astrologers the house of the sun ; afterwards Virgo, or the house of Mercury ; Libra of Venus ; Scorpius of Mars ; Sagittarius of Jupiter ;

and

and Capricornus or the house of Saturn. But from Capricorn in an inverse order, Sagittarius is attributed to Saturn; Pisces to Jupiter; Aries to Mars; Taurus to Venus; Gemini to Mercury; and last of all Cancer to the Moon. From among the number of these theologists consider Cancer and Capricorn as two ports; Plato calls them two gates. Of these, they affirm that Cancer is the gate through which souls descend, but Capricorn that through which they ascend, and exchange a material for a divine condition of being \*. Cancer is, indeed, northern and adapted

Aquarius

Capricorn

Galaxy

\* Macrobius in the 12th chapter of his comment on Scipio's dream, has derived some of the ancient arcana which it contains, from the present part of this admirable work. What he has farther added, I shall translate on account of its excellence, and connection with the above passage of Porphyry. "Pythagoras (says he) thought that the empire of Pluto, began downwards from the milky way, because souls falling from thence, appear already to have receded from the gods. Hence he asserts, that the nutriment of milk is first offered to infants, because their first motion commences from the galaxy, when they begin to fall into terrestrial bodies. On this account, since those who are about to descend, are yet in *Cancer*, and have not left the milky way, they rank in the order of gods. But when by falling they arrive at the *Lion*; in this constellation, they enter on the exordium of their future condition. And, because in the *Lion*, the rudiments of birth, and certain primary exercises of human nature commence; but *Aquarius* is opposite to the lion, and presently sets, after the lion rises: hence, when the sun is in *Aquarius*, funeral rites are performed to departed souls; because he is then carried in a sign, which is contrary, or adverse to human life. From the confine, therefore, in which the zodiac, and galaxy touch each other, the soul descending from round figure, which is the only divine form, is produced into a cone by its effluxion. And as a line is generated from a point, and proceeds into length, from an indivisible; so the soul from its own point, which is a monad, passes into the duad, which is the first protraction. And this is the essence which Plato in the *Timaeus*, calls indivisible, and at the same time divisible, when he speaks of the fabric of the mundane soul. For as the soul of the world, so likewise that of man will be found in one respect ignorant of division, if the simplicity of a divine nature is considered; and in another respect capacious of division, if we regard the diffusion of the former through the world, and of the latter through the members of the body."

As soon, therefore, as the soul gravitates towards body, in this first production of herself, she begins to experience a material tumult, that is, matter flowing into her essence. And this is what Plato remarks in the *Phædo*, that the soul is drawn into body, staggering with recent intoxication; signifying by this the new drink of matter's impetuous flood, through which the soul becoming defiled and heavy, is drawn into a terrene situation. But the starry cup, placed between Cancer and the Lion, is a symbol of this mystic truth, signifying that descending souls first experience intoxication in that part of the heavens, through the influx of matter. Hence, oblivion the companion of intoxication, there begins silently to creep into the recesses of the soul. For if souls retained in their descent to bodies, the memory of divine concerns of which they were conscious in the heavens, there would be no dissension among men, concerning divinity. But all, indeed, in descending drink of oblivion; though some more, and others less. On this account, though truth is not apparent to all men on the earth, yet all exercise their opinions about it: because a *defect of memory*, is the origin of opinion. But those discover most, who have drank least of oblivion: because they easily remember what they had known before in the heavens. Hence, that which is called *letho* by the Latins, is called by the Greeks *anamnesis*, or repeated knowledge; because when we learn any truths, we recognize what we naturally knew, before material influxion, rushing into the body, had intoxicated the soul. But it is this *hyle* or matter which composes all that body of the world, which we every where perceive adorned with the impressions of forms. Its highest and purest part, however, by which divine natures, are either sustained, or composed, is called *mētēr*, and is believed to be the drink of the gods: but its more inferior and turbid part, is the drink of souls. And this is what the ancients have denominated the river of Lethe. But according to the Orphic writers the *ὕδωρ βασιλεύς*, or *material intellect*, is Bacchus, who proceeding from that indivisible part, is divided into particulars. Hence, in the Orphic mysteries, he is reported to have been torn in pieces, by Titanic fury, and the fragments being buried, are said to have risen entire, and collected.

adapted to descent : but Capricorn, is southern, and accommodated to ascent. And, indeed, the gates of the cave which look to the north, are with great propriety said to be pervious to the descent of men : but the southern gates, are not the avenues of the gods, but of souls ascending to the gods. On this account the poet does not say it is the passage of the gods, but of immortals; which appellation is also common to our souls, whether in their whole essence or from some particular and most excellent part only, they are denominated immortal. It is reported that Parmenides mentions these two ports in his book, concerning the nature of things : as likewise that they were not unknown to the Egyptians and Romans. For the Romans celebrate their Saturnalia when the sun is in Capricorn, and during this festivity the servants wear the shoes of those who are free, and all things are distributed among them in common ; the legislator intimating by this ceremony, that those who are servants at present, by the condition of their birth, will be hereafter liberated by the Saturnalian feast, and by the house attributed to Saturn, i. e. Capricorn ; when reviving in that sign, and being divested of the material garments of generation, they return to their pristine felicity, and to the fountain of life. But since the path beginning from Capricorn is retrograde, and pertains to descent ; hence the origin of the word Januarius or January from Janua a gate, which is the space of time measured by the sun while returning from Capricorn towards the east, he directs his course to the northern parts. But with the Egyptians the beginning of the year is not Aquarius, as among the Romans, but Cancer. For the 2

collected into one ; because intellect by passing into a divisible from an indivisible nature, and again returning from divisible to indivisible, both accomplishes the duties of the world, and does not desert the arcana of its own nature.

The soul, therefore, falling with this first weight, from the zodiac, and milky-way into each of the subject spheres, is not only clothed with the accession of a luminous body, but produces the particular motions, which it is to exercise in the respective orbs. Thus in Saturn, it energises according to a ratiocinative and intellectual power, which they call λογικὴ and Σεμπτιαῖς: in the sphere of Jove, according to the power of acting, which is called ἀρχαῖς: in that of Mars, according to the ardour of courage, which is denominated δυναῖς: in the orb of the sun, according to a sensitive and phantastic nature, which they call αἰσθητικὴ and φανταστικὴ: but according to the motion of desire, which is denominated ἴσθμουτικὴ, in the planet Venus : of pronouncing and interpreting what it perceives, which is called ἐγνωστικὴ, in the orb of Mercury ; and according to a plantal nature, and a power of acting on body, which is denominated φυτικὴ, when it enters the lunar globe. And this sphere, as it is the last among the divine orders, so it is the first in our terrene situation. For this body, as it is the dregs of divine concerns ; so it is the first substance of an animal. And this is the difference between terrene and supernal bodies (under which last, I comprehend the heavens, the stars, and the other elements) that the latter are called upwards to be the seat of the soul, and merit immortality from the very nature of the region, and an imitation of sublimity ; but the soul is drawn down to these terrene bodies, and is on this account reported to die, when it is inclosed in this fallen region, and the seat of mortality. Nor ought it to cause any disturbance, that we have so often named the death of the soul, which we have pronounced to be immortal. For the soul is not extinguished by its own proper death, but is only overwhelmed for a time. Nor does it lose the benefit of perpetuity, by its temporal demission : since when it deserves to be purified from the contagion of vice, through its entire refinement from body ; it will be restored to the light of perennial life, and will return to its pristine integrity and perfection."

star,

star, so this borders on Cancer, which star the Greeks denominate Κύνος, or the Dog. When this star rises they celebrate the calends of the month, which begins their year; because this is the place of the heavens where generation commences, by which the world subsists. On this account the doors of the Homeric cavern, are not dedicated to the east and west, nor to the equinoctial signs, Aries and Libra, but to the north and south, and particularly to those ports or celestial signs which are the nearest of all to these quarters of the world: and this because the present cave is sacred to souls, and to nymphs the divinities of waters. But these places are particularly adapted either to souls descending into generation, or to such as are separating from it. On this account they assigned a place congruous to Mithras, near the equinoctial; and hence he bears the sword of Aries, because this animal is martial, and is the sign of Mars: he is likewise carried in the Bull the sign of Venus; because the Bull as well as Venus is the ruler of generation. But Mithras is placed near the equinoctial circle, comprehending the northern parts on his right, and the southern on his left hand. Likewise to the southern hemisphere they added the south, because it is hot, and to the northern hemisphere, the north, on account of the coldness of the wind in that quarter. Again, it was not without reason that they connected winds with souls sinking to generation, and again separating themselves from its stormy whirl: because, according to the opinion of some, souls attract a spirit, and obtain a pneumatic substance. Indeed, Boreas is proper to souls passing into generation: for the northern blasts recreate those who are on the verge of death; and refresh the soul reluctantly detained in the body. On the contrary, the southern gales dissolve life. For the north, from its superior coldness, collects into one, detains and strengthens the soul in the moist and frigid embraces of terrene generation: but the south dissolves the humid bands, and by its superior heat, having freed the soul from the dark and cold tenement of the body, draws it upward to the incorporeal light and heat of divinity. But since our habitable orb verges mostly to the north, it is proper that souls born in this turbulent region should be conversant with the north wind; and those who depart from hence with the south. It is, indeed, on this account that wind blowing from the north, is immediately on its commencement vehement; but the south, on the contrary, is more vehement towards the end. For the former hangs directly over the inhabitants of the north pole, but the latter is more distant, and the blast from places very remote, is more tardy than from such as are near; but when it is gradually collected it blows abundantly and with vigour. Hence, because souls enter into generation, through the northern gate, they have feigned this wind to be amatorial; and hence the poet:

\* "Boreas changed into the form of a horse mingled himself with the mares of Eriethonius; and they big with young produced twice six foal." And they report that he committed a rape on Orithyia, from whom he begot Zetis and Calais. But attri-

\* Iliad. lib. xx. l. 223, &c.

buting the south to the gods, when the sun is at his meridian, they draw the curtains before the statues of the Gods in temples; and conceal them from the view, observing the Homeric precept, that it is not lawful for men to enter temples when the sun is inclined to the south: "for this path is open to immortals alone."

† Hence when the god is at his meridian they place a symbol of mid-day and of the south in the gate of the temple. Besides, in other gates it was esteemed unlawful to speak at all times; because they considered gates as sacred. On this account too the Pythagoreans, and wise men among the Egyptians, forbade any person to speak while passing through gates or portals; for at that time the divinity who is the principle of the universe is to be worshipped in silence. But Homer was not ignorant that gates are sacred, because he represents Oeneus in the place of supplication knocking at the gate.

Before his gates the aged Oeneus came,  
And suppliant shook their well-compacted frame †.

Besides he knew that the gates of heaven were committed to the care of the hours, commencing in cloudy places; and which are opened and shut by the clouds: for he says,

" Whether they unfold, or close a dense cloud §."

Hence likewise they are said to resound because thunders roar through the clouds.

Heaven's gates spontaneous open to the powers,  
Heavens sounding gates kept by the winged hours ‡‡.

Besides Homer elsewhere makes mention of the gates of the sun, signifying by these Cancer and Capricorn: for the sun proceeds as far as these signs, when he descends from the north to the south; and from thence ascends again to the northern parts. But Capricorn and Cancer are situated about the milky circle, Cancer occupying the northern extremity of this circle, and Capricorn the southern. Again, according to Pythagoras *the people of dreams*\* are souls, which are reported to be collected in the milky way; the appellation.

† In the original ιστανεῖ καὶ σύμβολος τῆς μεσημέριας καὶ ταύτη, ἵπλ τῇ θύρᾳ μεσημέριαζον τῇ θύρᾳ. Which Holstenius translates most erroneously, as follows: "Austrum igitur meridiæ symbolum statuunt: cum Deus meridiano tempore ostio immineat."

‡ Iliad. lib. ix. l. 579.

§ Iliad. lib. viii. l. 395.

‡‡ Iliad. lib. viii. l. 393.

• This assertion of Pythagoras that *the people of dreams*, δῆμος ὄντων, are souls situated in the milky way, admirably contributes to elucidate the following passage in the 24th book of the Odyssey, respecting the descent of the suitors souls to the region of spirits:

Πάρ' οἵσαν οὐκενόν τε ρέας καὶ λευκάδα πίτην,  
Η' δὲ παρ' οὐλίοιο πύλας καὶ δῆμος ὄντων  
Ηίσαν, αἷψε δίκουτο κατ' ἀσφαλέστερα λευκῶν,  
Ἐνθά τε ναίσσει φυχαῖς ηδώλα καρμόγνων.

i.e. "But they passed beyond the flowing waters of the ocean, and the rock Leucas, and the gates of the sun, and *the people of dreams*: and they immediately came into meadows of asphodel, where souls the images of the dead

appellation of which is derived from souls, nourished with milk after their lapse into the whirls of generation. Hence those who desire to evocate departed souls, sacrifice to

dead reside." For it is evident from hence that the souls of the suitors passed through the galaxy, or the seats of the blessed, according to the most ancient theology: and I doubt not but Homer describes in these lines the complicated progression of an impure soul till it regains its original habitation in the stars, and again begins to gravitate to this terrene abode. This, I presume, will be manifest from the following elucidation of these admirable lines.

In the first place those souls are said to pass over *the flowing waters of the ocean*, and *the Leucadian, or white rock*. Now by this nothing more is meant than the flight of the suitors souls to the extremity of the earth, in order to a subterranean descent: for, according to the most ancient opinion, the earth is bounded by the ocean; and the Leucadian rock may, as Eustathius observes, be some rock on the earth's extremity, which receives the last rays of the sun. Afterwards they are said to pass through *the gates of the sun*, by which, as Porphyry informs us above, we must understand the tropics of Cancer and Capricorn: and as Capricorn is subterranean, and affords a passage to ascending immortals, we must conceive that they enter through this prior to the tropic of Cancer. But in order to comprehend the perfect propriety of this transition, we must observe that the souls of the suitors on account of their impurity, are punished in the recesses of the earth, before they enter the celestial tropics, and pass into the meadows of *Pluto*. This the poet evidently evinces by the *screaking noise*, which they utter, and the squalid paths, through which they descend: a noise of this kind as Proclus well observes, (in Plat. Repub.) "representing a species of life solely given to appetite and imagination." After they have been purified therefore by subterranean punishment, they are fit to ascend to *the people of dreams*, or the souls of the blessed situated in the milky way. However as the soul, on account of her middle nature, is incapable of a perpetual sameness of situation, but is formed for infinite circulations (as will be demonstrated in the following elements); hence Homer, without mentioning her duration among the gods, though it is doubtless very extended, agreeable to the mystic brevity of his Muse, makes her immediately pass into *the meadows of Asphodel*, *where souls the images of the dead reside*. Now these *meadows of Asphodel*, form the supreme part of *Plato's dominions*: for, according to Pythagoras, as we are informed by Macrobius, in the preceding note to page 287, the empire of Pluto commences downwards from the milky way; so that these meadows are most probably situated in *the Lion*, the constellation into which souls first fall, after they leave the tropic of Cancer. But the description of the Asphodel, perfectly corresponds with, and confirms the preceding exposition. For the Asphodel is a plant, bearing a naked *flarry flower*, which comprehends an ovarium, or *orbicular figure*, after the manner of a cup. And what can more aptly symbolize with the stars, than such a flower? It was usual too with the ancients to scatter this plant in the tombs of the deceased; occultly intimating most probably, the similitude of this flower to the pristine and proper habitations of the soul. And hence it derived its appellation from *ανθός*; *αφεις*, διά στήν καιρέσσων οὐρών, i. e. from the ashes of burnt dead bodies.

In these meadows, then, *the images of the dead* are said to reside. Now these *ἴδεα*, or *images*, are no other than those vehicles of the soul, which have been so copiously, and admirably discussed by Syncellus, in the preceding quotation from his book on dreams; and which, from their residence in the starry regions, must be luciform, ethereal, and pure. It is this phantastic spirit, or primary vehicle of the soul, which Virgil alludes to, in these beautiful lines:

exinde per amplum

Mittimur Elysium, et pauci lata arva tenenus:  
Donec longa dies perfecto temporis orbe  
Concretum exemit labem, purumque reliquit  
Ætherium sensum, atque aurae simplicis iguem.

i. e. "We are afterwards sent through ample Elysium, and a few of us possess the joyful plains: till a long period, when the revolving orb of time has perfected its circulation, frees the soul from its concrete stains, and leaves the ethereal sense pure, together with *the fire (or splendour) of simple ether*." For here he evidently *conjoins* the rational soul, or the ethereal sense, with its splendid vehicle, or *the fire of simple ether*; since it is well known that this vehicle according to Plato, is rendered by proper purgation *ἀβοτίς*, or *luciform*, and divine. It must here however be observed that souls in these *meadows of Asphodel*, or *summit of Pluto's empire*, are in a falling state; or other words through the secret influx of matter, begin to desire a terrene situation. And this explains the reason why Hercules in the infernal regions, is represented by Homer boasting of his terrene exploits and glory-

to them with milk sweetned with honey: convinced that by the allurements of pleasure, these souls would desire to pass into generation, with the very beginning of which milk is generally produced.

Besides the southern regions produce small bodies, because being attenuated by the heat they are diminished and dried up: and by a contrary reason all bodies generated in the north are large, as is evident in the Celte or Gauds, Thracians, and Scythians; and these regions are humid and abound with much pasture. For the word Boreas is derived from the Greek *βορεας*, which signifies aliment. Hence also the wind which blows from a land abounding in nutriments is called *βορειος*; or nutritive. From these causes therefore the northern parts are properly adapted to the class of souls obnoxious to mortality and generation; but the southern quarter to immortals, exempt from the mutability inseparable from the flowing realms of generation: in the same manner as the east is attributed to the gods, and the west to daemons. Hence since diversity is the origin of nature, the ancients considered every thing with a double entrance, as the symbol of nature. For the progression of things is either through an intelligible or a sensible nature. And if through a sensible nature, either through the sphere of the

ing in his pristine valour: why Achilles laments his situation in these abodes; and souls in general are engaged in pursuits, similar to their employments on the earth: for all this is the natural consequence of a propensity to a mortal nature, and a desertion of the regions every way lucid and divine. Let the reader too observe, that, according to the *arcana* of the Platonic doctrine, the first and truest seat of the soul, is in the intelligible-world, where she lives entirely divested of body, and enjoys the ultimate felicity of her nature. And this is what Homer divinely insinuates when he says:

Τὸν δὲ μετ', εἰσενόμοια θίνεν Ήρακληνίν,  
Εἴδωλον' αὐτὸς δὲ μετ' αθανάτους θεοῖς  
Τίγκεται, οὐ θαλήγε, καὶ ἔχει παλλαῖς φέρεν "Ηέλιον. — ο. τ. λ.:

i. e. "after this I saw the Herculian power, or *image*: but Hercules himself is with the immortal gods, delighting in celestial banquets, and enjoying the beautiful-footed Hebe." Since for the soul to dwell with the gods entirely separated from its vehicle, is to abide in the intelligible world, and to exercise, as Plotinus expresses it, the more sacred contests of wisdom.

Should it be enquired why departed souls, though in a state of felicity are compared by Homer to *dreams* and shadows, I answer with Porphyry (apud Stob. p. 132.) that they are shadows with respect to human concerns, both because they are destitute of body, and are void of memory: for after they have passed the Stygian river, they are entirely ignorant of their pristine life on the earth, though they recognize, and converse with each other, as is evident from the discourses between Patroclus, Ajax, and Antilochus. Indeed together with memory, they lose all knowledge of corporeal resemblances, which are rendered apparent through the ministry of the phantasy. For since the phantasy consists from memory, as Plato asserts in the Phelebus; whatever we imagine perishes with the memory; and when this is taken away all the perturbations of the soul are removed, so she then becomes wholly intellectual, and passes into a state divinely prudent and wise. However, by means of the blood, which, as we have before observed, is, according to Homer, the instrument of the phantastic soul, departed spirits recognize material forms, and recollect their pristine condition on the earth. And to the phantasy reasoning pertains; since it is nothing more than an aggregation of memory, collected through imaginations, into the judgment of universals. But this is very different from the *intellective energy*, acquired by the soul beyond Acheron, which Cocytus and Pyriphlegethon fill, from the whirling streams of the dreadful Styx. Let the reader, however, remember that the phantasy is twofold, communicating in its supreme part with the rational soul, and in its inferior part with sense; and that it is this inferior part which the soul deserts, when it acquires an intellectual condition of being.

fixed.

fixed stars, or through the orbs of the planets ; and again either with an immortal or a mortal motion. Likewise one centre or hinge of the world is above the earth, but the other is subterranean ; and one part of the heavens is eastern, and another western. In like manner some parts of the world have a dexter, and others a sinister position. Thus too night is opposed to day ; and the harmony of the universe consists from the amicable junction of contrary and not similar natures. Plato also makes mention of two gates, one of which affords a passage to those ascending into the heavens, the other to those descending on the earth : and theologists place the sun and moon as the gates of souls, which ascend through the sun and descend through the moon. So, according to Homer,

“ Two urns by Jove’s high throne have ever stood,  
The source of evil one, and one of good \*.”

But Plato, in his Gorgias, by vases understands souls, some of which are beneficent, and others malignant, and again some are rational and others irrational. But souls are denominated vases because they are capacious of certain energies and habits, after the manner of vessels. In Hesiod too we find one vase shut, but the other opened by pleasure, who diffuses its contents, and leaves nothing but hope behind. For in whatever concerns a depraved soul diffused about the dark and turbulent nature of matter, deserts the proper order of its essence ; in all these, it is accustomed to nourish itself with the pleasing though delusive prospects of hope.

Since then every twofold division is a symbol of nature, this Homeric cavern has with great propriety two gates, numerically different ; the one peculiar to gods and pure souls ; but the other to such as are mortal and depraved. Hence Plato took occasion to speak of bowls, and to substitute vases for Amphora, and two gates, as we have already observed, in the place of two ports. Also Pherecydes Syrus, mentions recesses, and dens, caves, gates, and ports, by which he insinuates the generation of souls, and their separation from a material nature. And thus much for an interpretation of Homer’s cave, which we appear to have sufficiently explained, without adducing any farther testimonies from ancient philosophers and theologists, which would give an unreasonable extent to our discourse.

One particular however remains to be explained, and that is the symbol of the olive at the top of the cavern ; since Homer appears to insinuate something egregious by giving such a position : for he does not merely say that an olive grows in this place, but that it flourishes at the head or vertex of the cave.

“ High at the head a branching olive grows,  
Beneath a gloomy grotto’s cool recess, &c.”

But the growth of the olive in such a situation is not fortuitous as some may suspect, since it finishes and contains the enigma of the cave. For as the world was not produced by the blind concurrence of chance, but is the work of divine wisdom and an intellec-

\* Iliad. xxiv. l. 527.

tual nature, hence an olive the symbol of divine wisdom, flourishes near the present cavern, which is an emblem of the material world. For the olive is the plant of Minerva, and Minerva is wisdom. And since this goddess was produced from the head of Jupiter, the theological poet gives a proper position to the olive, consecrated at the head of the port: signifying by this symbol that the universe is the offspring of an intelligible nature, separated indeed by a diversity of essence, though not by distance of place from his work; and by unremitting and ever present energies, not remote from any part of the universe, but situated as it were on its very summit, that is governing the whole with perfect wisdom from the dignity and excellence of his nature. But since an olive always flourishes, it bears a similitude peculiar and convenient to the revolutions of souls in this material region. For in summer the white part of the leaves is upwards, but in winter it is bent downwards. On this account also in prayers and supplications they extend the branches of an olive, presaging from this omen that they shall exchange the sorrowful darkness of danger for the fair light of security and peace. But the olive is not only of an ever-flourishing nature, it likewise bears fruit, which is the reward of labour, is sacred to Minerva, supplies the victors in athletic labours with crowns, and affords a friendly branch to the suppliant petitioner. Thus too the world is governed by an intellectual nature, and a wisdom ever flourishing and vigilant, who also bestows on the conquerors in the athletic race of life, the crown of victory, as the reward of severe toil, and patient perseverance: and the mighty builder who supports the universe by his divine energies, invigorates miserable and suppliant souls, contending for the most glorious of all prizes, the olympiad of the soul.

In this cave therefore, says Homer, all external possessions must be deposited; here, naked and assuming a suppliant habit, afflicted in body, and casting aside every thing superfluous, sense too being averse from needless possessions, it is requisite to sit at the foot of the olive, and consult with Minerva, by what means we may most effectually amputate and destroy that hostile rout of passions, which lurk in the secret recesses of the soul. Indeed as it appears to me it was not without foundation that Numenius thought the person of Ulysses in the Odyssey represented to us a man who passes in a regular manner over the dark and stormy sea of generation \*; and thus at length arrives at that region, where tempests and seas are unknown, and finds a nation

“ Who ne'er knew salt, or heard the billows roar.”

Again, according to Plato, the deep, the sea, and a tempest are so many symbols of the constitution.

\* This was no doubt fully proved by Porphyry, in some of his unfortunately lost writings; such as his book *on the philosophy of Homer*; or that, *on the Allegories of the Greek and Egyptian Theology*, of which we have already made mention. However, it does not seem impossible, from the hints afforded us in this excellent treatise, for a person conversant in the Platonic philosophy, to evince the truth of this assertion. Presuming, therefore, that an attempt of this kind will be acceptable to the literal reader, though my abilities are far inferior to those of Porphyry; I shall request his attention to the contents, and his pardon for the length of the ensuing discourse. I only premise, that I shall make use of a small treatise in Greek, *on the wanderings of Ulysses*, by an anonymous author, where he appears to have penetrated the sense of the allegory; and freely reject his interpretation.

constitution of matter: and on this account, I think, the poet called that port by the name of the marine god Phorcys."

" But

terpretation, when foreign from the leading character of Ulysses, above mentioned, according to Numenius and Porphyry.

The first apparently fabulous adventure then of Ulysses, is that of the Lotophagi; a coast upon which he was driven by the violence of the winds; and which produced a fruit called Lotos, whose enchanting taste caused oblivion, and a vehement desire of perpetually abiding with the Lotophagi. Or as it is beautifully described by Homer, in the elegant version of Mr. Pope:

The trees around them all their fruit produce,  
Lotos the name, divine, nectarious juice!  
(Thence call'd Lotophagi) which who so tastes,  
Infatiate riots in the sweet repasts,  
Nor other home, nor other care intends,  
But quits his house, his country, and his friends:  
The three we sent from off th' enchanting ground.  
We dragg'd reluctant, and by force we bound:  
The rest in haste forsook the pleasing shore,  
Or, the charm tasted, had return'd no more.

Lib. ix. l. 94. &c.

Here we view Ulysses subject to different desires, and blown about by contrary blasts of fortune, inconsiderately permitting the various affections of his soul to be engaged in improper employments; who, after they have tasted of sensual pleasures, and fraudulent manners, refuse to return to ruling intellect, and the natural good habits of the soul. Hence, fascinated by delight, they consign their country and kindred to oblivion; and desire to live forever lost in the intoxication of pleasure. Reason, however, no longer sustaining that her dignity should be obscured by depraved enjoyment, immediately recalls the affections from fraudulent deeds; which is represented by Ulysses withdrawing his men from the coasts of Lotophagi. After this she binds and restrains them, by the most weighty arguments, so as to prevent them from afterwards returning to evil. And indeed, she treats the affected parts of the soul, in a manner accommodated to their disease; and strengthens and fortifies such as are not affected, and removes them from the possession of noxious delight.

The next adventure of Ulysses is that of the Cyclops, whom he deprived of sight, and irritated with reproaches. Now, according to Porphyry in the above excellent treatise, this is no other than a malignant natal dæmon, who destroys by his stratagems the good affections of the soul; which is occultly signified by Ulysses loosing his companions, through the brutal appetite of the destructive dæmon. Ulysses, however, lamenting the dreadful effects of his voluntary confinement, meditates, and at length accomplishes his deliverance, together with that of the remaining good habits of his soul; and glories in resuming the liberty he has lost. But this fable indeed contains one of the greatest arcana of philosophy: for this is what Synesius occultly intimates, in the preceding quotation from his book on dreams; when he speaks of matter marking her votaries in her secret volume, and severely punishing such as desert from her service. Thus Ulysses after a voluntary submission to his natal dæmon, by indulging the irrational appetites and desires of his soul, flies from his base servitude; and adds irritations to his flight. He is, however, pursued by the anger of the marine and material dæmons, and punished for his escape. For he who blinds the eye of sense, and extinguishes its light, after his will has profoundly assented to its use, must expect punishment for the attempt; as necessary to his own private good, and the general order of the universe. Indeed, troubles and misfortunes resulting from such undertakings, not only contribute to appease the anger of their malevolent authors, but likewise purify and benefit the subjects of their revenge. He, therefore, who in the present life, perceives himself like Ulysses, attended by uncommon misfortunes, while he is in the road of virtue, and eagerly searching for wisdom, may safely conclude, that either here, or in a prior state of existence, he has voluntarily submitted to the power of his natal dæmon, and has now deprived him of sight; and that he has been profoundly delighted with the nature of matter, and is now abrogating the confessions which he made. This too is insinuated in the beautiful story of Psyche, by Apuleius, when the terrestrial Venus, sends Mercury, with a book in which her name is inscribed, to apprehend Psyche as a fugitive from her mistress. For this whole story relates to the descent of the soul into this terrene body, and its wanderings and punishments.

" But it is the port of the ancient marine Phorcys †".

Likewise his daughter Thoosa, is mentioned in the beginning of the Odyssey. But from Thoosa the Cyclops was born, whom Ulysses deprived of sight that he might by this

nishments till it returns to its true country and pristine felicity. And this I may probably demonstrate in some future period, by publishing a translation of this admirable fable, and a comment on the divine mysteries it contains. We must here, however, observe, that as the advancements of Ulysses in virtue are but moderate, till he leaves Calypso ; so the greatest of his troubles commence after that period, as our discourse will shortly evince.

In the next fable, which is that of Æolus, a being, as the name implies, various and versatile, though hospitable and benignant, we see Ulysses sorrowful and wandering, through the anger of his natal dæmon, enquiring after a refuge from misfortune ; though not as becomes one studious of piety, but committing himself to enchanters and magicians, and relying on their incantations for his deliverance from danger. He cannot, however, accomplish his end, by such undertakings, but remains frustrated of his hope, and filled with shame for his disappointment. The Poet too, by attributing the ill success of Ulysses to his sleep, egregiously insinuates that the rational soul was in a dormant state, when he confined in practices so incapable of producing the desired end, and so inconsistent with the goods which intellect confers. Such methods, indeed, as they increase the desire of success, so they present strongly to our view, the distant object of our pursuit ; but this is immediately succeeded by the sleep of reason, and the destruction of hope. And this is what Homer, appears to signify, by the following verses :

Nine prosp'rous days, we ply'd the lab'ring oar ;  
 The tenth presents our welcome native shore :  
 The hills display the beacon's friendly light,  
 And rising mountains gain upon our sight.  
 Then first my eyes by watchful toils oppress,  
 Compl'y'd to take the balmy gifts of rest ;  
 Then first my hands did from the rudder part,  
 (So much the love of home posses'd my heart.)

Lib. x. l. 28. &c.

And hence :

————— the thongs unbound,  
 The gushing tempest sweeps the ocean round ;  
 Snatch'd in the whirl, the hurried navy flew,  
 The ocean widen'd, and the shores withdrew.

After this succeeds the adventure of the Lefrigums which seems to indicate the yet imperfect condition of Ulysses' nature, unable to distinguish the coast of virtue, from the infamous regions of vice. Hence he becomes an involuntary prey to the depredations of depraved manners ; and is for some time incapable of exerting the power of reason. However, at length perceiving the magnitude of the evils with which he is surrounded, he cuts the detaining cables of vice, and flies from his dangerous situation ; deplored, indeed the ruined state of his better manners, but rejoicing that his principal part has escaped, and that he is not totally destroyed. And this the poet appears to me to insinuate by the following verses :

Whilst thus their fury rages at the bay,  
 My sword our cables cut, I call'd to weigh ;  
 And charg'd my men, as they from fate would fly  
 Each nerve to strain, each bending oar to ply.  
 The sailors catch the word, their oars they seize,  
 And sweep with equal strokes the smoky seas ;  
 Clear of the rocks th' impatient vessel flies ;  
 Whilst in the port each wretch encumber'd dies.

With

† Φορκυς δι της ιδι λημνη αλισσα γεγονος.

this means while sailing over the stormy ocean be reminded of his sins, till he was safely landed in his native country. On this account too, a seat under the olive is proper

With earnest haste my frightened sailors press,  
While kindling transports glow'd at our success;  
But the sad fate that did our friends destroy  
Cool'd every breast, and damp'd the rising joy.

Lib. x. l. 125, &c.

In the next fable, which is the beautiful allegory of Circe, we shall find some deep arcana of philosophy contained, exclusive of its connection with Ulysses. By the Æean island then, in which the palace of Circe was situated, we must conceive the region of sorrow and lamentation; for this word is evidently derived from the interjection *alas!* and the adjective *alas;*, *lamentable*. And by Circe we must understand the goddess of sense: for thus Porphyry in Stobæus p. 141. Ομηρος δὲ τοι επειδη περιστοι γέ περιφερει παλιγγενεσις, Κίρκη περιγενεσις, ηλιος παῖδα, τη πάστων φθορά γένεσι, γέ γενεσι ἀν παλιγγενεσις αλις γέ συναρπάζει. i. e. "Homer calls Circe, the daughter of the sun, the period and revolution of regeneration in a circle, who ever connects and combines all corruption with generation, and generation again with corruption." Hence, we may observe that the Æean isle, or this region of sense, is with great propriety called the abode of trouble and lamentation. In this region then, the companions of Ulysses, that is, the thoughts and natural powers of his soul, are changed by the incantations of the goddess; and his opinions and natural motions, rashly wandering from the authority of ruling intellect, are converted through the allurements of delight, into an unworthy and irrational habit. Ulysses, however, or the rational soul, is by the assistance of Mercury, or reason, prevented from destruction. Hence, intellect roused by its impulsive power, and recollecting the ills which its natural faculties endure; at the same time, being armed with prudent anger, and the plant moly, or virtue, which is able to repel the allurements of pleasure, wars on the goddess of sense, and prevents the effects of her fascinating charms. Nor is reason alone free from the dire incantations of delight, but it likewise restores to their proper form the powers of nature, which had been previously corrupted; and thus departs a gain by its loss. For he who returns to himself from the dominion of vice, derives at least this advantage in his return, that he becomes afterwards more prudent in resisting its incursions, and employs his first defeat as an incitement to the acquisition of virtue. It must here, however, be observed that Ulysses is an involuntary offender, in all his adventures posterior to that of the Cyclops. His passions, indeed, hurry him into various vices and misfortunes, but his will by no means concurs with their endurance. But his connection with his natal dæmon was voluntary; and after his departure from hence, he must be considered as in a gradual course of purification, though his progress in virtue is but small, till the latter part of his abode with Calypso.

But Homer's account of Circe, exclusive of its relation to Ulysses, contains, according to Porphyry (in Stob. p. 141), an admirable explanation of the soul. "For thus, (says he) Homer speaks:

No more was seen the human form divine;  
Head, face, and members, bristle into swine;  
Still cursèt with sense, there *minds* remain alone,  
And their own voice, affrights them when they groan.

Lib. x. l. 239, &c.

This fable, therefore, is the enigma of the opinions of Pythagoras and Plato respecting the soul; signifying that it is of an incorruptible and eternal nature, but not void of passion and mutability; since by dissolution and death, it is capable of being transmuted and changed, into other corporeal forms; and by its desire of pleasure it pursues a form adapted and allied to the condition of its life. And in this the assistance of learning and philosophy is perceived, if the soul, mindful of what is honest, and disdaining base and unlawful pleasures, can govern and defend herself from being changed into a beast; and from embracing a brutal and impure body, which increases and nourishes a nature dull and irrational, together with desire and anger, rather than reason. Indeed, the order and nature of this transmutation is predicated by the dæmon of Empedocles, when he says:

Σάρκας ἀλλήγωτι περιστάλλεται χιτώνι.  
Καὶ μετ' ἀμειχεύτα ταῖς φυξαῖς.

i. e. folding round the external garment of flesh, and afterwards investing souls with its covering. But the Æean isle,

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which

per to Ulysses, as to one who supplicates divinity, and would please his natal daemon with a suppliant branch. For indeed it will not be lawful for any one to depart from this

which receives the dead body is that part of the continent, into which souls first descending wander and lament, and are ignorant

what coast before them lies  
Or where the sun shall set, or where shall rise.

Indeed, since through the love of pleasure, they desire an association, and nourishment in the flesh, and in conjunction with its nature, they again fall into the confused mixture of generation, truly mingling things eternal and mortal, prudence and passion, celestials, and terrestrials; ensnared and fascinated by pleasures, again leading to the fluctuating realms of generation. And in this case, souls particularly require the greatest felicity and prudence; lest pursuing the most base concerns, and becoming bound to their parts and passions, they obtain an unhappy and brutal life. For that which is called the *τριάδα*, or triple path of Hades, is perceived in the *rational*, *irrational*, and *desiderative* parts of the soul; each of which contains the principle of a life convenient to its nature. And these assertions are not to be reckoned the figments of fables and the inventions of poets, but are to be esteemed as true and natural discourses. For those whose *desires*, in their mutation and generation, obtain the principality, will be changed into affinine bodies, and an impure life, through the dominion of gluttony and lust. But when the soul raging with weighty contentions, and odious cruelties, seeks a second generation, it betakes itself full of recent severity, into the nature of a wolf, or lion; acquiring a body of this kind as a defensive organ, adapted to its ruling affection. Hence, it is requisite that every one should be pure with respect to death, as in the sacred mysteries of initiation, by banishing every depraved affection, mitigating every desire, and expelling envy and anger from all connection with the body. And this is the true Mercury with his rod of gold, the clear indicator of honest conduct, who entirely prohibits and restrains the soul from the mixture of generation; or if she should drink the envenomed potion, preserves her in a human life, as long as can possibly be effected."

After this follows the allegory, respecting the descent of Ulysses into the infernal regions, which, exclusive of its connection with Ulysses, contains likewise some of the greatest arcana of the Grecian theology. As it respects Ulysses, it appears to me to insinuate, his flying to the assistance of necromancy, in order to know the result of the ill with which he is surrounded, through the anger of his natal daemon. Hence Tiresias is nothing more than a departed spirit evoked by magical art, for the purpose of disclosing the secrets of futurity, and informing Ulysses how he may return to the true empire of his mind. The success, however, was not answerable to the certainty of the information; and perhaps Homer meant to intimate by this allegory, that the end of such illicit practices is never correspondent to the desires by which they are undertaken. Hence he plainly indicates the dangers of such a conduct, by the consequences which may possibly attend its execution; and by the horror which forced Ulysses to hasten its conclusion: for thus Ulysses speaks:

Curious to view the kings of ancient days,  
The mighty dead that live in endless praise,  
Resolv'd I stand; and haply had survey'd  
The god-like Theseus, and Perithous' shade;  
But swarms of spectres rose from deepest hell,  
With bloodless visage, and with hideous yell,  
They scream, they shriek; sad groans and dismal sounds  
Stun my scar'd ears, and pierce hell's utmost bounds.  
No more my heart the dismal din sustains,  
And my cold blood hangs shiv'ring in my veins;  
Left Gorgon rising from th' infernal lakes,  
With horrors arm'd, and curls of hissing snakes,  
Should fix me stiffen'd at the monstrous sight,  
A stony image, in eternal night!      Lib. ii. l. 627. &c.

Indeed

this sensible life in a regular way and in the shortest time, who blinds and irritates his material dæmon; but he who dares to do this, will be pursued by the anger of the marine

Indeed by such a conduct, he becomes impious, profane, and execrable; till he returns to that condition of mind, in which the judgement of reason, and the light of intellect emerges through the gloom of impiety and folly; and prudent cogitations dance round the liberated soul. For in this case it may be truly said:

Here the gay morn resides in radiant bow'rs,  
Here keeps her revels with the dancing hours. Lib. xii. l. 3. & 4.

With respect to the recondite wisdom contained in the description of the infernal regions, I shall only observe from Porphyry (ap. Stob. p. 131.) that the reason why departed spirits, are represented as possessing no knowledge of human concerns, till they inhale the vapour of blood, is because according to Homer and many of his successors, human intelligence or prudence consists in blood. And this says Porphyry is confirmed by the testimony of most writers posterior to Homer, who inform us that when the blood is inflamed by a fever or the bile, imprudence and foolishness is produced. But Empedocles considered the blood as an instrument of prudence, when he says:

Αἵματος οὐ πλάγεσι τετραμύλιον αὐτόθεντος,  
Τὸν τούτου μάλιστα κακίσκεται αὐθόποτον.  
Αἷμα γαρ ἀνθρώποις περικέρδιον οὐτούτοις.

The sense of which is, "that the blood surrounding the heart is the seat of intelligence in men."

But we must now view Ulysses passing from sense to imagination; in the course of which voyage, he is assailed by various temptations, of surprising power, and destructive effect. We shall perceive him victorious in some of these, and sinking under others; but struggling against the incursions of all. Among the first of these is the enchanting melody of the Sirens,

Whose song is death, and makes destruction please.

By which the poet evidently signifies alluring and fraudulent pleasures, which charm the soul in its passage from a sensible life, with flattering and mellifluous incantations. These delights however will be vanquished by him, who, imitating the example of Ulysses, closes, with divine reasons and energies as with wax, the powers of the soul, and the organs of sense; so that every passage being barred from access, they may in vain warble the song of extacy, and expect to ruin the soul by the enchanting strain. It will however be requisite that besides this, the corporeal assaults, should be restrained by the bands of philosophy, and rendered irresistible by external machines: for thus like Ulysses, we shall employ the senses, without yielding to their impetuous invasions; and experience delight without resigning the empire of reason to its fascinating controul.

Ulysses having escaped the danger of the Syrens, passes on to the rocks of Scylla and Charybdis, of terrific appearance, and irresistible force. By these two rocks, the poet seems to signify the affections compressing human life on both sides; and which every one must experience, who proceeds like Ulysses in a regular manner to an intellectual state of existence. Some of these, which are conversant with the soul, are like Scylla, of a lofty malignity; fraudulent yet latent, and obscure, as concealed in the penetralia of the mind. And such is pride, and other depraved affections of the soul. In these recesses, a dæmon, the prince of such affections resides, a dire and enraged dog, who partly exposes his own malice, and partly hides it in impenetrable obscurity. Hence he is capable of producing mischief in a twofold respect: for he privately hurts by malignant stratagems, openly ravishes the soul, on the lofty rock of haughtiness, and rends it with the triple evil of deadly teeth, I mean revolt, hatred of humanity, and haughty arrogance. Indeed a dæmon of this kind will be perpetually vigilant, in endeavouring to destroy, at one time the whole; and at another time a part of the soul, struggling like Ulysses against passion, and yielding reluctantly to its invasions.

But the other affections which belong to the body, are indeed lofty, and evidently destructive, but far inferior to the others; since their baseness is conspicuous, and not concealed by ostentation. A wild fig-tree, that is the will, is produced on the top of this rock; wild indeed, on account of its free nature, but sweet in

Q. q. 2 fruition

rine and material gods, whom it is first requisite to appease, by sacrifices, labours, and patient endurance; at one time by contending with perturbations, at another time by employing

fruition; and under which, often through the day, the impetuosity of the boiling body, are accustomed to absorb, and disturb the man, agitating upwards and downwards inflamed desire, so that mighty destruction both to soul and body, is produced by their mutual concert. But it is highly proper that a rock of this last kind, should be anxiously avoided by one, who like Ulysses is labouring to return to his true country and friends. Hence if necessity requires he will rather expose himself to the other: for there the energy of cogitation, and of the souls simple motions, is alone necessary to be exerted; and it is easy to recover the pristine habit of the soul. In short the poet, seems to represent by this allegory of the two rocks, as well the dangers spontaneously arising from the soul, as those which happen from the external mass of matter; both of which must be sustained, or one at least by a necessary consequence. For it is impossible that neither of them should be experienced by any one, who is passing over the stormy ocean of a sensible life.

After this succeeds the allegory of the Trinacrian isle, containing the herds sacred to the god of day, which were violated by the companions of Ulysses; but not without destruction to the authors of this impiety, and the most dreadful danger to Ulysses. By the result of this fable, the poet evidently shews that punishment attends the sacrilegious, and the perjured; and teaches us that we should perpetually reverence divinity with the greatest sanctity of mind, and be cautious how we commit any thing in divine concerns contrary to piety of manners, and purity of thought. But Homer by attributing sense to the flesh and hides of the slain herds, manifestly convinces, that every base deed, universally proclaims the iniquity of its author: but that perjury and sacrilege are attended with the most glaring indications of guilt, and the most horrid signatures of approaching vengeance, and inevitable ruin. We may here too observe that the will of Ulysses, was far from consenting to this impudent; and that though his passions prevailed at length over his reason, it was not till after frequent admonitions had been employed, and great diligence exerted to prevent its execution. This indeed is so eminently true, that his guilt was the consequence of surprize, and not of premeditated design; which Homer appears to insinuate, by relating that Ulysses was asleep, when his associates committed the offence.

In the next fable, we find Ulysses impelled, by the southern wind, towards the rocks of Scylla and Charybdis; in the latter of which he found safety, by clinging on the fig-tree which grew on its summit, till he refuted the mast, on which he rode after the tempest. But the secret meaning of the allegory appears to me as follows: Ulysses who has not yet taken his leave of a sensible life, is driven by the warmth of passion, represented by the southern gales, into the dire charybdis of insane desires, which frequently boiling over, and tossing on high the storms of depraved affections, plunges into ruin, the soul obnoxious to its waves. However he is far from shameful idleness, and careless security; but perceiving the danger to which he is exposed, when the base storm begin to swell, and the whirlpools of depravity roar, he seizes the helm of temperance, and binds himself fast to the solid texture of his remaining virtue. The waves of desire are indeed tempestuous in the extreme; but before he is forcibly merged by the rage of the affections, into the depths of depravity, he tenaciously adheres to his unconsenting will, seated as it were on the lofty summit of terrene desire. For this like the wild fig-tree affords the best refuge to the soul struggling with the billows of base perturbations. Hence he by this means recovers the integrity which he had lost, and afterwards swims without danger over the waves of temptation; ever watchful and assiduous while he sails through this impetuous river of the flesh, and is exposed to the stormy blasts of heated passion and destructive vice. Hence too while he is thus affected, and anxious lest the loss from unworthy affections should return upon himself, he will escape being lacerated by the teeth of arrogance, though he should terribly and fiercely bark in the neighbourhood of desire, and endeavour like Scylla, to snatch him on her lofty rock. For those who are involuntarily disturbed like Ulysses by the billows of desire, suffer no inconvenience from the depraved rock of pride: but considering the danger of their present situation, they relinquish confident conceit, for modest diffidence, and anxious hope.

Hitherto we have followed Ulysses in his voyage over the turbulent and dangerous ocean of sense; in which we have seen him struggling against the storms of temptation, and in danger of perishing through the tempestuous billows of vice. We must now attend him in the region of imagination, and mark his progress from the enchanted island, till he regains the long lost empire of his soul. That the poet then by Calypso occultly signifies

employing stratagems of various kinds, by all which he transmutes himself into different forms; so that at length being stripped of the torn garments by which his true person was

fies the phantasy, is I think evident from his description of her abode, (for the anonymous Greek author, affords us no farther assistance.) For she is represented as dwelling in a cavern, illuminated by a great fire; and this cave, is surrounded with a thick wood, is watered by four fountains, and is situated in an island, far remote from any habitable parts, and environed by the mighty ocean. All which particulars correspond with the phantasy, as I presume the following observations will evince. In the first place then, as the phantasy is situated between sense and cogitation, it communicates with each in such a manner that its beginning is the end of the cogitative power, and its end is the commencement of the senses. Hence on account of its twofold nature it partakes of a twofold light; receiving in its supreme part the splendor of cogitation, and in its inferior part, a light corresponding to that of sense. Now it is this inferior part or the common phantasy, which is represented by the cave of Calypso, for its light is artificial and external like that of fire: and this correspondence is evident from the etymology of the phantaſy, which is derived from φάντα, or light. In the next place the island is said to be surrounded with a thick wood, which evidently corresponds to a material nature, or this humid body, with which the phantasy is invested: for θάρ, or a wood, implies matter according to its primary signification. But the four fountains by which the cave is watered occultly intimate those four gnostic powers of the soul discovered by the Pythagoreans, and embraced by Plato; *intelligence, cogitation, opinion, and imagination*. And these fountains are said, with great propriety and correspondence to communicate with each other. In the last place the island is said to be environed with the ocean, which admirably agrees with a corporeal nature, forever flowing without admitting any periods of repose. And thus much for the secret agreement of the cavern and island with the regions of imagination.

But the poet by denominating the goddess, Calypso, and the island, Ogygia, appears to me, very evidently to confirm the preceding exposition: for Calypso is derived from καύπισσα, which means to cover as with a veil; and Ogygia, is from ὀγυγίς, ancient. Now, as we have been previously informed by Synesius, the phantastic spirit, is the primary vehicle of the rational soul, which it derived from the planetary spheres, and in which it descended to the corporeal world. It may therefore with great propriety be said to cover the soul, as with a fine garment, or veil; and it is no less properly denominated *ancient*, when considered as the first vehicle of the soul.

In this region of the phantasy then, Ulysses is represented as an involuntary captive; continually employed in bewailing his absence from his true country, and ardently longing to depart from the fascinating embraces of the goddess. For thus his situation is beautifully described by the poet;

But sad Ulysses by himself apart,  
Pour'd the big sorrows of his swelling heart;  
All on the lonely shore he sat to weep,  
And roll'd his eyes around the restless deep;  
Tow'r'd his lov'd coast, he roll'd his eyes in vain,  
"Till dim'd with rising grief they stream'd again.      Lib. v. l. 82. &c.

His return however, is at length effected by means of Mercury, or reason, who prevails on the goddess to yield to his dismission. Hence after her consent, Ulysses is said with great propriety, to have placed himself on the throne, where Mercury had sat: for reason now resumes her proper seat, and begins to exercise her authority with undisturbed controul. But Homer appears to me to insinuate something egregious, when he represents Ulysses on his departure from Calypso, sailing by night, and contemplating the order and light of the stars, in the following beautiful lines:

And now rejoicing in the prosp'rous gales,  
With beating heart Ulysses spread his sails;  
Plac'd at the helm he sat, and mark'd the skies,  
Nor clos'd in sleep his ever watchful eyes.  
There view'd the Pleiads, and the northern team,  
And great Orion's more resplendent beam,  
To which around the axle of the sky  
The bear revolving, points his golden eye.

Who.

## RESTORATION OF THE

was concealed, he may recover the ruined empire of his soul. Nor will he even then be freed from molestation, till he has entirely passed over the raging sea, and taken a long farewell

Who shines exalted on th' æthereal plain,  
Nor bathes his blazing forehead in the main,  
Far on the left those radiant fires to keep  
The nymph directed, as he sail'd the deep.  
Full seventeen nights he cut the foamy way ;  
The distant land appear'd the following day :  
Then swell'd to sight Phæacia's dusky coast,  
And woody mountains half in vapour lost ;  
That lay before him, indistinct and vast,  
Like a broad shield amid the wat'ry waste.

Lib. v. l. 269, &amp;c.

Indeed as Ulysses is the image of a man passing in a regular manner from a sensible life, and advancing from darkness to light, he is very properly represented as sailing by the splendor of the stars, and directing his course by the most conspicuous of these illustrious orbs. For star-light corresponds to the light of the mathematical sciences, which are the proper employment of one who is departing from the sensible phantasy, and her doting charms. And the stars themselves correspond to ideas, from which the light of science is derived. Ulysses therefore who is hastening to an intellectual life, contemplates these lucid objects with vigilant eyes, rejoicing in the illuminations and assistance they afford him, while sailing over the dark ocean of a material nature.

But as he is now earnestly engaged in departing from sense, he must unavoidably be pursued by the anger of Neptune, whose service he has forsaken ; and whose offspring he has blinded by stratagem, and irritated by reproach. Hence in the midst of these delightful contemplations, he is almost overwhelmed by the waves of misfortune, roused by the wrath of his implacable foe. He is however through divine assistance or Lencothes, enabled to sustain the dreadful storm : for receiving from divinity, the immortal fillet of true fortitude, and binding it under his breast, (the proper seat of courage) he encounters the billows of adversity, and bravely shoots along the boisterous ocean of life.

Ulysses therefore having with much difficulty escaped the dangers arising from the wrath of Neptune, lands at length on the island of Phæacia, where he is hospitably received, and honourably dismissed. Now as it is proper that he who like Ulysses departs from the delusions of imagination, should immediately betake himself to the more intellectual light of thought, the land of Phæacia, ought to correspond to the realms of cogitation : and that this is the case the following discourse will I persuade myself abundantly evince. In the first place then this island is represented by the poet, as enjoying a perpetual spring ; which plainly indicates, that it is not any terrene situation. Indeed the critical commentators have been so fully convinced of this, that they acknowledge Homer describes Phæacia, as one of the Fortunate Islands ; but they have not attempted to penetrate his design, by such a description. Now if we consider the perfume, liberty, unsading variety, and endless delight, which the regions of cogitation afford, we shall find that it is truly the fortunate island of the soul. In the next place the poet by the description of the palace of Alcinous, the king of this island, egregiously insinuates the pure and splendid light of cogitation : for thus he speaks :

The front appear'd with radiant splendors gay,  
Bright as the lamp of night, or orb of day.  
The walls were massy brafs : the cornice high  
Blue metals crown'd, in colours of the sky :  
Rich plates of gold the folding doors incase ;  
The pillars silver, on a brazen base ;  
Silver the lintels deep projecting o'er,  
And gold the ringlets that command the door.  
Two rows of stately dogs, on either hand,  
In sculptur'd gold, and labour'd silver stand.  
These Vulcan form'd intelligent to wait  
Immortal guardians at Alcinous' gate.

Lib. vii. l. 84, &amp;c.

And

farewell of its storms; till though connected with a mortal nature, through deep attention to intelligible concerns, he becomes so ignorant of marine and material operations, as to mistake an oar for a corn-van.

Nor

And he represents it, as no less luminous internally, by night.

Refulgent pedestals the walls surround,  
Which bays of gold with flaming torches crown'd;  
The polish'd ore reflecting ev'ry ray;  
Blaz'd on the banquets with a double day.

For this palace is not like the cavern of Calypso naturally obscure, but remarkably bright and resplendent. Indeed Homer by his description of the outside of this palace, sufficiently indicates its agreement with the planet Mercury, who is the god of speech; and cogitation as Plato beautifully observes in the *Theætetus*, is nothing more than inward discourse. For, according to astronomers, the planet Mercury is resplendent with the colours of all the other planets. Thus Baptista Porta in *Cœlest. Physiog.* p. 88. *Videbis in eo Saturni luridum, Martis ignem, Jovis candidum, Veneris flavum, nec-non utriusque nitor, hilaritasque, et ob id non peculiaris formæ, sed eorum formam capit, cum quibus associatur, ob id in describendo ejus colore astrologi different.* That is, "you may perceive in this planet the pale colour of Saturn, the fire of Mars, the whiteness of Jupiter, and the yellow of Venus: likewise the brilliancy and hilarity of each; and on this account it is not of a peculiar form, but receives the form of its associates, and by this means causes astrologers to differ in describing its colour."

But that the island of Phæacia, is the region of cogitation, is indisputably confirmed by Homer's account of the ships fabricated by its inhabitants: for thus he beautifully describes them:

Ορφα στο την πάμποις πεπανδρευσας φρεστήσε.  
Οὐ γέρε Φαινόσσει πελεγρεύεις θασοι.  
Οὐδὲ τι πεδάλι ήτι τάστ' ἀλλας πάτε ξεωστι.  
Ἄλλ' αὐται θασοι πούμεναι καὶ φρίνας αιδρῶν.  
Καὶ πάτους θασοι πέλασε καὶ πίνας ἀγρούς  
Ἄθρωποιν. καὶ λαΐτμα τάχιστ' ἀλλός επιτρέψονται.  
Μῆρι καὶ νιφίλα παπαλυμενίας οὐδὲ πετεί εφιν.  
Οὐτέ τι παραπόναις έπει δίος οὐδὲ απολίσθαις." Lib. viii. l. 556, &c.

i. e. "That ships intently directed by intellect, may send you to your country. For the Phæacians have no pilots, nor have the ships helms like others: but they know the thoughts and minds of men. They likewise know the cities and fertile fields of all men; and swiftly swim over the waters of the sea, covered with darkness and clouds: for they never are afraid of sustaining any damage, or of being utterly lost." Or in verse:

Se shalt thou instant reach the realm affign'd,  
In wond'rous ships self-mov'd, instinct with mind;  
No helm secures their course, no pilot guides,  
Like man intelligent they plow the tides,  
Conscious of ev'ry coast, and ev'ry bay,  
That lies beneath the sun's all seeing ray;  
And veil'd in clouds impervious to the eye,  
Fearless and rapid thro' the deep they fly.

Now it is absurd to suppose that Homer would ever employ such an hyperbole, in merely describing the excellency of the Phæacian ships: for it so eminently surpasses the bounds of probability, and is so contrary to the admirable prudence, which Homer continually displays, that it can only be admitted as an allegory, pregnant with latent meaning, and the recondite wisdom of antiquity.

It must here however be observed, that as the energies of cogitation are twofold, according to the objects on which they are employed (for they are either sensible or intellectual); so the manners of Alcinous and his nobles, are perfectly opposite to those of the other inhabitants. For these latter are thus described by the poet:

Nor is it proper to believe that interpretations of this kind are forced, and are nothing more than the conjectures of ingenious men : but when we consider the great wisdom of antiquity,

A race of rugged mariners are these ;  
Unpolish'd men, and boistrous as their seas ;  
The native islanders alone their care,  
And hateful he that breathes a foreign air.  
These did the ruler of the deep ordain  
To build proud navies, and command the main ;  
On canvas wings to cut the wat'ry way ;  
No bird more light, *no thought more swift than they.*

Lib. vii. l. 33, &c.

The last of which lines, so remarkably agrees with the preceding account, that I presume no stronger confirmation can be desired. Nor is the original less satisfactory :

*Tōv vīc̄ dīst̄as dīst̄a w̄t̄p̄t̄ h̄d̄ n̄q̄a.*

i. e. "The ships of these are swift as a wing, or as a *conception of the mind.*" But the inhabitants of the palace, are represented as spending their days in continual festivity, and unceasing mirth : in listening to the harmony of the lyre ; or in forming the tuneful mazes of the joyful dance. And this distinction of manners, admirably agrees with the difference between vulgar, and intellectual cogitations : for the former of these are boisterous, and rough, selfish and proud ; skilled indeed in rapidity, but groveling and unpolished. But the latter are constantly employed in intellectual festivity and mirth ; in tuning the melodious lyre of divine recollection, or forming the responsive dance of refined imaginations. It was with the greatest reason therefore, that Ulysses exclaimed on this occasion :

How sweet the products of a peaceful reign ?  
The heav'n taught poet, and enchanting strain :  
The well fill'd palace, the perpetual feast,  
A land rejoicing, and a people blest.  
How goodly seems it ever to employ  
Man's social days in union, and in joy ?  
The plenteous board high-heap'd with cates divine,  
And o'er the foaming bowl the laughing wine. Lib. ix. l. 3, &c.

And here we may observe how much the behaviour of Ulysses at the palace of Alcinous, confirms the preceding exposition, and agrees with his character as a man passing in a regular manner from the delusions of sense, to the realities of intellectual enjoyment. For as he is now seated in the palace of cogitation, it is highly proper that he should call to mind his past conduct, and be afflicted with the survey ; and that he should be wakened to sorrow by the lyre of reminiscence, and weep over the follies of his active life. Hence when the divine bard Demodocus, inspired by the fury of the muses, sings the wrath of Ulysses and Achilles, on his golden lyre ; Ulysses is vehemently affected with the relation. For :

Touch'd at the song, Ulysses strait resign'd  
To lost affliction all his manly mind :  
Before his eyes the purple vest he drew,  
Industrious to conceal the falling dew :  
But when the music paus'd, he ceas'd to shed  
The flowing tear, and rais'd his drooping head. Lib. viii. l. 83, &c.

And when the inhabitants of the palace, or refined cogitations, transported with the song, demanded its repetition :

Again Ulysses vail'd his pensive head,  
Again unmann'd a shower of sorrow shed.

For

antiquity, and how much Homer excelled in prudence and in every kind of virtue, we ought not to doubt, but that he has secretly represented the images of divine things under

For reminiscence is delightful to the former, because purified from guilt: but afflictive to the latter, because he has not yet arrived at the summit of virtue, and acquired the perfection of contemplative good.

But while Ulysses is at the palace of Alcinous, Homer takes occasion of introducing, some admirable mythological arcana, in the story of Mars and Venus; the explication of which from Proclus, on Plato's Republic, page 388. will I persuade myself be acceptable to the Platonic reader, and vindicate Homer from acting improperly by introducing this excellent fable. "Both Vulcan and Mars then (says Proclus) operate about the universal world. And Mars indeed separates, perpetually nourishes, and constantly excites the contrarities of the universe, that the world may exist perfect and entire from all its parts. But Vulcan adorns the whole sensible machine by his artifice, and fills it with certain reasons, proportions, and powers of nature. Hence he is feigned by Homer to construct twenty tripods about the heavens, that he may adorn them with the most perfect of many-sided figures: and he is likewise said to fashion a multitude of sublunary forms, variously figured, and artificially adorned; as when he says:

Chains, bracelets, pendants, all their toys I wrought.      Iliad. xviii.

But each of these deities requires the assistance of Venus; Mars that he may insert order and harmony in contrary natures; but Vulcan that he may induce beauty, in sensible operations, and thus render the world beautiful and divine. However since Venus is every where, Vulcan indeed always enjoys her, according to the superior orders of beings. As for instance, if Vulcan is super-mundane, Mars is mundane; and if the former is in the heavens, the latter enjoys Venus, in the regions subject to the moon. Hence Vulcan is said to have taken Venus for his wife according to the will of Jove; but Mars is reported to have committed adultery with Venus. For the cause of beauty, and conciliation, is naturally conjoined with the artificer of sensible concerns: but is after a manner foreign from the presiding deity of division and contrariety: for the collective, are opposed to the separating genera of gods. And hence fables have given the appellation of adultery, to this consent of dissimilar causes. But a consent of this kind was necessary to the world, that contraries might join in mutual conciliation, and the war of the universe terminate in peace. Indeed as in the celestial regions, beauty, elegance of form, and the fabrications of Vulcan, are particularly conspicuous: but in the realms of generation, elementary war, opposition, contrariety of natural powers, and the gifts of Mars, reside: on this account I say, the sun beholding from the heavens the congress of Mars and Venus, betrays it to Vulcan, with all whose actions he unites in amicable co-operation.

After this, Vulcan entangles them with all-various bonds, invisible to other gods, that he may connect the mundane genera, with artificial reasons; and may produce one constitution from the contrarities of Mars, and the conciliating benefits of Venus. For both are required in generation. But according to Timæus, there are some bonds of celestial, and others of sublunary concerns: and the latter are dissoluble, but the former of a perpetual nature. Hence Vulcan at last dissolves the bonds, with which he had confined Venus and Mars; and this he effects through the persuasion of Neptune.

Who sues importunate to loose the god.

For Neptune desirous that the perpetual continuity of generation, may be preserved, and that the circle of mutation may revolve into itself, causes begotten natures to be corrupted, and such as are corrupted to be renewed. Is there any reason then, why we should wonder at Homer's feigning Mars and Venus to be fettered with the bonds of Vulcan, since Timæus also gives the appellation of bonds, to the demiurgic reasons, by which the celestial gods constitute generated natures? Is it not likewise congruous to the nature of things, that these bonds should be again dissolved, since they are the bonds of generation? Indeed the first, and most perfect artificer of things seems to have composed the world from contrary elements, and to have given them friendship in a certain proportion, by uniting the effects of Vulcan, Venus, and Mars, in association with each other. For producing the contrarities of the elements, he operates within himself, according to Mars: but when he fabricates friendship, he energizes according to the power of Venus; and by connecting the natures of Venus and Mars, he appears to contain in himself the primitive exemplar of Vulcan's art. Hence he is all, and operates with all the gods. Besides the junior artificers imitating their father, produce mortal animals, and again receive them when they perish, fabricating at the same time with Vulcan, mundane bonds, and primarily containing in themselves

der the concealments of fable. For it is not possible that this whole exposition could be devised, unless from certain established truths; an occasion of fiction had been given:

But

the causes of their dissolution. For every where, he who possesses a bond, knows also the necessity of its solution."

We may here too observe that Ulysses, with the greatest propriety relates his past adventures in the palace of Alimone: for as he now betakes himself to the intellectual light of thought, it is highly necessary that he should review his past conduct, faithfully enumerate the errors of his life, and anxiously solicit a return to true manners, and perfect rectitude of mind. But the description of his departure from Phæacia is no less pregnant with philosophical mystery, than poetical beauty. For as he is now passing by the pure energy of thought, to his true country, the rational soul; he is represented as departing by night, and falling into so profound a sleep in his voyage, as to be insensible for some time of its happy consummation. For thus according to the poet:

He climb'd the lofty stern; then gently prest  
The swelling couch, and lay compos'd to rest. Lib. xiii. l. 75.

And the vehemence of his thoughts, is finely represented by the rapidity of the vessel:

Now plac'd in order the Phæcian train  
Their cables loofe, and launch into the main :  
At once they bend, and strike their equal oars,  
And leave the finking hills, and lef'sning shores.  
While on the deck the chief in silence lies,  
And pleasing slumbers steal upon his eyer.  
As fiery coursers in the rapid race  
Urg'd by fierce drivers thro' the dusty space,  
Tois their high heads, and scour along the plain ;  
So mounts the bounding vessel o'er the main.  
Back to the stern the parted billows flow,  
And the black ocean foams and roars below.  
Thus with spread sails the winged galley flies ;  
Lesl swift an eagle cuts the liquid skies ;  
Divine Ulysses was her sacred load,  
A man in wisdom equal to a god !  
Much danger long and mighty toils he bore,  
In storms by sea, and combats on the shore ;  
All which soft sleep now banish'd from his breast,  
Wrapt in a pleasing, deep, and death-like rest.

By the night, therefore, Homer intimates the stillness and tranquillity which attends intellectual contemplation: and by the sweet and death-like sleep of Ulysses, his being abstracted from all sensible concerns, while merged in the profound and delightful energies of thought. For he has now bid adieu to the storms of passion, and the conflicts of desire; and is hastening to expel these dangerous foes, from the secret recesses of his soul.

Nor is it without reason that the poet represents Ithaca as presenting itself to the mariners view, when the bright morning star emerges from the darkness of night. For thus he speaks:

But when the morning star, with early ray,  
Flam'd in the front of heav'n, and promis'd day ;  
Like distant clouds the mariner descries  
Fair Ithaca's emerging hills arise. Lib. xiii. l. 93, &c.

Since it is only by the dawning beams of intellect, that cogitation can gain a glimpse of the native country, and proper empire of the soul.

But when Ulysses awakes from the delightful sleep of his corporeal energies, and, through the assistance of Minerva,

But rejecting the discussion of this to another work, we shall here finish our proposed explication of the cave of the nymphs.

## SECTION

serua, recognizes his native land, he immediately enters into a consultation with the goddesses, how he may effectually banish the various perturbations and inordinate desires which yet lurk in the penetralia of his soul. For this purpose it is necessary that he should relinquish all external possessions, mortify every sense, and employ every stratagem, which may finally destroy these malevolent foes. On this occasion the garb of poverty, the wrinkles of age, and the wants of life, are symbols of mortified habits, desertion of sensible pursuits, and an intimate conversion to intellectual good. For the sensitive eye must now give place to the purer sight of the rational soul; and the strength and energies of a corporeal nature must yield to the superior vigour of intellectual exertions, and the severe labour of moral investigation. And this Homer appears most evidently to insinuate by the following beautiful lines :

Now seated in the olive's sacred shade,  
Confer the hero and the martial maid,  
The goddesses of the azure eyes began :  
Son of Laertes ! much-experienc'd man !  
The suitor train thy earliest care demand,  
Of that luxurious race to rid the land :  
Three years thy house their lawless rule has seen,  
And proud addresses to the matchless queen.  
But she thy absence mourns from day to day,  
And inly bleeds, and silent wastes away :  
Elusive of the bridal hour she gives  
Fond hopes to all, and all with hopes deceives.

Lib. xiii. l. 372.

Hence :

It fits thee now to wear a dark disguise,  
And secret walk unknown to mortal eyes,  
For this my hand shall wither ev'ry grace,  
And ev'ry elegance of form and face,  
O'er thy smooth skin a bark of wrinkles spread,  
Turn hoar the auburn honours of thy head,  
Disfigure every limb with coarse attire,  
And in thy eyes extinguish all the fire;  
Add all the wants and the decays of life,  
Estrange thee from thy own ; thy son, thy wife ;  
From the loath'd object ev'ry sight shall turn,  
And the blind suitors their destruction scorn.

Lib. xiii. l. 397, &amp;c.

After this follows the discovery of Ulysses to Telemachus, which is no less philosophically sublime than poetically beautiful. For by Telemachus we must understand intellectual virtue, the true progeny of Ulysses, or the rational soul. Hence Ulysses, while employed in the great work of mortification, recognizes his legitimate offspring, and secretly plans with him the destruction of his infidels foes. The assistance however of Minerva, or wisdom, is requisite to this discovery, who beautifies and adorns the rational soul, and restores it on this occasion, to its pristine dignity and excellence of form. But it is necessary that this should be nothing more than a temporary change, till the enemies of reason are destroyed, and the dominion of intellect regained. With great propriety, therefore, is Telemachus represented as exploring his absent father, and impatient for his return : for the rational soul then alone associates with true virtue, when it withdraws itself from sensible delights, and earnestly meditates a restoration of its fallen dignity, and original sway.

And now Ulysses presents himself to our view in the habits of mortification, hastening to his long deserted palace, or the occult recesses of his soul, that he may mark the conduct, and plan the destruction of the malevolent passions, who are secretly attempting to subvert the empire of his mind. Hence the poet very properly and pathetically exclaims :

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## S E C T I O N III.

IT is now requisite that we should direct our attention to Iamblichus, the celebrated disciple of Porphyry, who, on account of the sublimity of his genius, and his admirable proficiency in theological learning, was surname, *the divine*. This extraordinary man who appears to have been born for the advancement of theology, though zealously attached

And now his city strikes the monarch's eyes,  
Alas! how chang'd! a man of miseries;  
Propt on a staff, a beggar old and bare,  
Is tatter'd garments, flutt'ring with the air! Lib. xvii. l. 201, &c.

However as this disguise was solely assumed for the purpose of procuring ancient purity and lawful rule, he divests himself of the torn garments of mortification, as soon as he begins the destruction of occult desires; and resumes the proper dignity and strength of his genuine form. But it is not without reason that Penelope, who is the image of intellectual purity, furnishes the instrument by which the hostile root of passions are destroyed: for what besides the arrows of purity can be sufficient to extirpate the leading bands of impurity and vice? Hence as soon as he is furnished with this irresistible weapon, he no longer defers the ruin of his insidious foes, but:

Then fierce the hero o'er the threshold strode;  
Stript of his rags, he blaz'd out like a god.  
Full in their face the lifted bow he bore,  
And quiver'd death a formidable store;  
Before his feet the rattling shov'r he threw,  
And thus terrific to the suitor crew. Lib. xxii. l. 1, &c.

But Homer represents Penelope as remaining ignorant of Ulysses, even after the suitors are destroyed; and he is seated on the throne of majesty, anxious to be known, and impatient to return her chaste and affectionate embrace. For thus he describes her:

Then gliding through the marble valves in state,  
Oppos'd before the shining fire she sat.  
The monarch by a column high enthron'd,  
His eye withdrew, and fix'd it on the ground  
Anxious to hear his queen the silence break:  
Amaz'd she sat, and impotent to speak;  
O'er all the man her eyes she rolls in vain,  
Now hopes, now fears, now knows, then doubts again. Lib. xxiii. l. 88, &c.

Nor ought this to appear strange, for purity has been so long absent from his soul, that it is difficult to obtain a recollection of their pristine union, and legitimate association with each other. However in order to facilitate this discovery, he renders all harmonious and pure, within the recesses of his soul; and by the assistance of Minerva, or wisdom, resumes the garb and dignity which he had formerly displayed.

Then instant to the bath, (the monarch cries)  
Bid the gay youth and sprightly virgins rise,  
Thence all descend in pomp, and proud array,  
And bid the dome resound the mirthful lay;  
While the sweet lyrist airs of raptures sing,  
And forms the dance responsive to the strings. Lib. xxiii. l. 131, &c.

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attached to the Platonic philosophy, yet explored the wisdom of other sects; particularly of the Pythagoreans, Egyptians, and Chaldeans; and formed one beautiful system of recondite knowledge, from their harmonious conjunction. There is a short life of this philosophic hero extant, by Eunapius, the substance of which is as follows: Iamblicus was descended of a family equally illustrious, fortunate, and rich. His country was Chalcis, a city of Syria, which they denominate Cælen. He associated with Anatolius, who was the second to Porphyry, but he far excelled him in his attainments, and ascended to the very summit of philosophy. But after he had been for some time connected with Anatolius, and most probably found him insufficient to satisfy the vast desires of his soul, he applied himself to Porphyry, to whom (says Eunapius) he was nothing inferior, except in the structure and power of composition. For his writings were not so elegant and graceful as those of Porphyry: they were neither agreeable, nor conspicuous; nor free from impurity of diction. And though they were not entirely involved in obscurity, and perfectly faulty; yet, as Plato formerly said of Zenocrates, he did not sacrifice to the *mercurial Graces*. Hence he is far from detaining the reader with delight, or inviting him to a perusal of his works; but he rather seems to avert and dull the attention, and frustrate the reader's expectation. However, though the surface of his conceptions is not covered with the flowers of elocution, yet his thoughts contain a most admirable depth, and his imagination is truly divine. He shared in an eminent degree the favour of divinity on account of his cultivation of justice; and obtained a multitude of associates and disciples, who came from all parts of the world; for the purpose of participating the streams of wisdom, which so plentifully flowed from the sacred

*And afterwards Ulysses is described, as appearing through the interposition of Minerva, like one of the immortals:*

So Pallas his heroic form improves  
With bloom divine, and like a god he moves.      Lib. xxiii. l. 163.

For indeed he who, like Ulysses, has destroyed the power of his passions, and purified himself from their deluding nature, no longer ranks in the order of mortals, but assimilates with divinity itself. And now that he may become entirely known to intellectual purity, that chaste Penelope of the soul, he need only relate the secrets of their mystic union, and recognize the bower of celestial love. For then perfect recollection will ensue; and the anxiety of suspense will be changed into transports of assurance, and tears of rapturous delight.

And thus we have attended Ulysses through his various wanderings and woes, till he recovers the ruined empire of his soul. We must however remember, according to the beautiful observation of Porphyry, in the above treatise, that he is not freed from molestation till he has passed over the raging sea of a material nature, has entirely appeased the anger of divinity, and is become so insensible to the concerns of a corporeal life, as to be ignorant of their nature and use. For :

Then heav'n decrees in peace to end his days,  
And steal himself from life by slow decays;  
Unknown to pain, in age resign his breath,  
When late stern Neptune points the shaft of death;  
To the dark grave retiring as to rest;  
His people blessing, by his people blest.      Lib. iii. l. 281, &c.

fountain

fountain of his wonderful mind. Among these was Sopater \* the Syrian, of the greatest eloquence, both in composition and discourse; Eustathius the Cappadocian; and of the Greeks, Theodorus and Euphrasius. All these were excellent for their virtues and attainments, as well as many others of his disciples, who were not much inferior to the former in eloquence; so that it seems wonderful, how Iamblichus, could attend to them all, with such gentleness of manners and benignity of disposition.

He performed some few particulars relative to the veneration of divinity, by himself, without his associates and disciples; but was inseparable from his familiars in most of his operations. He imitated in his diet the frugal simplicity of the most ancient times; and during his repast exhilarated those who were present by his behaviour, and filled them as with nectar by the sweetness of his discourse. Some of these inflamed with an unwearied desire of hearing his wisdom, and incapable of being satiated with its pleasure, were his constant guests, and once addressed him as follows: "Why, O divine master, do you thus act alone, without communicating to us your most consummate wisdom? Yet it has been reported to us by your servants, that you have been seen, while engaged in prayer, elevated more than ten cubits from the ground, your body and garments at the same time being changed into a golden colour; and that when your prayers have been finished, your body has returned to its pristine form, and descending to the earth you have associated and discoursed with us as before." Upon this Iamblichus laughed (though he was not addicted to laughter) and replied: "He who invented this false relation, was not unpleasant; but in future, nothing shall be transacted without you."

The two following circumstances, relative to the theurgical powers of this wonderful man, are related by Eunapius, which the reader may credit or reject as he pleases. At that season of the year, when the sun rises in conjunction with the dog-star, Iamblichus went with his disciples to sacrifice, in one of the suburbs of the city; and after the sacrifice was performed they returned to town, gently walking along, and discoursing concerning the gods, as a subject very proper for the occasion. Then Iamblichus, who was perfectly lost in thought in the midst of the discourse, whose voice was fallen, and eyes immovably fixed on the earth, turned to his companions and exclaimed: "Let us take another road, for not far from hence there is a funeral procession". Iamblichus accordingly chose a purer way, and was accompanied by some who were ashamed to forsake their master: but the greater part, among whom was Aedesius, obstinately persisted in the former road, ascribing the affair to the vanity and superstition of the man, and tracing the event with avidity and caution. In the mean time, those whose office consists in burying the dead approached, contrary to the expectation of his disciples; and upon enquiring whether they had taken that road from the first, they answered in the affirmative, and that no other path led to the place of their destination.

\* This Sopater, succeeded Plotinus in his philosophical school. Vide Sozom. Hist. Eccles. 3, 5.

But the second relation is far more wonderful than the present: for in the first (says Eunapius) perhaps the sight and smell of Iamblichus was more powerful than that of his disciples. His associates therefore, not satisfied with this testimony of his extraordinary powers, were desirous to try him in a greater affair, and upon soliciting Iamblichus for this purpose, he replied that a proof of this kind was not dependant on his own will, but must be referred to a proper opportunity. In a short time after this, they all went to Gadara in Syria, a place so famous for baths, that, after Baiae in Campania, it is the second in the Roman empire. Here a dispute concerning baths arising while they were bathing, Iamblichus smiling, said to them: "Though what I am about to disclose is not pious, yet for your sakes it shall be undertaken;" and at the same time he ordered his disciples to enquire of the natives, what appellations had been formerly given to two of the hot fountains, which were indeed less than the others, but more elegant and graceful. Upon enquiry, they found themselves unable to discover the cause of their nomination; but were informed that the one was called *eros, eros, or love,* and the other *anteras, anteras, or the god who avenges the injuries of lovers.* Iamblichus immediately touching the water with his hand (for he sat perhaps on the margin of the fountain) and murmuring a few words, raised from the bottom of the fountain, a fair boy, of a moderate stature, whose hair seemed to be tinged with gold, and the upper part of whose breast was of a luminous appearance. His companions being astonished at the novelty of the affair, let us pass on says he, to the next fountain; and at the same time he arose, fixed in thought, and performing the same ceremonies as before, called forth the other love, who was in all respects similar to the former, except that his hair, scattered in his neck was blacker, and was like the sun in resplendence. At the same time both the boys, eagerly embraced Iamblichus, as if he had been their natural parent: but he immediately restored them to their proper seats, and when he had washed departed from the place. After this affair, the astonishment of his familiars and disciples was so great, that they submitted to the doctrines of Iamblichus with implicit assent. Eunapius observes that other extraordinary particulars were related of Iamblichus, but that they had too much the appearance of fables to be combined with historical veracity. He adds that he should fear the preceding relations, were delusive and fictitious, if they had not been confirmed by men who were eye-witnesses of their reality.

A celebrated philosopher named Alypius, lived at the same time as Iamblichus, who was deeply skilled in dialectic; but was of such a short stature, and so slender in body, that he exhibited the appearance of a pygmy. However his great abilities amply compensated for this trifling defect; and he might be said to have emigrated into soul alone, by which he was possessed as by some inspiring god. This Alypius had many followers, but his mode of philosophising, was confined to private conference and disputation, without committing any of his dogmata to writing. Hence his disciples gladly applied themselves

themselves to Iamblichus, desirous to draw abundantly from his copious mind, as from a perennial and overflowing fountain. The fame therefore of each continually increasing, they once accidentally met like two resplendent stars, and were surrounded by so great a crowd of auditors, that it represented some mighty museum. While Iamblichus on this occasion waited rather to be interrogated than to propose a question himself, Alypius, contrary to the expectation of every one, relinquishing philosophical discussions, and seeing himself surrounded with a theatre of men, turned to Iamblichus, and said to him: *tell me O philosopher, is the rich man unjust, or the heir of the unjust? For in this case there is no medium.* But Iamblichus hating the acuteness of the question, replied, “This kind of disputation, O illustrious man, relative to external concerns, is foreign from our philosophical mode; since we alone propose as subjects of speculation, characters replete with philosophic virtue.” After he had said this he departed, and at the same time all the surrounding multitude was immediately dispersed. But Iamblichus collecting himself when alone, and admiring the acuteness of the question, often privately resorted to Alypius, whom he vehemently extolled for the subtlety of his judgment, and the sagacity of his genius; and whose life he historically and copiously delineated. This Alypius was an Alexandrian by birth, and died in his own country, worn out with age: and after him Iamblichus\*, leaving behind him many roots and fountains of philosophy; which, through the cultivation of succeeding Platonists, produced a fair variety of vigorous branches and copious streams.

The writings of this extraordinary man, though inestimably valuable, are not numerous; and the greater part are unfortunately lost. The only one which is preserved, relative to the Platonic theology, is the following:

*On the mysteries of the Egyptians, Chaldeans, and Assyrians:* or an answer to the epistle of Porphyry to the prophet Anebo. This admirable book contains many of the greatest arcana, of the ancient theology, respecting gods and dæmons, their cultivation and commerce, and the conjunction of the soul with divinity. It fully solves all the doubts concerning the impassivity of a divine nature; demonstrates its omnipresence, and never-failing energy; shews that we are continually surrounded with its light; and that all the divinities subsist in indivisible union, and indissoluble consent. There is an excellent Greek and Latin edition of this work, published, with copious notes, by the learned Gale: and it is greatly to be wished, though but little to be expected, that it was once translated into English, accompanied with a philosophical comment, which might both disclose its beauties, and reveal the sacred mysteries it contains.

Among the lost writings of Iamblichus, respecting theology, we may reckon in the first place, *three books, concerning the physics, ethics, and theology of arithmetic:* or φυσικά καὶ θεολογικά ἀριθμητικά. These three books, form the fifth, sixth, and seventh,

\* The exact time of Iamblichus' death is unknown; it is however certain, that it was during the reign of Constantine; and according to the accurate Fabricius, prior to the year of Christ 333. *Biblioth. Græc.* Tom. 4. p. 283.

of a great work by Iamblichus, in ten books, entitled, *a collection of the Pythagoric dogmata*. And the seventh book, Fabricius thinks is still extant.

2. *Concerning the gods.* From this work the emperor Julian derived most of the dogmata contained in his elegant oration to the sun.

3. *Commentaries on the Parmenides, Timaeus and Phaedo of Plato.* The inestimable value of the first and second of these commentaries is sufficiently evident from the frequent mention made of them by Proclus, in his writings on these dialogues; and from the admirable passages contained in them, which he has fortunately preserved.

4. *Concerning the perfection of the Chaldaic philosophy.* The twenty-seventh book of this great work, is cited by Damascius, in his MS. treatise *περὶ μεγίστων*: and this whole discourse was studied with avidity by Proclus, and enabled him as we are informed by Marinus, to ascend to the very summit of theurgic virtue. And thus much for the works of Iamblichus relative to the Platonic theology.

But here we may observe with wonder how the deepest mysteries of this theology, became more and more explicitly unveiled, in proportion as the Roman empire was hastening to its dissolution; and Christianity to an universal establishment. Though the works of Plotinus and Porphyry contain all the arcana of theology, yet they contain them occultly and concisely. Their depth is in a great measure latent, and their fire condensed. But in Iamblichus we find greater copiousness and precision: theology is rendered more easy of access, and her light is more widely diffused. The profundity of barbarian theology is more accurately explored, and its consent with that of Pythagoras and Plato more abundantly and distinctly evinced. We find in his works, mystery united with bright evidence, religion with sublime philosophy, and science with divine illumination. Now this difference in the mode of unveiling the Platonic theology, is perfectly agreeable to the state of the Roman empire, and the new religion, at the periods when these modes were adopted. In the times of Plotinus and Porphyry, when Galienus, and Dioclesian swayed the sceptre of the world, Rome was in the middle of her course to destruction and Christianity had nearly accomplished one half of her journey to ecclesiastical empire. However as neither the fall of Rome, nor the establishment of Christianity, were then absolutely certain, these philosophers were cautious in disclosing all, that a baser period might require. This period Iamblichus was destined to see approach under the reign of the emperor Constantine; when the new religion was established, and the old treated with ridicule and contempt. Indeed the new religion had no sooner ascended the throne, and assumed the reins of arbitrary power, but she was surrounded with myriads of unphilosophic converts, and in her progress to despotism, drew after her the capital of Rome; and at once fixed the destruction of its ancient empire. And thus we may see that the writings of Iamblichus were perfectly correspondent to the depravity of the times.

The most celebrated disciple of Iamblichus, appears to have been one Oedesius a Capadocian, who was of noble birth; but, as is generally the case with philosophers,

possessed but a slender estate. According to Eunapius who wrote his life, he was not much inferior to Iamblichus, except in a divine afflatus, which seems to have been peculiar to that illustrious hero. To Cœdésius we may add Maximus and Dexippus, both disciples of Iamblichus; and frequently cited by Simplicius in his elaborate commentary on the predicaments of Aristotle. But here we must regret, that none of the immediate successors of Iamblichus, contributed any thing to the advancement of the ancient theology. They revered indeed the arduous flights, and divine genius of their master; but never attempted even to imitate, what they could not equal, and were content to grovel without presuming to soar. The iniquitous times indeed of the emperor Constantine, may afford a reasonable apology for the decay of genius, and the languor of philosophy. The destructive rod of ecclesiastical empire was already extended; and its lethargic influence was already felt on the active spirit of liberal investigation. Religious faction had now started from the bosom of delusion; and *holy* persecution, was hastening from the infernal seats, to massacre the nations, and deluge Europe and Asia in blood. The peaceful and instructive disputes of philosophers, were now beginning to be exchanged for the jargon of orthodox and heterodox sectaries; and the calm voice of ancient theology, was silenced by the barbarous and tumultuous sounds, of Arian and Trinitarian clamours. This alarming change however, checked only for a short period, the generous ardour of the philosophic genius: for the æra was now at hand in which theology was destined to display the full blaze of her celestial light. *Sacred* zeal indeed presumed to hurl the darts of *faith*, against her venerable person: but her arm was destitute of vigour, and her weapons fell innocent to the ground. The buckler of *true* theology was not to be transpierced, by such imbecil darts; and the attempt was like that of weak old Priam, against the strong and youthful Pyrrhus.

— telumque imbellē sine ictu  
Conjecit: rauco quod protenus ære repulsum,  
Et summo clypei necquicquam umbone pependit.

But the order of discourse now brings us to a survey of the last branch of the *theological tree*, in which we may discover amidst numerous ramifications, and elegant foliage, exhaustless vigour, and luxuriant fruit. The source of this illustrious branch was the great Athenian Plutarch, of whom such honourable mention has been made, in the preceding life of Proclus \*. To Plutarch succeeded Syrianus and Olympiodorus; and to these Hermeas and Proclus. It was by the labours of this last philosophical hero, that theology received the consummation of excellency, and exhibited diffused elegance, combined with majesty and strength. This will be evident from perusing his life, and studying his more abstruse writings, among which the following elements may be deservedly ranked. Though Marinus as we have observed in the life of Proclus, was his immediate successor, yet Asclepiodotus the master of Damascius, was his best disciple;

\* See vol. I. of this work.

and

and was most capable of receiving the exuberant streams of wisdom, which vigorously flowed from his philosophic mind. After Asclepiodotus an illustrious series of philosophers succeeded, who terminated the golden chain of Platonists, and were the last advocates for the dignity of ancient wisdom and theology. These great men were Zenodotus, Severianus, Ammonius Hermias, Hierius, Asclepius, Simplicius, Isidorus, Damascius, Diogenes, Eulalius and Priscian. But of all these, none except Damascius, appears to have contributed any thing to the perfection of theology : for the works of the rest consist for the most part in excellent commentaries on Aristotle ; but Damascius, in his book, on principles, has preserved a most valuable store of recondite wisdom, and unfolded some of the sublimest mysteries of ancient theology. This inestimable work is however still in manuscript, and is not likely in the present age to emerge from its shameful concealment.

Seven of the preceding illustrious heroes, who were united by friendship as well as philosophy, Damascius the Syrian, Simplicius of Cilicia, Eulalius the Phrygian, Priscian the Lydian, Hermias and Diogenes of Phoenicia, and Isidorus of Gaza, disgusted with the religion of their sovereign Justinian, determined to seek from Chosroes the Persian king, that liberty of conduct which their native country denied. Chosroes though a barbarian, was deeply skilled in the philosophy of Plato and Aristotle ; and was so imbued with the dogmata of Plato, that not one of his abstruse dialogues escaped his penetrating genius. The ill success however of these philosophers in their journey to Persia, gives us reason to suspect that the philosophic attainments of Chosroes, were influenced more by pride than the love of truth : and that he affected the name without possessing the requisites of a sage. The return of these philosophers was precipitate, and their disappointment extreme. They derived however a considerable advantage from their expedition ; and the conduct of Chosroes in this particular will confer immortal honour on his character and name. He was the means of procuring for the seven sages, an exemption from the barbarous penal laws of Justinian against the Pagans ; and thus enabled them to end their days in security and peace, and in the enjoyment of that liberty of conscience which no religion before the Christian, ever attempted to destroy.

The reign of Justinian, indeed, as it firmly established the Christian religion, terminated the glorious empire of philosophy, by suppressing the schools of Athens, and suspending the ecclesiastical sword over the heads of heathen theologists. But the fall of philosophy was naturally succeeded by the darkness of delusion and ignorance ; by the spirit of wild fanaticism, and intolerant zeal ; by the loss of courage and virtue ; and by the final dissolution of the empire of the world. She was ruined indeed but not without revenge. War, pestilence, and famine, were the scourges of a prince who had presumed to demolish her schools, and intercept the diffusion of her sacred light : and his reign was disgraced by an irreparable decrease of mankind, in the most fertile regions

gions of the earth. We may add too that his dominions were alarmed with the dreadful blaze of two mighty comets, whose malignant light foretold approaching calamities and war; and signified perhaps the establishment of religious anarchy, and the commencement of *barbarous impiety* and folly. And to complete this catalogue of prodigies and desolation, every year of his reign was marked with violent earthquakes of uncommon duration, and incredible extent. The whole surface of the Roman empire was agitated with horrid internal convulsions; and enormous chasms were formed by the earth's strong vibrations. Large bodies were discharged into the air, and the sea concurring in the general ruin, overflowed or deserted its natural bounds, by alternately advancing and retreating with accumulated majesty and strength: and a mountain was torn from Libanus, and hurled into the waves, amidst the dreadful tossings of the deep. History after this period exhibits nothing but religious dissensions, despicable *councils*, and bigotted sects; the enmity of *saints*, and the discord of Nestorians and Jacobites, Maronites and Armenians, Copts and Abyssinians. *Religious war*, and *pious rebellion* succeeded to philosophical theory; and Nestor and Cyril led the confused and clamorous dance of ecclesiastical disputation.

It would neither be consistent with the design of this history, pleasing to the author, nor entertaining to the Platonic reader to trace the rapid increase of barbarism and ignorance, after the abolition of the Athenian and Alexandrian schools. It will be sufficient to observe, that the jargon of innumerable sects, established a tyranny unknown to the Pagan world, the tyranny of religious despotism; and finally extirpated from the earth, the dominion of ancient wisdom and virtue. From the incredible multitude of different persuasions, Christianity lost all appearance of a *revelation*; and by the conduct of its professors, seemed rather calculated to confound than illuminate mankind. The same infatuated spirit has indeed marked its progress to the present day; and we find that in proportion as this baneful zeal prevails, knowledge retires, virtue droops, and magnanimity is destroyed: hypocrisy becomes the substitute for generosity; and whining cant succeeds the decent confidence, inspired by genuine dignity and worth. As the rapidity of a river is increased by the contraction of its channel; so its vigour is diminished by the multiplication of its streams. In a similar manner, the influence of any religion is lessened, when it is divided into various streams of opinion, by the discord of party, and the zeal of profession. The energy of the whole is lost by diffusion; and the river of the *Church* is weakened by the numerous and narrow rivulets of *Dissenters*. Experience unfortunately shews, that the professors of a national religion, are generally men of greater integrity, than those who compose the dissenting sects; and the fact may be supported by a rational theory. The trifling employments, grovelling cares, and contemptible fame which are necessarily connected with religious dissension, unavoidably debilitate the mind, and contract the heart. The whole attention is engrossed in regarding the little concerns, and supporting the narrow opinions of a party; and that strength of understanding, and integrity of character, which are requisite to acquire eminence

eminence in science and virtue, are lost in imbecile exertions and hypocritical cant. It is on this account that I should prefer a dissenter in Scotland, and a papist in France, to a dissenter and catholic in England: for in those countries they cease to be sectaries, and may consequently in some degree become virtuous and wise.

It seems at first sight surprising that there should be no sects among the Grecian polytheists: they were unanimous in their belief of a multitude of gods subordinate to one supreme: their mode of worship was uniformly the same; and they appear to have had no conception of religious innovation. Shall we say that a religion is false in proportion to its unity; that truth may be branched out into an endless variety of discordant streams; and that error alone resists the power of copious and confused division? Such a speculation is indeed curious, but not safe; and its result would perhaps be more logical than *orthodox*, and more informing than discreet! Let us therefore direct our attention, to a more important subject, and consider the excellence of the Christian religion with respect to the commercial interests of mankind. That Christianity is not favourable to philosophy (I mean that of the ancients) is evident from its causing the destruction of the ancient schools; which it has not yet restored, though more than a thousand years have elapsed since their dissolution. Indeed the wisdom of a sage, is not likely to coincide with the doctrine of a *fishterman*; and implicit faith, ill suits with liberal doubt, and severe investigation. However, the spirit of *meekness*, which Christianity so admirably inculcates, though opposite to the *dignity* of philosophy, promotes the humility of merchandize, and facilitates the emoluments of trade. It enables men to suppress their passions from considerations of interest; teaches them to refer every thing to private advantage; and to consider magnanimity as a dangerous and arrogant virtue. It is to this spirit, that we must ascribe the great extent of commerce, in all the civilized parts of the world; and that Europe is much richer, though less wise than of old. The spirit of meekness by gradually suppressing the noble ardour of ancient heroism, and withdrawing the attention from abstract investigations, as daring and presumptuous, has given birth to innumerable discoveries in the arts, unknown to the speculative genius of antiquity. Hence the luxuries of life have received an immense improvement; and the spirit of meekness though not calculated to soar, has wandered over the surface of the earth, and diffused its *humble* blessings even to the remote regions of the poles. Penetrating and smooth, it has crept like oil, through the communities of mankind; and increased the activity, by lubricating the joints of the flexible body of Commerce. As oil too allays the fury of the sea, and calms its agitated waves; so meekness suppresses the effervesce of desire, restrains the restless spirit of enquiry, and calms the impetuosity of genius. Hence though we are no longer surprised with the daring exploits, and prodigious talents which distinguished the ancient world, yet we can boast a greater uniformity of character, a more general equality in moderate attainments, and a more interested spirit. In consequence of this universal mediocrity, our capacity,

capacity for commerce is increased, and our abilities enlarged, for accumulating wealth by groveling pursuits. But the most important advantage acquired by the spirit of meekness, is that mentioned by the great apostle of the Gentiles, "of becoming all things to all men." The benefits indeed which such a pliability of temper confers on a commercial kingdom, compose so great a part of the arcana of traffic, that revelation alone could have made mankind sensible of their importance. Meekness like Proteus assumes every possible appearance which the interest of concealment may require; and philosophy alone can trace it, through its multiform shapes, and vanquish its transforming power.

But though we excel the ancients in the virtue of meekness, and its attendant arts, we are infinitely below them in the cultivation of intellectual philosophy. By the invention of the microscope and telescope, we have indeed discovered the structure of the subtle parts of body, and beheld stars invisible to the ancient world. Hence our knowledge of particulars has received, and is continually receiving an immense increase: but we forget that particulars are infinite, and that while they produce the fleeting fabric of opinion, they are incapable of forming the steady and permanent basis of science. The doctrine of causes, was the object of ancient investigation: the enumeration of effects is the busy employment of the moderns. Experimental inquiries have enabled the philosopher of the present day to solve partial phænomena, and to deceive the importunities of doubt by the intervention of secondary causes. However, arguments derived from the modifications of matter, can only satisfy superficial enquirers; and will be indignantly rejected, by the profound and contemplative genius. So far from deriving any illumination by accumulated experiments, the professors of this philosophy confess their ignorance of principles; and neglect their investigation under the specious pretext of declining hypotheses. On the contrary the philosophers of antiquity impelled by intellectual dignity and strength, ascended to principles, as the pillars of the universe, and the sources of conviction and repose. Hence they gloried in asserting and vindicating the capacious powers of the soul: and by severe investigation, experienced the serene splendours of knowledge, and banished the anxieties of doubt. The intellectual philosophy refines the morals while it enlightens the mind, and improves the heart while it exalts the powers of imagination and thought. On the contrary the mechanical philosophy produces opposite consequences, by introducing the darkness of ignorance, and debasing the energies of the soul.

But there cannot I think be a more egregious instance of the barren state of philosophy at present, than the prevailing opinion that the most valuable knowledge is derived from common life, and the general conduct of mankind. The manners of the multitude, so far from affording any really valuable information, exhibit nothing but specimens of folly and vice, astonishingly various, and differently combined. A knowledge of this kind may indeed be necessary to the man who wishes to accumulate wealth, and acquire popular honours; but is infinitely remote from the possession of true wisdom, and the

true cultivation of human understanding. The best, as well as the most exalted knowledge, is as we have already proved,\* that which is desirable for its own sake ; which confers felicity on its possessor, and gives a final respite to the arduous labour of mental investigation. The knowledge of common things, is alone the province of common, or uncultivated minds ; and men of great genius in every age, have been distinguished by their happy ignorance of the trifling pursuits, and empty attainments of the vulgar. Indeed he who mixes much with the multitude, necessarily imbibes false opinions and engages in puerile occupations : the strength and activity of his mind, is continually weakened, or unworthily exerted, by a general diffusion ; and he at length loses all that intellectual energy, which nature first implants, but retirement calls forth into the blossoms of elegance, and the perfection of vigor.

The late Dr. Johnson is a striking instance of the truth of these observations ; and a lasting example of the wretchedness of a mind unenlightened by philosophy. His talents were indeed vast and uncommon, but degraded by false cultivation and ruined through neglect. Hence he employed himself solely on subjects of vulgar speculation, and thought deeply on nothing but the vices and follies of the illiterate and the base. Like a giant in the dark, his strokes were indeed powerful, but often ineffectual ; and were never directed by the hand of wisdom, or assisted by the irradiations of truth. Thus he constantly displayed strength without skill, and exertion without knowledge, abilities without genius, and grandeur without a grace. He appears to resemble indeed nothing so much as the eyeless Polyphemus. Deprived of the cheering light of science and philosophy, he wandered in the caverns of sense, wretched through the want of sight, and avoided by the timid multitude who trembled at his strength. To approach him too near was generally destructive of the order of society, and often fatal to the peace of bold but ignorant individuals.

His piety too as well as his literary talents shews how little of felicity is to be expected where philosophy is wanting. For though he professed to believe in the immortality of the soul, he was a perpetual slave to the dread of death : and though he was continually exercised in the externals of religion, he could find no consolation when alone. There is nothing indeed whose certainty is so generally admitted in discourse as the soul's immortality ; and yet nothing at present is so generally disbelieved. For I will not disgrace the word belief, by supposing it possible that a man can be firmly assured of this important truth, and yet continually seek for arguments in defence of its reality. This is however the case with modern believers. They profess reverence for the decisions, and faith in the doctrines of revelation ; but are glad to seek for conviction in the arguments of philosophy. Faith is found sufficient to support the mind, while it reclines on the bosom of the church, or clings round the pillars of orthodox opinion.

\* Vol. I. Sect. 4.

But

## RESTORATION OF THE

But when it is once shaken by enquiry and staggered by doubt ; when it leaves the enchanted enclosure of faith, and ventures on the wide ocean of enquiry ; it can alone find security in the harbour of reason, and rest in the embraces of philosophy.

Dr. Johnson is however celebrated by his female biographer as "a man good beyond the imitation of mortals." As if goodness could ever reside in a soul perpetually harassed with fears, and agitated with passion ; distracted with the prospects of futurity, and afraid of retiring into itself. Is it not ridiculous to suppose that a consciousness of virtue and worth can ever be combined with misery and fear ; or that the steady and serene light of truth can ever dwell enshrined in the gloom of despondence, or beam through a mind disturbed and clouded with care ? "The good man says Plotinus is ever tranquil and serene, undisturbed by passion, and superior to grief :" and that religion is but of little worth, which confers on its votary nothing but the torments of anxiety from consciousness of inward folly or vice ; and the dread of dissolution from the uncertainty of its result. We may rest assured that no one can be truly worthy who is wretched in himself : for to be truly good is to resemble the divinity : and to suppose that misery can be combined with such a character, is to ascribe imperfection to deity, and unhappiness to the fountain of good. For the exemplar cannot be contrary to its image, though it may be infinitely superior in excellence and dignity of nature.

And thus much for a history of the restoration of the Platonic theology by the latter Platonists. I only add, that I am in no respect a debtor to the gratitude of the public : for my writings hitherto, have neither been attentively studied, nor liberally received. Solely influenced by the love of truth, I have endeavoured to disseminate the wisdom of Greece, and to draw aside the mystic veil of recondite theology : but experience has convinced me that the period of philosophy is past ; and that some fortunate revolution can alone restore its fallen honours, and establish its original sway. Should the present work survive the literary wreck, which will probably precede the revival of philosophy, I shall consider myself amply rewarded for the toil of its execution : and I am not ashamed of owning, that the pleasing hopes of such an event have inspired me with the patience and vigour requisite to so laborious an undertaking. In short whatever may be its immediate or future success, my views have been liberal in the publication, and my mental advantages considerable from the study of ancient philosophy. Amidst the various storms of a life distinguished by outrage and disease, it has been a never-failing support, and an inviolable retreat. It has smoothed the brow of care, and dispelled the gloom of despondence ; sweetened the bitterness of grief, and lulled agony to rest. After reaping such valuable advantages from its acquisition, I am already rewarded, though my labours should be unnoticed by the present and future generation. The lyre of true philosophy is no less tuneful in the desert than in the city ; and he who knows how to call forth its latent harmony in solitude, will not want the testimony of the multitude to convince him that its melody is extatic and divine.

E L E M E N T S  
O F  
T H E O L O G Y.

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## O N T H E O N E.

## P R O P O S I T I O N I.

All multitude participates in a certain respect of *the one*.

**F**OR if it in no respect participates of *the one*, neither will the whole be one whole, nor each of the many from which the multitude is composed: but each of these will also be multitude, and this will be the case in infinitum; and each of these infinites will again be infinite multitude. For if it in no respect participates of *the one*, neither according to its whole self, nor according to each of its parts, every where, and throughout there will be infinite multitude. For each of these *many*, which

\* I have already mentioned this admirable work, with the praise it so justly deserves, in my account of the life and writings of Proclus. (Vol. I. of this work, p. 38.) I now add, with great pleasure that I have been no less fortunate with respect to the translation of these Elements than in translating the commentaries of Proclus. For the Greek text, is very frequently defective in parts essential to the meaning; and consequently necessary to the perfection of the whole. This defect I have been able to supply from a Latin version of Patricius (Ferrar. 1583) who appears to have had a perfect manuscript in his possession. But notwithstanding this assistance, I must freely own, that I never translated any thing which required so much intense thought, and severe labour in its execution. This indeed must necessarily be the case, if the abstruseness of the subject, the difficulty of finding proper terms, and the defects of the original, are properly considered. But the learned reader may be experimentally convinced of the truth of this assertion, if he only compares the Greek text with the Latin version of Amelius Portus; in which I am sorry to say, he will scarcely find one proposition, in which Portus has not mistaken the sense of his author. Indeed were I disposed to entertain the critical reader, I might swell the volume with copious notes from the blunders of Portus; and display the superiority of my own version by contrasting the English with the Latin. But I consider verbose criticisms of this kind as both useless and pedantic; as remote from the philosophical genius; and as alone calculated to fill up the leisure hours of men, who have ruined their understandings in the study of words. The genuine Platonist who may be ignorant of Greek, will I per-

which ever you assume, will either be *one* or *not one*, will either be multitude or nothing. But if each of these many be nothing, that which is composed from them shall also be nothing. But if each be many, then each shall consist from infinite infinites. But these consequences are impossible. Since no being is composed from infinite infinites\*. For there is nothing greater than infinite. But that which is constituted from all is greater than each particular. Nor can any thing be composed from nothing. All multitude therefore participates in a certain respect of *the one*.

## PROPOSITION H.

**E**very thing which participates of the *one*, is both *one* and *not one*.

**F**OR if it be not *the one itself* (*αὐτὸν*) (since it participates of *the one*) because it is something different from *the one*, it suffers *the one*, by participation, and sustains itself to become *one*. If then it be nothing else besides *the one*, it is *one* alone, and does not participate of the *one*, but is *the one itself*. But if it be something different from *the one*, which is not *the one*, but its participant, it is both *not one* and *one*, not indeed the self-subsisting *one*, but *one being*, as participating of *the one itself*. This then is neither *one*, nor does it subsist as *the one*, but it is *one being*, at the same time participating of *the one*, and on this account, since it is not the self-subsisting *one*, it is both *one* and *not one*, because it is something different from *the one*. For so far as it abounds it is *not one*; but so far as it is passive from participation, it becomes *one*. Every thing therefore which participates of *one*, is both *one*, and *not one*.

suade myself, rejoice to see this invaluable treasure in his native tongue: and those who have been led to consider the theology of the heathens as delusion, and absurdity, will doubtless be surprized to find, that it is replete with the sublimest knowledge, and the most important truths. Yet I must admonish the reader, that these Elements cannot be understood by any one, who is not a thorough adept in the preceding Commentaries on Euclid: for the propositions relate to the most abstract subjects that can be conceived; and the demonstrations are uncommonly subtle and profound. Indeed if opportunity permitted, I should attempt a commentary on every proposition: but this if ever I should be able to accomplish it, must be reserved for some more auspicious period. In the mean time I hope that my occasional elucidations, will be acceptable to the Platonic reader, and assist him in the study of this inestimable work. I only add, that these Elements form an admirable introduction to the six books of Proclus on Plato's theology.

\* The absurdity of this consequence consists in this, that from the hypothesis of unity being excluded, infinite infinites would subsist separate from each other, without any bond of conciliating unity; which is evidently impossible. For though every line contains infinite infinites in capacity, because the smallest part of every line is capable of infinite division; yet these infinites in capacity are connected by the power of unity, and form one finite line.

## PROPOSITION III.

Every thing which becomes one, becomes so through the participation of one; and is one, so far as it suffers the participation of one.

FOR if things which are not one, become one, it must be by a conjunction, and communication with each other: and they will sustain the presence of one, without being *one itself*. Hence they will participate one, so far as they suffer themselves to become one. For if they are already one, they will not become one: since that which is, does not become that which it already is. But if they are formed from non-one, and privation, they will in the first place possess one, from some *one*, being engenerated in their nature.

## PROPOSITION IV.

Every thing united, is different from the *one itself*.

FOR if it is united it will participate of one, so far as it is called united. And that which participates of one is both one, and non-one. But *one itself*, is by no means one and non-one. For if this also was both one and non-one, the one which it contains will also possess both, and this in infinitum; since there is no *one itself* in which the progression can stop; but every thing will be both one and non-one. That which is united therefore is something different from one. For if that which is united was the same with one, one would be infinite multitude; and in like manner each of the parts from which the united nature is composed.

## PROPOSITION V.

All multitude is posterior to the *one itself*.

FOR if multitude is prior to the one, the one indeed will participate of multitude, but the multitude which is prior to the one, will not participate of the one: since it is multitude prior to the subsistence of the one. For it cannot participate that which is not: because that which participates of the one, is both one and non-one. But the one does not yet subsist, since multitude is the first. It is however impossible, that there

should be any multitude, which in no respect participates of the one. Multitude therefore is not prior to the one. But if multitude subsists together with the one, it will be of the same order with the one : for time cannot hinder such a conjunction. Hence neither the one can be essentially many, nor multitude one, because they are at the same time contra-distinguished ; since neither is prior, or posterior to the other. Multitude therefore will not be essentially one, and every thing it contains will be a non-one, and this in infinitum, which is impossible. Hence it naturally participates of the one, nor can any part of it be assumed which is not one : for if any part is not one, it will be an infinite composed from infinites, as we have demonstrated. And hence it entirely participates of the one. But if the one which is one itself, in no respect participates of multitude, multitude will be perfectly posterior to the one ; participating indeed of the one, but not participated by the one. But if the one should participate of multitude in such a manner as to exist as one according to subsistence, but as not one according to participation ; the one itself will be multiplied, in the same manner as multitude is united by the one. Hence the one will communicate with multitude, and multitude with the one. But things which coalesce, and communicate after a manner with each other, if they are congregated by something else, that something must have a prior existence. But if they connect themselves, they are not opposed to each other : for opposites do not hasten to a mutual conjunction. But if the one, and multitude have a contrary division, and multitude, so far as multitude, is not one, and the one so far as one, is not multitude ; hence the one cannot subsist in the other : for they would be at the same time both one and two. But if there be any thing prior to the one, and multitude which collects them into one, this will either be one, or non-one. And if non-one, it will either be many, or nothing. But it is not many ; lest multitude should be prior to the one. Nor is it nothing : for how can that congregate which is nothing ? Hence it is the one alone. For this one is not also many, lest we should advance in an infinite progression. It is therefore the one itself ; and all multitude proceeds from the one.

## C O N C E R N I N G U N I T Y .

### P R O P O S I T I O N VI.

**E**very multitude is either composed from things united, or from unities.

**F**OR that every one of things many is not multitude alone, is evident ; and it is likewise clear that each part of this multitude again, is not multitude alone. But if it be not multitude alone, it is either united, or unities. And indeed if it participates of unity it is united : but if it be composed from things primarily united, it is unities. For if

If there be one itself, there is also that which primarily participates the one, and is primarily united. But this is composed from unities. For if it is composed from things united, these are again united from certain others, and this will take place in infinitum. But it is requisite that a nature which is primarily united, should be composed from unities. And thus we have discovered what we proposed in the beginning.

### Concerning PRODUCING CAUSES, and THINGS PRODUCED.

#### P R O P O S I T I O N VII.

**E**very thing productive of another, is more excellent than the nature of the thing produced.

**F**OR it is either more excellent, or worse, or equal. Let it be in the first place, equal. That which is produced from this therefore, will itself also, either possess a power productive of some other, or it will be entirely barren. But if it be barren, it will on this account be worse than its producing cause: and because of its inefficacy, it will be unequal to that which is prolific, and possesses a productive power. But if it be productive of other natures, it will either produce that which is equal to itself (and this will be the case in all things, and all beings will be equal to each other, and nothing will be more excellent than another, since the productive nature, always constitutes the thing produced equal to itself), or that which is unequal. But in this case, it will not be equal to its producing cause: for it is the property of equal powers to fabricate equal effects. But the productions of these are unequal to each other, since on this hypothesis the producing cause, is equal to that which is prior to itself, but that which is posterior is unequal to it. It is requisite, therefore, that the thing produced should not be equal to its producing cause.

But neither can the producing cause be ever worse than the thing produced. For if the producing cause, confers essence on the thing produced, it bestows power also, according to essence. And if it is productive of all the power which that posterior to itself possesses, it can also make itself such as its production. But if it can do this, it will also make itself more powerful: for impotence cannot hinder, since a fabricative power is present, nor a defect of will: for all things naturally desire good. Hence if it can form any thing else more perfect, it will also perfect itself, before it perfects that which is posterior to itself. The thing produced, therefore, is neither equal to, nor more excellent than its producing cause: and hence the producing cause is entirely more excellent than the nature of the thing produced.

On

**On the FIRST GOOD, which is called THE GOOD ITSELF.**

**PROPOSITION VIII.**

The first good which is no other than good itself, precedes all the participants of good.

FOR if all beings desire good, it is evident that the first good is above beings. For if he is the same with any one being, either being is the same with the good, and so this particular being, will no longer be desirous of good, since it is the good itself; for that which desires any thing, is indigent of that which it desires, and by its desire is different and foreign: or being and the good are different; and the good, will indeed participate of being, and being of the good. It is, therefore, a particular good resident in some particular participant, but not good simple and universal, and which all beings desire: for this is the common object of desire to all beings. But that which is generated in another, participates alone of that, in which it is generated. The first good therefore is nothing besides good. For if you add any thing else, by addition you diminish the good itself; effecting a particular good, instead of good simple and universal. For that which is added, since it is not good itself, but something less, diminishes by its essence the good itself.

**Concerning that which is SUFFICIENT TO ITSELF.**

**PROPOSITION IX.**

Every thing sufficient to itself, either according to essence, or according to energy, is more excellent than that which is insufficient, and the cause of whose perfection depends on another cause.

FOR if all beings, naturally desire good, and one thing supplies itself with good, but another is indigent of something else; the former, will indeed have the cause of good present, but the latter separate and apart. By how much the nearer, therefore, that is which affords the object of desire, by so much the more excellent will it be than that which requires a separate cause, and externally receives the perfection of its being or energy. Besides, that which is sufficient, is both similar and diminished, and more similar to the good itself. It is diminished, because it participates good, and because it is not the first good. Yet it is in some respect allied to the good, because it can possess good from itself. But that which participates, and participates through another, is more distant from the first good, which is nothing else than good.

PROPO-

## PROPOSITION X.

**E**very thing sufficient to itself is worse than that which is simply good.

**F**OR what is that which is sufficient, than that which from itself, and in itself, possesses good? But this is now full of good, which also it participates: but it is not the simply good. For that, as has been demonstrated, is more excellent than the participation and repletion of good.. If then that which is sufficient fills itself with good, that from which it fills itself will be more excellent than that which is sufficient, and will be superior to sufficiency: for that which is simply good is not indigent of any thing. For it does not desire any other; since by desire it would be imperfect; and thus would be full of good, and not the first good.

## CONCERNING CAUSE.

## PROPOSITION XI.

**A**ll beings proceed from one first cause.

**F**OR either there is no cause of beings, or the causes of all terminated beings revolve in a circle, or there is an infinite ascent of causes; so that one thing is the cause of another, and the prior subsistence (*πρότερος*) of essence, no where stops its progression. But if there be no cause of beings, neither will there be an order of things second and first; of the perfecting, and perfect; of the adorning, and adorned; of the generating, and generated; of the active and passive: nor will there be any science of beings. For the knowledge of causes is the employment of science: and we then assert that we know, when we know the causes of beings. But if causes revolve in a circle, the same causes will be both prior and posterior, more powerful and more debile. For every thing which produces is more excellent than the nature of the thing produced. But there is no difference, whether we conjoin the cause with the thing caused by many, or fewer mediums, and place the thing caused as subordinate.. For the cause is more excellent than all the intervening natures of which it is the cause: and by how much the greater the number of mediums, by so much the more is it a cause. But if there be an addition of causes in infinitum, and one always proceeds from another; on this hypothesis likewise science cannot subsist: for there is no knowledge of infinites. But causes being unknown, neither can there be any science of things

things subsequent to causes. If, therefore, it is requisite that there should be a cause of beings, and causes are distinct from things caused, and there can be no infinite ascent; there will be a first cause of beings, from which as a root particulars proceed, some of which exist in propinquity, and others at a distance from his nature. For that it is necessary there should be one principle, is demonstrated; because all multitude is secondary to the one.

## PROPOSITION XII.

The principle and first cause of all beings is the good.

FOR if all things proceed from one cause, it is requisite to call that cause, either the good, or more excellent than the good. But if it be more excellent than the good, we ask whether *any thing* emanates from this cause into beings, and into the nature of beings, or *nothing*? And indeed if nothing, it will be absurd: for we cannot, on this hypothesis, any longer preserve it in the order of a cause; since it is every where requisite that something should be present from the cause to the things caused, and especially from the first cause, from which all things depend, and through which every being exists. But if there is a participation of this first cause in beings, in the same manner as there is of the good, there will be something more excellent than goodness, penetrating into beings from the first cause. For since it is more excellent, and superior to the good, it cannot bestow on secondary natures any thing worse than the benefits distributed by that which is posterior to itself. But what can be more excellent than goodness itself? Since we apply the term *more excellent* to that which participates more of the good. If then that which is non-good, is not more excellent, it must be posterior to the good. But if, likewise, all beings desire good, how can any thing be prior to this cause? For if good also desires, how can it be good in the most eminent degree? But if it does not desire, must not all beings desire that cause of all, from which they proceed? And if it is the good itself, from which all beings depend, the good must be the principle and first cause of all.

## PROPOSITION XIII.

*Every Good is endued with a power of uniting its participants, and every union is good; and the good itself, is the same with the one.*

FOR if the good itself is the preserver of all beings, and on this account is desirable by all, but the one itself, preserves and contains the essence of each: (for all things

are

are preserved by the one, and dispersion removes every thing from essence) hence the good causes those things to be one, to which it is present, and contains them by union. But if the one is endued with a congregating and containing power, it perfects every being by its presence: and hence it is good to all things to be united. But if union is essentially good, and good is unific (or unifying), the simply good, and the simply one is the same: at the same time uniting, and benefiting beings. Hence it is, that things which in a certain respect fall from good, are also deprived of the participation of one: and that things which are destitute of the one, because they are replete with separation, are after the same manner likewise deprived of good.

#### C O R O L L A R Y.

Hence both goodness is union, and union is goodness; and the good is the one, and the one is the first good.

### Concerning an IMMOVEABLE, and SELF-MOTIVE PRINCIPLE OR CAUSE.

#### P R O P O S I T I O N XIV.

Every being is either immoveable, or moved; and if moved, it is either moved by itself, or by another.

**I**N the first place if it is moved by itself, it is self-motive, but if by another, it is alter-motive (*ἀλλομόρφως*.) Every being therefore, is either immoveable, or self-motive, or alter-motive. For it is necessary, that since there are alter-motive natures, there should be something immoveable, and between these, a self-motive nature. For if every thing alter-motive, when in motion is moved by another, motions are either performed in a circle, or in infinitum. But they can neither subsist in a circle, nor in infinitum, since all beings are terminated by a principle, and the motive nature, is more excellent than the thing moved. Something immoveable therefore will be the first mover. But if this be the case, it is necessary that there should be something self-motive. For should all things stand still, what will that be which is first moved? It cannot be the immoveable itself, for motion is not natural to this. Nor the alter-motive, for it is moved by another. It remains therefore that the self-motive, must be that which is first moved; since it is this which unites the alter-motive to the immoveable, existing as a medium, moving, and at the same time moved. For of those, the one moves alone, and the other is alone moved. Every being therefore, is either immoveable, or self-motive, or alter-motive.

#### C O R O L L A R Y.

From hence also it is evident, that of things which are moved, a self-motive nature is the first, but of things motive, an immoveable nature.

### Concerning an INCORPOREAL ESSENCE, and its PROPERTIES.

#### P R O P O S I T I O N X V.

Every thing which is converted to itself, is incorporeal.

**F**OR no body is naturally adapted to be converted to itself. For if that which is converted to any thing, is conjoined to that to which it is converted, it is evident that all the parts of a body must be conjoined with all the parts of that which is converted to itself: since self-conversion then takes place, when that which is converted becomes one with that to which it is converted. But this is impossible in body, and in all partible natures. For the whole of that which is partible, is not conjoined with the whole, on account of the separation of the parts which are differently situated. No body therefore is naturally adapted to self-conversion, so that the whole may be converted to the whole. And hence whatever is self-convertive is incorporeal and impartible.

#### P R O P O S I T I O N X VI.

Every thing which is converted to itself, has an essence separate from all body.

**F**OR if it be inseparable from any body, it will not possess some action separable from body. For if essence is inseparable from body, it is impossible that an essential energy should be separable: since in this case energy would be more excellent than essence; because the latter would be indigent of bodies, but the former would be sufficient to itself, without requiring the assistance of body. If then anything be inseparable according to essence, it must be so likewise according to energy, or indeed more inseparable. But if this be the case, it is not converted to itself. For that which is converted to itself, as it is different from body, has an energy separate from body, neither subsisting through body, nor in conjunction with its nature: since action, and that to which action is directed, is not indigent of body. Hence that which is converted to itself, is entirely separable from bodies.

#### P R O P O S I T I O N X VII.

Every thing which moves itself primarily, is endued with a self-convertive power.

**F**OR if it moves itself its motive energy also is resident in its nature, and the thing moving is at the same time one with the thing moved. For either it moves with a part, but is moved in a part, or the contrary. But if one part is motive, and another part

part is moved, it will not be essentially self-motive, because it will subsist from non-self-motive natures: and it will appear indeed self-motive, but will not be so essentially. But if the whole moves, and a part is moved, or the contrary, there will be some part in each, which, according to one, will be at the same time both moving and moved\*: and this will be primarily self-motive. But if one and the same moves and is moved, it will possess with itself the energy of moving, because it is self-motive; but it will be converted to that in which it energises. Every thing therefore primarily self-motive is converted to itself.

## PROPOSITION XVIII.

**E**very thing which supplies being to others, is that primarily which it bestows on the things supplied.

**F**OR if it gives being, it procures the communication from its own essence. But that which it gives is worse than its own essence: and that which it is, is more excellent and perfect. For every artificer of any thing, is more excellent than the nature of the thing fabricated: and hence that which pre-exists in the donor, is more sublime than the gift; for the one is primary, but the other secondary and subordinate. For it is necessary, either that both should be the same, and that there should be one reason of both; or that nothing should be common, or the same in both; or that this should be first, but that the second. But if there be, one and the same reason, or definition, the one will no longer be cause, and the other effect; nor this in itself, but that in the thing given; nor will this be the efficient, but that the effect. But if they have nothing the same, the remainder will not subsist, in consequence of the existence of the other, because it will communicate nothing to its being. It remains therefore, that this which bestows is first; but that which is bestowed is second; among which the being of the one is supplied from the other.

## PROPOSITION XIX.

**E**very thing which is primarily inherent in any of the natures among beings, is present to all the beings distributed according to that nature, in one reason, and after the same manner.

**F**OR if it be not present to all after the same manner, but to these, and not to those; it is evident that it will not be primarily inherent in that nature. But it will be present with some primarily, and in others which participate sometimes but not always,

\* Because, since the whole is motive, the part which is moved will also be motive, and so will be at the same time both moving and moved. And if a part moves, and the whole is moved; then because of the motion of the whole, the part will be both moving and moved.

secondarily. For that which sometimes subsists, and sometimes not, does not exist primarily, nor essentially, but is adventitious, and extrinsically accedes to the natures in which it resides.

### Concerning the GRADATION OF BEINGS.

#### PROPOSITION XX.

**The essence of soul is superior to all bodies; and an intellectual nature is superior to all souls: and the one itself is superior to all intellectual essences.**

**F**OR every body is moved by another, but is naturally incapable of moving itself. But it is moved by itself, through the participation of soul: it likewise lives through soul, and by means of its presence, is after a manner self-motive; but when soul is absent it is alter-motive; because it is essentially endued with this nature, but soul is allotted a self-motive essence. For it imparts self-mobility to whatever it supervenes. But soul is much prior to that which it essentially imparts. It is therefore above bodies, as a self-motive essence: since these become self-motive through participation. Again, soul which is moved from itself, has the second order from an immovable nature, existing immovable in energy; because a self-motive nature precedes all things that are moved, but an immovable essence, all moving natures. If therefore soul which is self-motive, moves others, it is requisite that an immovable mover, should be prior to soul. But intellect moves, existing immovable, and always energising according to the same. For soul participates through intellect of eternal intelligence; in the same manner as body participates through soul of a self-motive nature. For if eternal intelligence, was primarily resident in soul, it would be inherent in all souls; in the same manner as a self-motive nature. And hence this is not primarily inherent in soul. It is therefore requisite, that a first-intellective nature should be prior to soul. Intellect therefore is prior to souls. But the one is prior to intellect. For intellect though immovable, is not the one: since it understands, and energises about itself. But all beings of whatever kind, participate of the one, but all do not participate of intellect. For it is necessary that those natures should participate of knowledge, to whom a portion of intellect is present; because intellectual cognition, is the principle, and first cause of knowledge. Hence the one is superior to intellect; nor is there any thing superior to the one: for the one is the same with the good. But the good is the principle of all things, as we have demonstrated.

That

## That INTELLECT is not the FIRST CAUSE.

## PROPOSITION XXI.

**E**very order beginning from unity proceeds into some multitude co-ordinate to unity: and multitude of every order is reduced to one unity.

**F**OR unity possessing the relation of a principle, generates a multitude proper to itself. Hence one series, and one universal order descends from unity into multitude. For there would neither be any order, nor series, if unity was essentially barren. But multitude is again reduced into one common cause of all co-ordinates. For that which is the same in every multitude, does not proceed from one of the things which multitude contains: since that which emanates from one out of many, is not common to all, but is alone peculiar to the property of that one. Since therefore according to every order, there is both a certain communion, coherence, and identity, on account of which *these* are said to be co-ordinate, and *those* of another order, it is evident that the sameness of every order proceeds from one principle \*. There is therefore in every order an unity prior to multitude, affording one reason, and series to the things ordered in itself, as well with respect to each other, as likewise to the whole. For admitting that among things contained under the same series, one thing is the cause of another, yet it is necessary that before all things, there should be a cause that the series is one, and that from it all things should be generated as co-ordinates; not that every thing may be a particular something, but may exist of this particular order.

\* The truth of this may be exemplified in light. Thus for instance we see many species of light; one kind emanating from the sun, another from fire and the stars, another from the moon, and another from the eyes of many animals. But this light though various, is every where similar, and discovers in its operations a unity of nature. On account of its uniformity therefore it requires one principle, and not different principles. But the Sun is the only principle of all mundane light. And though there are many participants of light posterior to the solar orb, yet they scatter their uniform light, through one solar nature, property and power. But if we again seek for the principle of light in the sun, we cannot say that the solar orb is the principle of light: for the various parts of this orb diffuse many illuminations. There will therefore be many principles. But we now require one first principle of light. And if we say that the soul of the sun generates light, we must observe that this is not effected by her animal multiplicity, or she would diffuse different lights: and hence we must assert that she generates visible by intellectual light. But this production again, does not subsist through intellectual variety, but rather by a unity of intellect: and this unity is a symbol of that simple unity which is the principle of the universe. And to this principle the solar intellect is united by its unity: and through this it becomes a god. This divine unity of the sun, therefore, is the principle of the uniform light of the world; in the same manner as simple unity and goodness is the author of intelligible light to all intelligible natures.

COROL.

## COROLLARY.

From hence it is evident that both one and multitude, is inherent in the nature of body, and that one nature has many coherent natures, and that many natures depend on the one nature of the universe. And this property belongs to the order of souls, to begin from one first soul, and to descend into a multitude of souls, and to reduce multitude into one. And to an intellectual essence it is peculiar to possess an intellectual unity, and a multitude of intellects proceeding from one intellect, and intimately converted to its nature. And to the one prior to all things, a multitude of unities is present, and to unities themselves a return to the one. Hence after the first one, unities \* subsists; intellects after the first intellect; souls after the first soul; and natures after universal nature.

## PROPOSITION XXII.

Everything which subsists primarily, and according to the nature of a principle, is in every order one; and is neither two, nor more than two, but is universally self-begotten.

FOR if possible let it be two; since the same absurdity will ensue should more than two be admitted. Then if it be two, it is either that which is composed from both unities; and in this case the first will be one and not two: or it is each of the unities. But in this case, either one of each, and not both, will be the first; or both will be equally the first. But if equally, neither of them will be the first: for if the one is first, but this one is not the same with the other, what order will it possess? For that subsists primarily, which is nothing else than what it is denominated. But each of these being different, the one from the other, each at the same time is, and is not that which it is said to be. But if these differ from each other, yet not with respect to that which is called first (for this primarily suffers identity), both will not be first, but that through the participation of which, both are said to be first.

## COROLLARY.

From hence it is evident that the first being is one alone, and that there are not two, or more first beings. And that the first intellect is one alone, but that there are not two

\* This will be evident by considering that *the one*, or the first principle of all, must produce that which first proceeds from himself, by *union* (*καθ' ινειν*). And as his first production must be the most similar of all things to himself, and must be at the same time multitude (or in what respect would it differ from the one); hence it is necessary, that this progression must be no other than self-perfect unities. In consequence therefore of this sublime doctrine, as Proclus beautifully observes (Theol. Plat. p. 123,) there is one god, and many gods; one unity, and many unities prior to beings: and one goodness, and many goodnesses after this first good. It likewise follows that the first principle of all is a super-essential one; and that after this one, there are many super-essential unities. And we may consider every unity of being, as the flower of some certain being; and as the summit and centre about which every being subsists. For a farther account and confirmation of this sublime doctrine, study the third book of Proclus on Plato's Theology.

first intellects. And that the first soul is one ; and in every particular species, the same conclusion will result, as for instance the first beautiful, first equal, and similarly in all the rest. In like manner there is the same demonstration, with respect to one first form of animal and man.

### Concerning the imparticipable \* (or that which is without participation), and that which participates.

#### PROPOSITION XXIII.

Every imparticipable produces from itself participants, and all participated hypostases, or subsistencies, are reduced to imparticipable essences.

FOR an imparticipable possessing the relation of unity, as depending on itself, and not on another, and as separated from participants, generates things able to participate. For either it remains in itself barren, and possesses nothing honourable ; or it gives something from itself. And that which receives from this imparticipable participates, and that which is given subsists in a participated manner. But every thing participating of another by which it is generated, is posterior to that nature which is similarly present to all things, and which fills all things from itself. For that which subsists in one thing is not in others : and that which is similarly present to all things, that it may illustrate all, is not in one thing, but before all. For it either subsists in all, or in one of all, or before all. But that which subsists in all, because distributed into all, again requires another, which may unite its divided nature : and all things will no longer participate of the same, but this will participate one thing, and that another, if the one is distributed into many. But if it subsists in one of all, it will no longer be common to all, but to one. If therefore it is common to things capable of participating powers, and is likewise common to all, it will be prior to all things. But this is imparticipable.

\* By *imparticipable*, in these Elements, we must understand that which participates nothing belonging to the characteristic of its nature, and which is likewise participated through proper mediums by all subordinate natures. Thus for instance *imparticipable being*, is that which participates nothing of *being*, but is the source of *being*, to others : and at the same time it is not participated without a medium by subordinate essences ; since the participation of *being*, in subordinate natures, is accomplished through the immediate progeny of *being itself*. And so of the rest. Nor ought it to seem wonderful that an imparticipable should preside over every series of causes : for this is a necessary consequence, if we consider that every cause must be that to its following progeny, which the first cause of all, is to the universality of things. But the first cause of all is perfectly imparticipable ; since he would not be purely *the one*, if he was mingled with *many*, and received the condition of a subordinate nature. As the divine unities therefore are participated by all the following orders of things, and are the mediums, by which every being is conjoined with the first cause of all ; so the immediate progeny of every primary cause conjoins all the subsequent progressions with that cause, which from its similarity to *the first* is properly denominated an imparticipable.

## PROPOSITION XXIV.

**E**very participant is inferior to that which it participates, and that which is participated is subordinate to an imparticipable.

**F**OR the participant being imperfect prior to participation, but becoming perfect through participation, is entirely posterior to that which it participates; so far as it is perfect through participation. For so far as it was imperfect, it is inferior to that which it participates and which is the cause of its perfection. But that which is participated because common to some one, and not to all, is on this account allotted a subsistence inferior to that which is common to all, and not to some particular one: for the latter is more allied, but the former, less, to the cause of all things. Hence an imparticipable, precedes things participated; and these last precede participants. And in short that which is imparticipable is one prior to many; that which is participated is one in many; and every participant is at the same time non-one, and one.

## CONCERNING THE PERFECT.

## PROPOSITION XXV.

**E**very thing perfect proceeds to the procreation of such offspring as it is able to produce, imitating the one principle of the universe.

**F**OR as the principle of the universe, on account of his goodness is uniformly constitutive of all beings (for the good is the same with the one, and on this account that which is endued with the form of good, is the same with that which is uniform); so things posterior to the principle, on account of their proper perfection hasten to generate other things subordinate to their own essence: and this perfection is a certain portion of the good; and the perfect so far as perfect imitates the good. But this is constitutive of all things. Hence the perfect also is naturally productive of things within its power; and that which is more perfect, by how much the more perfect it is, by so much the more is it the cause of more numerous productions: for that which is more perfect, participates more of the good. But this is nearer to the good, is more allied to the cause of the universe, and is the cause of more numerous productions. But the imperfect, by how much the more imperfect it is, by so much the more is it the cause of less numerous effects: for existing more remote from the producer of all things, it becomes the cause of fewer effects. For to constitute and adorn, or perfect, or contain,

or

or vivify, or fabricate all things, and to effect each of these in many, is allied to the principle of all. But if this is accomplished in a few it, becomes more foreign from the principle.

## C O R O L L A R Y.

From hence it is evident that matter which is most distant from the principle of the universe, is barren, and the cause of nothing. For if it should generate any thing, it would have something posterior to itself, and it would not be the most remote. But that which it produces, would be more distant than itself, and because it produces and imitates the productive cause of all beings, it would be nearer to the principle of the universe.

## CONCERNING THAT WHICH PRODUCES.

## P R O P O S I T I O N   XXVI.

**E**very cause productive of other things, abiding in itself produces natures posterior and subsequent to itself.

**F**OR if it imitates the one itself, but that immovable generates natures posterior to itself, hence every productive nature will in a similar manner possess the cause of producing. But that the one itself immovable generates, is evident from hence. For if he generates through motion, either motion will be resident or non-resident in his nature, and that which is moved will be no longer one; because it will be changed and moved from one. Hence the one will either produce in infinitum\*, or immovably. And every producing nature will imitate the one productive cause of the universe. For from that which is first, that which is not first every where emanates. And hence from that which is productive of all things, that which is productive of some things will proceed. Every producing cause, therefore, produces subsequent natures, abiding in itself: and while productive natures abide in themselves undiminished, secondary natures are produced from them. For that which is in any respect diminished, cannot abide such as it is.

\* Because if the one generates through motion, an infinite motion must take place, through the want of an immovable nature.

## PROPOSITION XXVII.

**E**very producing nature, on account of its perfection, and abundance of power is productive of secondary natures.

**F**OR if it produces, not on account of its perfection, but through a defect of power, it cannot preserve its own proper order immovable. For that which affords being to another, through its defect and imbecillity, confers on it essence by a mutation and alteration of itself: but every producing nature abides such as it is, and while it abides, that which is posterior to itself proceeds into being. Hence existing full and perfect, it procreates secondary natures immovably, and without diminution: at the same time existing such as it is, neither changing itself into its progeny, nor diminishing its nature. For the thing produced is not a dissection of the parts of the producing cause: since this is neither proper to generated natures, nor to generating causes. Nor is it a transition; since it is not the matter of that which proceeds to generation. For the producing cause abides such as it is, and the production is different from itself. That which generates therefore abides without alteration, and without diminution, multiplying itself through its prolific power, and supplying from itself secondary subsistencies.

## PROPOSITION XXVIII.

**E**very producing nature generates things similar to itself, prior to such as are dissimilar.

**F**OR since the producing cause is necessarily more excellent than the thing produced, these can never be mutually the same simply considered, or equal according to power. But if they are not the same and equal, but different and unequal; they are either entirely separated from each other, or they are both united and separate. But if they are entirely separate, they cannot be conciliated with each other, and the thing produced will not sympathize with its cause. Hence the one will not participate of the other, because they are entirely different. For that which is participated, imparts a communication to its participant, with respect to that of which it participates. But it is necessary that the thing caused should participate of the cause, as that from which its essence is derived. But if that which is produced is in one respect separated, and in another united to its producing cause; if it equally suffers both, it equally participates and does not participate of its producing cause. It will therefore both possess and not possess

possess an essence from it, after the same manner. But if it should be more separated, the thing generated will be more foreign from its generating cause than is proper, and will be to itself more inelegant than elegant, and more deprived than endued with sympathy of nature. If then generated natures, are both allied according to essence, and in sympathy with their causes, naturally depending upon, and desiring a contact with them, pursuing good and obtaining the object of desire through cause, it is evident that things produced are more united with their producing causes than separated from them. And things which are more united are more similar than dissimilar to the natures with which they are especially united. Every productive cause therefore generates things similar prior to such as are dissimilar.

## PROPOSITION XXIX.

**E**very progression is accomplished by a similitude of things secondary to such as are first.

**F**OR if a producing cause generates similars prior to dissimilars, similitude generates things produced from their producing causes. For similars become similar through similitude, and not through dissimilitude. If, therefore, progression in its diminution preserves the identity of the thing generated to its generator, and exhibits that which is posterior to itself secondarily, such as itself is primarily, it will possess its essence through similitude.

## PROPOSITION XXX.

**E**very thing immediately produced from another, both abides in its producing cause, and proceeds from it.

**F**OR if every progression is effected while things first abide, and is perfected through similitude, things similar subsisting prior to the dissimilar, the thing produced will abide in a certain respect in its producing cause: since that which has entirely proceeded from its cause, possesses nothing the same with that which abides, but is perfectly separated. But if it possesses any thing in common, and united with its abiding cause, it will also abide in its cause, in the same manner as that abides in itself. But if it abides only without proceeding, it differs nothing from its cause, nor will it while that abides, be effected something different: for if it is something else, it will be distinct and separate. But if separate, and its cause abides, it proceeds from its cause that it may be separated from its abiding generator. So far, therefore, as the thing produced possesses any thing

the same with its producing cause, it abides in this cause: but so far as it possesses something different, it proceeds from its cause. But on account of its similitude, it is at the same time in a certain respect both the same with, and different from its producing cause. It abides therefore, and at the same time proceeds with its cause, and neither is separate from the other.

## PROPOSITION XXXI.

**E**very thing proceeding from another essentially, is converted to that from which it proceeds.

**F**OR if it should indeed proceed, without being converted to the cause of its progression, it will not desire this cause: since every desiderative nature (*τὸς ἄργοντος*) is converted to the object of its desire. But every being desires good; and its acquisition takes place through a cause proximate to particulars. Hence particulars desire their cause: For well-being is distributed to any particular being, through the same cause as being itself. But desire is primarily directed to that cause from which well-being proceeds: and conversion is directed to that, to which desire primarily tends.

## PROPOSITION XXXII.

**E**very conversion is effected by a similitude of the converted natures to the object of their conversion.

**F**OR every thing converting itself to another, hastens to conjoin itself with the object of its conversion, and desires its communion and conjunction. But similitude collects all things, in the same manner as dissimilitude separates and divides. Conversion therefore is a certain communion and contact. But every communion and every contact is caused by similitude. And hence every conversion is effected by similitude.

## PROPOSITION XXXIII.

\* **E**very thing proceeding from, and returning to another, has a circular energy.

**F**OR if it is converted to that from which it proceeds, it conjoins the beginning with the end. And there is one and a continued motion: this commencing from, and

\* In order to understand this proposition the reader must observe that the hypothesis requires that both the progression and regression subsist together. And this hypothesis is no less proper than true; for unless effects were continually converted to their causes, they could not exist; since they depend on these for their subsistence, and this can only be procured by conversion.

that

that proceeding to the abiding nature. Hence all things proceed circularly, from causes to causes. But these circles of regression are greater and less: since some conversions proceed to things proximately placed above them, but others to things more superior, and so on to the principle of all. For all things proceed from this, and to this finally return.

### PROPOSITION XXXIV.

**E**very thing which is naturally converted, converts itself to that from which it derives the progression of its peculiar subsistence.

**F**OR if it is naturally converted, it possesses an essential desire towards that to which it is converted, and it directs all its being towards that to which it makes an essential conversion. And it is also essentially similar to the object of its conversion; and on this account is in sympathy with it according to nature, because allied to it by essence. But if this be the case; either the essence of both is the same; or the one proceeds from the other; or both derive their similitude from some other one. But if the essence of both is the same, how can the one be naturally converted to the other? And if both proceed from one; both will be naturally converted to this one. It remains therefore that the one must derive its essence from the other. But if this be the case, progression also must originate from that to which there is a conversion according to nature.

### C O R O L L A R Y.

From hence it is evident that intellect is the object of desire to all things: that all things proceed from intellect; and that the whole world, though eternal, derives its essence from intellect. For it is not because eternal excluded from proceeding from intellect. Nor is it because established in perpetual order, excluded from conversion. But it both perpetually advances and is eternal according to essence; and it is perpetually converted and indissoluble according to its invariable order.

### PROPOSITION XXXV.

**E**very thing caused (or produced by a cause), both abides in its cause, proceeds from, and is converted to it.

**F**OR if it alone abides, it will differ in nothing from its cause; from which it will be indistinct. For progression subsists together with distinction. But if it alone proceeds, it will be unjoined with its cause, and in no respect commun-

cate with it according to a sympathetic affection. But if it is alone converted, how can that which does not derive its essence from its cause, be naturally converted to that which is foreign from its nature? But if it abides and proceeds, and is not converted, how can the natural desire of every thing to well-being, and to good, and an excitation to its generator, arise? But if it proceeds indeed, and is converted, but does not abide, how, since it is distant from its cause, can it hasten to be conjoined with it? Since it would be unconjoined prior to its departure. For if it was conjoined, according to this conjunction it would entirely abide. But if it should abide and be converted, but should not proceed, how, since not separated, can it be converted? For every thing returning to its cause, in the act of returning is assimilated to that from which it is essentially divided. But it is necessary that the thing caused, should either alone abide, or be alone converted, or alone proceed; or that the extremes should be conjoined with each other; or that the medium between these, should be united with each extreme; or that all these should take place together. It remains, therefore, that every thing caused must abide in, proceed from, and be converted to its cause.

### P R O P O S I T I O N   XXXVI.

Of all things which are multiplied in progression, such as are first are more perfect than such as are second, and such as are second than those of a posterior order, and so on in continual succession.

**F**OR if progressions distinguish things produced from their causes, and are the subordinations of secondary natures to such as are first; first progressions will be more conjoined with their causes, from which they are produced, and of which they may be considered as the blossoms. But secondary progressions are more remote from their causes; and this will be the case in a continual succession. But things nearer, and more allied to their causes, are more perfect: for causes are more perfect than things caused. But such as are more remote are more imperfect, because on this account dissimilar to their causes.

### P R O P O S I T I O N   XXXVII.

Of all things which subsist according to conversion, the first are more imperfect than the second; and the second than those in succession. But the last are the most perfect.

**F**OR if conversions are produced in a circle, and conversion tends to that from which the progression began; but progression is from the most perfect, conversion also will tend to the most perfect. And if that to which progression tends is the last, the first conversion

conversion will originate from this. But progression to the last is the most imperfect; and conversion commences from the most imperfect. In things, therefore, subsisting according to conversion, the first are the most imperfect, but the last the most perfect.

## PROPOSITION XXXVIII.

**E**very thing proceeding from a certain number of causes, is converted to them by the same number as it proceeds from them; and every conversion subsists through the same cause as progression.

**F**OR since both are produced through similitude, that which immediately proceeds from any cause, is immediately converted to it: for the similitude was immediate. But that which requires a medium in its progression, requires also a medium in its conversion. For it is requisite that both progression and conversion should subsist about the same. It will, therefore, be first converted to the medium, and afterwards to that which is more excellent than the medium. Hence well-being is distributed to every thing through the same number of causes as being; and the contrary of this is likewise true.

## PROPOSITION XXXIX.

**E**very being is either alone essentially converted, or vitally, or according to a gnostic mode ( $\gamma\omega\sigma\pi\kappa\omega\varsigma$ ).

**F**OR it either possesses being alone from its cause, or life together with being, or it receives from thence a gnostic power. So far, therefore, as it is *being* alone, it makes an essential conversion. But so far as it likewise *lives*, a vital conversion. And so far as it *knows*, a gnostic conversion. For according to its progression, such is its conversion; and the measures of its conversion are defined by the measures of its progression. Hence some are endued with desire according to being alone; this desire being adapted to the participation of causes. But others according to life; and this vital desire is a motion to more excellent natures. And others according to cognition, which desire is a perception of the goodness of causes.

## PROPOSITION XL.

**S**elf-subsistent natures antecede all things proceeding from another cause.

**F**OR if every thing sufficient, is more excellent, either according to essence, or according to energy, than that which depends on another cause, but that which produces itself, because productive of its own being, is sufficient to itself. But that which

is alone produced from another is not sufficient. Likewise since that which is sufficient is more allied to good; and things more allied and similar to causes, subsist from cause prior to dissimilars: hence things self-productive, and self-subsistent, are more ancient, than such as proceed to being from another alone. For either nothing is self-subsistent; or the good itself is such; or things which are the first subsistents from the good. But if nothing is self-subsistent, there will not be a true sufficiency in any thing. For this cannot reside in the good, since that is more excellent than sufficiency, subsisting as the one; and being *the good*, but not possessing good. But neither on this hypothesis can sufficiency reside in natures posterior to the good. For all things will be indigent of that which is prior to their nature. But if the good is self-subsistent, because it produces itself, it will not be one. For that which proceeds from one, is not one: and it will proceed from itself, if it subsists by itself. And hence the one itself will be at the same time one, and not one. It is necessary, therefore, that a self-subsistent nature should be posterior to the first: and it is evident, that it must likewise be prior to things alone proceeding from another cause. For it is more principal than these, and, as we have demonstrated, is more allied to the good.

## PROPOSITION XLI.

Every thing residing in another is alone produced by another. But every thing residing in itself, is self-subsistent.

FOR that which abides in another, and is indigent of a subject, can never be generative of itself. For that which naturally generates itself, does not require a foreign seat, since it is contained by itself, and is preserved in itself separate from a superior. But that which is capable of abiding, and of being established in itself, is productive of itself: since it proceeds into itself, and contains its own nature; and abides in itself; as a thing caused in its cause. For it does not abide as in place, nor as in a subject. For place is different from that which subsists in place, and that which resides in a subject is different from its subject. But this is the same with itself. For it is self-subsistent, and abides in itself, as the thing caused in its cause.

## PROPOSITION XLII.

Every thing self-subsistent is converted to itself.

FOR if it proceeds from itself, it will also return to itself. For that which is the source of progression to particulars, is likewise the end of a conversion coordinate to the progression. For if it should alone proceed from itself, but should not

not be converted by a progression into itself, it will never desire its own proper good, and that which it is able to afford itself. But every cause is capable of conferring on its progeny, together with the essence it affords, well-being, which is conjoined to the essence which it distributes. Hence it can confer this on itself. And, therefore, this is the proper good of a self-subsistent nature. But this, according to the hypothesis, will not be desired by that which is converted to itself : and because it does not desire this, neither will it pursue it ; and in consequence of not pursuing, it will be imperfect and insufficient. But if sufficiency and perfection belong to any thing, they must be proper to a self-subsistent nature. And hence it will pursue and desire its own proper goods, and will be converted to itself.

## PROPOSITION XLIII.

**E**very thing converted to itself, is self-subsistent.

**F**OR if it be naturally converted to itself, and is perfect in its self-conversion, it will also possess essence from itself. For that to which conversion according to nature tends ; from this also the essential progression of every thing proceeds. If, therefore, it affords to itself *well-being*, it will also indeed afford to itself *being* ; and it will be the lord of its own subsistence. Hence that which is capable of being converted to itself, is self-subsistent.

## PROPOSITION XLIV.

**E**very thing converted to itself according to energy, is also converted to itself according to essence.

**F**OR if it can be converted to itself according to energy, but is not converted essentially, it will be more excellent according to energy than according to essence ; since the former is convertible, and not the latter. For that which depends on itself, is more excellent than that which depends on another. And that which preserves itself, is more perfect than that which is only preserved by another. If, therefore, it is converted to itself, according to that energy which proceeds from essence, it will likewise be allotted a converte essence ; so that it will not only energize to itself, but will likewise depend on itself, and will be contained and perfected by itself.

## PROPOSITION XLV.

**E**very thing self-subsistent is without generation.

**F**OR if it be generated, because generated it will be essentially imperfect, and independent of that perfection which proceeds from another. But because it produces itself, it is perfect and sufficient. For every thing generated is perfected by another, which brings it into existence from a non-existent state. Since generation is the passage from that which is imperfect to its contrary, the perfect. But if any thing produces itself, it is always perfect; because it always coheres to its own cause, or rather inheres in that which is perfective of its essence.

## PROPOSITION XLVI.

**E**very thing self-subsistent, is incorruptible.

**F**OR if it may be corrupted, it may desert itself, and exist separate from itself. But this is impossible. For, on account of the unity of its nature, it is at the same time both a cause and the thing caused. But every thing which is corrupted, is corrupted by a departure from its cause. For so far as any thing depends on that which contains and preserves it, so far it is contained and preserved. But that which is self-subsistent will never desert its cause, because it will not desert itself. For it is its own cause. And hence every thing self-subsistent is incorruptible.

## PROPOSITION XLVII.

**E**very thing self-subsistent is imitable (i. e. without parts) and simple.

**F**OR if that which is self-subsistent is partible, it will constitute itself partible, and the whole will be converted to itself \*, and all will be in all itself. But this is impossible. Hence that which is self-subsistent is imitable. But it is likewise simple. For if composite, it will contain both that which is excellent, and that which is base †; and the more excellent will proceed from the more base, and the more base from the

\* This is absurd, because every partible nature must be converted to something different from itself, on account of its parts. So likewise since a self-subsistent nature resides in itself, if such a nature was partible, one divisible whole would be in another, not different from itself.

† Because every composite consists of matter and form; the former of which is base, and the latter more excellent.

more

more excellent, since, according to hypothesis, the whole proceeds from the whole itself. Besides it will not be sufficient to itself, because it requires the elements of itself, from which it is composed. Every thing therefore which is self-subsistent is simple.

### Concerning that which is Eternal, for the Purpose of demonstrating the Eternity of the World:

#### PROPOSITION XLVIII.

**E**very thing not eternal, is either a composite, or subsists in another.

**F**OR it is either capable of being dissolved into its component parts, and is entirely composed from the parts into which it is dissolved; or being indigent of a subject, and deserting its subject, it passes into non-entity. But if it is simple, and abides in itself, it will be indissoluble and incapable of dissolution.

#### PROPOSITION LXIX.

**E**very thing self-subsistent is eternal.

**F**OR there are two modes according to which it is necessary, that any thing should be non-eternal. The one flows from composition, and the other from residing in a subject. But that which is self-subsistent, is neither compound, but simple; nor does it abide in another, but in itself. And hence it is eternal.

#### PROPOSITION L.

**E**very thing which is measured by time is *generation*, either according to essence, or according to energy, so far as it is measured according to time.

**F**OR if it is measured by time, essence or energy, according to time, is proper to its nature. And it is likewise proper that the terms, *it was*, and *it will be*, should be different from each other. For if *it was*, and *it will be*, were numerically the same, that which is measured by time would suffer nothing from time proceeding, and always having something prior and posterior. If then *it was*, and *it will be* differ from each

Y y 2 other,

other, that which is measured by time is *in generation* (*γενέσει*, *ιτι*) and never truly is \*, but proceeds together with time, by which it is measured, existing in a state of tendency

\* The truth of this reasoning may be evinced by the following considerations: Every thing which is measured by time, and such is every corporal nature, depends on time for the perfection of its being. But time is composed of the past, present, and future. And if we conceive that any one of these periods is taken away from the nature with which it is connected, that nature must immediately perish. Time, therefore, is so essentially and intimately united with the natures which it measures, that their being, such as it is, depends on the existence of time. But time, as is evident, is perpetually flowing, and this in the most rapid manner imagination can conceive. It is evident, therefore, that the natures to which it is so essential, must subsist in a manner equally transitory and flowing. As we cannot therefore affirm with propriety of any part of time, that it *is*; since even before we can form the assertion, the present time is no more; so with respect to all corporeal natures (from their subsistence in time) before we can say that they exist, they lose all identity of being. But as it appears to me this flowing, and evanescent nature of things existing in time, may be aptly illustrated by the following similitude. Conceive a line, all whose parts are in a continual flux, like the waters of the most rapid river: and let it be distinguished by three different colours, so profoundly mingled with each other, that every part of the line may possess these colours, in the same manner as the whole line. Now a line of this kind, will very properly represent time, distinguished by past, present, and future, to which the three colours correspond. If then we conceive another line moving on this coloured line, with an equable uniform motion, corresponding to the motion of the parts of the coloured line, and so that in every instant of its motion it may be tinged with one of these colours in regular succession; we shall see the condition of a nature whose being is measured by the progressions of time. For as the perpendicular line, in its uniform motion, no sooner assumes one colour, than it deserts it for another, and cannot on this account be said to possess any colour; so every corporeal nature, from its being profoundly mingled with the fleeting and unreal essence of time, cannot be said to possess any true and substantial being.

Such then is the unreal condition of every thing subsisting in time, or of every thing corporal, and entangled with matter. But this shadowy essence of body is finely demonstrated by Plotinus in the sixth book of his third Ennead, as follows: "Being (says he) properly so called is neither body, nor is subject to corporeal affections: but body, and its properties, belong to the region of non-entity. But you will ask, how is it possible, that visible matter, should possess no real being: that matter, in which stones and mountains reside, the solid earth, and bodies which mutually resist; since bodies which impel each other, confess by their collision, the reality of their existence? You will likewise ask after what manner things which neither strike against, nor resist each other; which neither externally act, nor internally suffer, nor are in any respect the objects of sight, viz. soul and intellect, are to be reckoned true and real beings? We reply, that, on the contrary, things more corpulent are more sluggish and inert, as is evident in bulky masses of earth: but whatever is less ponderous is more movable and alert; and the more elevated the more movable. Hence fire, the most movable of all the elements, flies in a manner from a corporeal nature. Besides, as it appears to me, whatever is more sufficient to itself, disturbs others less, and brings less inconvenience: but such things as are more ponderous and terrene, unable, from their defect of being, to raise themselves aloft, and becoming debile and languid, strike and oppress surrounding bodies, by their falling ruin and sluggish weight. Since it is evident, that bodies destitute of life, fall with molestation on any proximate substance, and more vehemently impel and pain whatever is endued with sense. On the contrary, animated beings, as participating more of entity, by how much the more of being they possess, by so much the more harmless they impinge on their neighbouring bodies. Hence motion, which is a kind of life, or soul, or an imitation of life in bodies, is more present with whatever is less corpulent; as if more of body was necessarily produced where a defect of being happens in a greater degree. Again, it will more manifestly appear from passivity, that whatever is more corpulent is more passive; earth in a greater degree than the other elements; and the rest in a similar proportion. For some things, when divided, suddenly return to their former union, when no obstacle prevents their conjunction; but from the section of a terrene body, the divided portions, always remain separate; as if destitute of natural vigour, and without any inherent desire of union and consent.

dency to being, or in becoming to be (*is τη γίνομαι δι*). Nor does it stop in the same state of being, but always receives different being; so that its *now* becomes perpetually different on account of the progression of time, and prevents its existing totally at the same period of time. For it subsists in a dispersion of temporal extension, and is co-extended with time. But this is no other than to possess being, in non-being. For that which is in generation, or a state of becoming to be, is not generated. *Generation*, therefore, is that which subsists in a flowing existence..

## PROPOSITION LI.

**E**very thing self-subsistent is exempt from things measured by time according to essence.

**F**OR if that which is self-subsistent is without generation, it cannot be measured by time according to essence. For generation subsists about a nature measured by time. Hence nothing self-subsistent subsists in time.

## PROPOSITION LII.

**E**very thing eternal, is at once total.

**F**OR whether it only possesses an eternal essence, it will possess the whole at once present. Nor will it have one of its parts already constituted, but another which remains to be constituted, because not yet in existence; but as much as is possible it possesses the whole, without diminution, and without extension. Or whether it possesses an energy with respect to essence, and this collected into one, and abiding in the same measure of perfection, and established as it were according to one and the same

consent. Hence they are ready, by every trifling impulse, to remain as they are impelled; to rush from the embraces of *bound*, and hasten into multitude, and non-entity. So that whatever becomes corporeal in an eminent degree, as falling fast into non-entity, has but little power of recalling itself into one. And on this account ponderous and vehement concussions are attended with ruin, when by mutual rushing one thing impels another. But when debility runs against debility, the one is valid against the other, in the same manner as non-entity rushing on non-entity. And this we think a sufficient confutation of their opinion, who only place being in the genus of body, persuaded by the testimony of impulses and concussions, and the phantasms perceived through the senses, which testify that sense is alone the standard of truth. Such as these are affected in a manner similar to those in a dream, who imagine that the perceptions of sleep are true. For sense is alone the employment of the dormant soul: since as much of the soul as is merged in body, so much of it sleeps. But true elevation, and true vigilance, is a resurrection from and not with the dull mass of body. For indeed a resurrection with body, is only a transmigration from sleep to sleep, and from dream to dream, like a man passing in the dark from bed to bed. But that elevation is perfectly true, which entirely rises from the dead weight of bodies: for these possessing a nature repugnant to soul, possess something opposite to essence. And this is farther evident, from their generation, their continual flowing and decay; properties entirely foreign from the nature of being, substantial and real."

*bound,*

bound, immovably, and without progression, it will still possess the whole at once present. For if it be eternal, as its name denotes, it is a perpetual being (*τὸ αἰώνιον*.) But *to be sometimes, and to have an existence in becoming to be*, is different from that which always is. And hence it is requisite that an eternal nature should not possess any thing prior and posterior : for in this case it would become generation and non-entity. But where neither prior, nor posterior, not *it was*, and *it will be* have any subsistence, but being alone, there an essence at once total abides \*. And every thing energizes according to its essence.

## C O R O L -

\*. The account of eternity given by Plotinus, in which its *total* nature is copiously demonstrated, is so imimitably sublime and profound, that I cannot refrain from presenting the reader with the following paraphrased translation, of the most considerable part of the seventh book of his third Ennead, relative to the nature of eternity. Proclus (in Theol. Plat. p. 149.) observes that the definition of eternity given by Plotinus in this discourse, viz. "Infinite life, which at once openly exhibits itself, and manifestly declares its own being," is a definition framed in a most divinely inspired manner (by *Diogenes*.)

" Since we reckon eternity and time to be different from each other, and that the one respects a nature which always is, and that the other is conversant with things in generation, unstable and flowing, from which this sensible universe is composed; we are apt to believe that from our natural ingenuity, and a sudden view of intelligence, an instinctive knowledge concerning time and eternity, is profoundly inherent in our souls; since we always affirm the same properties of these, and never vary in the appellations we assign them. But when we endeavour to approach nearer to these, and to explain their natures in a more perfect manner, we are immediately perplexed with doubts; and receiving the different sentiments of the ancients on this subject, according to the difference of our opinions, and perhaps conceiving the same sentiment, in a manner different from other men, we advance no farther in our enquiries; esteeming it sufficient, if when interrogated, we are able to relate the opinions of antiquity on this important subject. Indeed it is proper to believe that some of the ancient, and blessed philosophers, have discovered the truth: but it is highly necessary to enquire what method they pursued in their investigation, and how we may arrive at the knowledge of the same exalted truths. And first, it will be proper to enquire what they think of eternity, who judge it to be different from time: since from the knowledge of that which is, as the exemplar, we may perhaps more plainly understand, the nature of its image, which is denominated time. But if any one previous to his contemplation of eternity, should conceive the nature of time in his imagination, he may perhaps happen, by advancing into the intelligible world, and by a certain reminiscence, to comprehend that to which time is assimilated: since time possesses a similitude to eternity.

What then shall we say eternity is? Is it intelligible essence itself, in the same manner, as some are reported to affirm, that time is the whole heavens, and the world? For since we imagine and understand eternity to be something especially venerable, which we likewise affirm of an intelligible nature, we are not able to discern which of these is most venerable. And as that which is superior to these, ought not to be described by appellations of this kind, perhaps eternity, and an intelligible nature, may with propriety be received as the same. Again, both eternity, and the intelligible world, comprehend in themselves beings entirely the same. But when we say that the one abides in the other, placing intelligibles in eternity, and when we pronounce every thing intelligible eternal, as in the Timaeus of Plato, where it is written, " If the nature of the exemplar is eternal, &c." in these cases, I say, we affirm eternity to be something different from an intelligible nature; at the same time confessing that eternity dwells about, or is resident in, or is present with an intelligible nature. For indeed it is not necessary that because both are venerable, they should be perfectly the same: since perhaps the venerableness of the one, is derived from the other. And what is farther added to establish their identity, that the same things are contained by each, is to be thus explained: that an intelligible nature contains every thing belonging to itself, as parts; but that eternity comprehends at the same time the whole, not as a part, but because all things of this kind are called eternal, from its continual presence. But ought we to judge of eternity according to permanent station in the intelligible world, as in this sensible world we are accustomed to consider time according to motion?

## C O R O L L A R Y.

From hence it is evident that eternity is the cause of total existence. Since every thing eternal, has either its essence or its energy totally present with itself, either according to essence, or according to energy.

## P R O P O-

motion? But it may be properly enquired, in the first place, whether eternity, is the same with station belonging to being. For if it is the same with station, we cannot with more propriety, speak of a station which is eternal, than we can of an eternal eternity: since that is eternal which participates of eternity. Besides, if station is the same with eternity, how can motion be eternal? For in this case, it must lose its nature, and become permanent and stable. Besides, how is the notion of station contained in the *ever itself?* I do not mean the *ever*, or *always*, which respects time, but that which we understand when we speak of eternity. But if the *ever itself*, is contained in the station of being, we must consequently segregate the other genera of beings from eternity. Besides, it is requisite to conceive of eternity, not in station alone, but as abiding in *one*; and to preserve its indistinct property, lest it should become entirely the same with time. But station, considered simply as station, neither contains in itself the intelligence of one, nor of that which is indistinct. Hence we predicate of eternity, that it abides in *one* and consequently it participates of station, without being station itself.

What then is the nature of that, by which we call the whole of the divine world eternal and perpetual? And what is perpetuity itself? Is it the same with eternity, or does eternity subsist according to the perpetual? Perhaps we ought to conceive of eternity, as a certain one collected from many; viz. either as one intelligence, or one nature; whether consequent to things in the intelligible world, or existing together with it, or beheld as situated in the depths of its essence. All these, I say, reduced into eternity as one, which is also many, and is endued with a various capacity. Indeed he who beholds a various capacity, when he considers it as a subject, denominates it essence; but so far as he perceives life, he denominates it motion, and afterwards station, considered as abiding in a manner entirely the same. He will likewise behold difference and sameness, so far as they are many, bound in one. So that he who contracts the difference, subsisting in things which are many, into one life alone, and contemplates an unceasing sameness of energy, never passing its intelligence, or life, from one thing into another, but ever abiding in the same manner in itself, far remote from all distance; he, I say, who beholds all these, contemplates eternity, viewing life ever possessing a present whole, where all things abide together in sameness, without the order of first and last, and are comprehended in an indivisible bound. Where all things are collected into one, as into a point, not yet proceeding into a linear flux, but abiding in sameness, that is, in itself, in an *ever present now*: because nothing of its nature is past, nothing in it is future; but what it is, it always is. Hence eternity is not a subject, but that which beams as it were from its subject, according to the possession of an *ever present identity*; promising itself, that its *ever abiding nature*, will never be changed. For what should happen to this in future, which it is not at present? Since it is a perfect and present plenitude of all things. Nor can the term *was*, the appellation of time past, belong to eternity. For what can that be, which *was* present with its nature, and *is not*? It is in like manner independant of all connection with futurity. And hence eternity is that which neither *was*, nor *will be*, but alone *is*, which it possesses in a stable manner; because it is neither changed into a future, nor altered from a past duration. So that the eternity, which we are now investigating, is like total and full, abiding in its essence about being; and is every where indistinct and one.

Nor indeed is it to be thought, that eternity happens extrinsically to the first essence; but we should consider it, as dwelling in this essence, as emanating from, and abiding together with it. For it is beheld as profoundly resident in its essence; because from perceiving every thing else which we affirm to abide there, as perfectly intimate, we say that all things emanate from essence, and abide together with essence. For it is requisite that first natures should abide with, and reside in first beings, and in these have the whole of their essence, because the beautiful dwells in these, and emanates from them: and this also may be asserted of truth. And partly indeed the whole itself, is as it were in a part, but partly, as parts in the whole: as if this true nature was a whole, not congregated from parts, but itself generating parts, that on this account it may be truly all. Besides, truth which flourishes there, is not a certain consonance towards another, but is peculiar to every thing of which it is the truth. Indeed it is requisite that this total truth, if total, should not only be total, so far as it is all things, but should

## PROPOSITION LIII.

*Eternity itself has an existence prior to all eternal natures, and time itself exists before all temporal natures.*

FOR if every where participated natures are prior to their participants, and imparticipables before such as are participated, it is evident that an eternal being is different from the eternity in an eternal nature, and different from eternity itself. For the first of

should likewise so possess the whole, that nothing may be wanting to itself. And if this is the case, no affection of futurity can possibly belong to this being : for if the circulation of time can add any thing to its essence, it must be defective till such an accession takes place. Besides, nothing contrary to nature can happen to this true being ; for it suffers nothing. So that since nothing is generated in it, its being is independent of all temporal connections ; and hence it is neither *about to be*, nor *will be*, nor *was made*, nor *is made*. If indeed the affections of futurity are taken away from generated natures, because they are conversant with perpetual acquisition, they cease to exist. But to things of a nature essentially different, the addition of any future circumstance of being causes them to fall from the stable seat of essence. So that it is manifest from hence, that being itself could not reside in these natures, if they were composed from the past, and future processions of time. Since essence in generated natures appears to be nothing more than a certain procession from *real being*, from the beginning of generation, till it arrives at the extremity of time, when it ceases any longer to exist. And that which is called the being of these, subsists in such a manner that by diminishing any part of this extension, life is likewise diminished : for in this manner it is necessary that every thing which participates of futurity should exist. Hence it continually hastens, by a natural desire, to what is future, without desiring to repose ; since it continually draws fresh supplies of being to itself, while it performs one thing after another, and is moved in a certain orb of progression, by its desire of essence. And thus we have discovered the existence of temporal beings, and the cause of their motion to eternal duration, through the assistance of futurity. But to such things as are first, and to beings truly blessed, the desire of futurity is unknown : for they are already total ; and whatever of life they ought to possess, they now totally possess : so that they seek after nothing, because nothing which is requisite to the full energy and perfection of their essence is future, and consequently nothing happens to them connected with futurity. The absolute and total essence, therefore, of a being which is not separated into parts, but is total in all its parts, and in no one circumstance of being deficient ; and to which nothing of non-entity can possibly happen : (for it is not only requisite that all beings should be present with that which is universal, and total, but that every thing appertaining to non-entity, should be always excluded) is eternity : for that is called eternal which always is."

And shortly after this, he adds : " Hence eternity is something especially venerable, and is the same with a god, as inherent intelligence affirms. But intelligence likewise dictates that eternity is the same with that god, whom we denominate being and life. And it may with the greatest propriety be said, that eternity is a deity shining and unfolding himself in intelligible light, such as he is in his essence : in an essence, I say, perfectly unchangeable, and the same, and thus firmly abiding in an unceasing energy of life. Nor ought any one to wonder that we speak of eternity, as composed from many. For every thing which abides in the intelligible world, is called many, on account of its infinite power ; since infinite there receives its denomination, because it never falls off from the consummate intellectual plenitude of its nature. And indeed it is particularly called after this manner, because it loses nothing of its own : and if any one should describe eternity after this manner, as life already infinite because universal, and because it never deserts the integrity of its nature ; (since it cannot be diminished by the past, nor increased, by the future, because it is a perfect whole) he who thus describes eternity, will approach very near to its true definition. For what is afterwards added, that it is a perfect whole, and loses nothing of its integrity, is only a certain exposition of the definition which affirms it to be infinite life. But because a nature of this kind, thus all-beautiful and eternal, abides about the *one itself*, enlivening and in no respect departing from it, but ever abiding about and in it, and living with it, in indissoluble union ; hence it is said by

of these subsists as a *participant*, the second as a thing *participated*, and the third as an *imparticipable*. In like manner with respect to time, one thing exists as a *participant*, and the time which it contains, as a thing *participated*, and the time prior to this, as an *imparticipable*\*. And each of these consists from *imparticipables* every where, and in all things the same. But participated time alone exists in the natures by which it is participated. For there are many eternal and temporal natures, in all of which eternity abides according to participation. And since such a time is *indivisible*, there is also a divided time. And there is one time prior to these. And *eternity itself* is *an eternity of eternities*, but *time itself*, is a *time of times*; and they are sustainers of participated natures.

## PROPO-

by Plato, not rashly, but in a manner truly beautiful, and profound, *that eternity abides in one*: so that he not only reduces that which eternity contains into one; but the life of being in like manner reduces itself, about *the one self*. This then is what we investigate, and that is eternity, which thus abides. For that which is the *energy* of life abiding from itself, and residing in the depths of unity, without any deception, either in essence or life, is without all controversy eternity. Since truly *to be*, is never *not to be*, and to possess no diversity of being. But when in discoursing on eternity, we use the term *ever*: also when we say, it is not sometimes *being*, and sometimes *non-being*, we must consider these appellations as adopted only for the purpose of explanation; because the term *ever* is not perhaps principally assumed, but is employed, in order to shew, an incorruptible, and never-failing nature.

But here it is proper to enquire, whether we affirm all this concerning eternity, by means of a power foreign from its nature, and as of a foreign concern? But how is this possible? For how can intelligence take place in the soul, unless by an intimate union of intellect, with the object of its intellection? And after what manner can this profound union subsist, by an intelligence of such things as are foreign from our intellectual nature? It is therefore requisite, that we should be partakers of eternity; or we could not thus define its nature, and exhibit the properties which it contains."

\* The nature of *time itself*, which *every thing* in generation participates, has been a subject of much subtle and profound speculation to the Pythagoric, Platonic, and Peripatetic philosophers. According to Archytas the Pythagorean (as we are informed by Simplicius, on Aristotle's Categories) "time is a generative number, proceeding from the motion of the mundane soul, as from unity." But others of the ancients considered it as a certain extension of soul about intellect. Others confessed, that time was the period of soul and intellect. According to the determination of others, it is a natural extension about intellect. And others described time, by circulations in a circular nature; all which the Pythagoric sect comprehends. Time, therefore, according to Archytas and the most ancient philosophers, is a self-moving motion, and the number of temporal unity: for as eternity abides in *one*, so time is moved in *number*. According to Plato, "time is a certain image flowing in number, of eternity abiding in one." But according to Aristotle, "it is the number, or measure of motion according to prior, and posterior." But the opinion of Plotinus respecting time, is perhaps the most profound of all: and though the preceding definitions when attentively considered contain an abundance of intellectual truth, yet the doctrine of Plotinus on this subject *rekindles* a greater light in our intellectual conceptions. In his book then on eternity and time, which we have cited in the preceding proposition, after many admirable doubts concerning the nature of time, he enters on its definition, as follows:

"But it is requisite to recal ourselves again to that nature which we have said resides in eternity: a nature immutable and total, and a life infinite and indeclinable, and abiding in, and about the one. But time was not as yet, or at least was not among these true beings, but was about to follow, by a certain reason and nature of a posterior energy. Supernal natures therefore acting quietly among themselves, if any one desires to know how time first fell, he will not perhaps improperly call upon the Muses, who then were not, for the important explanation. Perhaps also he will opportunely invoke them, since the Muses then existed, according to true conceptions of their nature. And perhaps such a one will find time already generated, such as it now is, and manifests itself to our intellectual view. But he will speak concerning it after the following manner:

"Before priority originated, and was indigent of that which is posterior, time not yet existing according to pro-

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cession,

## PROPOSITION LIV.

Every *eternity* is the measure of eternal natures, and every *time* of temporal natures; and these are the only two measures of life and motion in beings.

FOR every thing which measures, either measures according to a part, or measures the whole, when it is accommodated to that which is measured. But that which measures according to the whole is eternity; and that which measures according to parts is time. There are therefore only two measures, this of eternal, and that of temporal natures.

cession, repos'd together with priority in being itself, with a tranquil and intellectual energy. But at length time desiring a nature engaged in a multiplicity of affairs, and anxious to govern itself, and become independent, and always chusing to possess more of that which is present, began to be agitated, and at the same time commenced a flowing duration, subsisting in continual succession, and a transition into different periods of existence. And by conceiving that from this motion, a certain longitude of procession is effected, we establish time, as an image of eternity. For since it is a certain restless power of the mundane soul, desirous of transferring what it there beholds, into another, it cannot sustain that a collected whole should be at once present with its nature: but as the spermatic reason, unfolding itself from a certain quiet seed, acts according to an abundant discursive energy; dispersing in the mean time that which is called abundant, by a certain division, and instead of the one which abides in itself, unfolding and consuming a non-abiding one, and thus proceeding into a more debile extension: so likewise the nature of the mundane soul fabricating the sensible world in imitation of that which is superior, and agitated with a motion, not such as the intelligible world contains, but rather similar to the motion which is there; with a motion I say affecting to represent its image, it renders itself in the first place temporal, as a substitute for eternity. But afterwards it endeavours to become sub servient to time, because it compels the world to reside in time, comprehending all its transitions in its forming nature. For the world is moved in this nature; since there is no other place for the universe than the mundane soul; and in the time of this soul, it is agitated without end. For this soul applying one energy after another in succession, generates that which is consequent, together with its energy, and at the same time proceeds with a cogitation posterior to this energy, and which before this had no existence, because cogitation was not yet in energy. Its present life too, is now not entirely similar to the preceding life: and hence together with this energy, another life succeeds; and in consequence of this another time is produced. Time, therefore, is allotted a distance and protraction of life; and a perpetual prorogation of life, possesses a perpetual time. If any one, therefore, should say that time is an energy of the mundane soul, perpetually proceeding in a certain transitory motion, from one life to another, he will perhaps appear to assert something correspondent to the truth. For if eternity is life abiding in station, and in a habit similar and the same, and already infinite; and if it is requisite that time should be the image of eternity, in the same manner as this universe is the image of the intelligible world, certainly instead of the life which flourishes there, another life must reside in a certain power of the mundane soul, equivocal with respect to intelligible life; and instead of intellectual motion, it is requisite that there should be a motion of some part of the soul. Likewise instead of identity, and a similar and abiding habit, that which does not abide in the same, but always pursues a different energy must succeed. And again instead of a property indistinct, and one, a resemblance of one possessing unity in a continued succession. Besides instead of that which is infinite and total, that which perpetually propagates itself in infinite succession. Lastly, instead of that which is a collected whole, that which sustains parts, and is always about to be total and perfect. For thus it will imitate that which is total, collected, and infinite, if it wishes its essence to consist in a continual pursuit of being; and thus resembles the being of eternity itself. But it is requisite not to receive time external to soul, in the same manner as eternity is not external to being itself. Nor again, must we conceive that time exists as an appendage, or any thing posterior, in the same manner as conceptions of this kind, are not proper to eternity: but we must contemplate it as beheld seated in the recesses of soul, with which it exists in conjunction, just as eternity subsists in being itself."

PROPO-

## PROPOSITION LV.

Every thing subsisting according to time, either subsists in an eternal time, or has its subsistence in some part of time.

FOR if all progressions subsist by similitude, and prior to things perfectly dissimilar, things similar are more proximate to first natures than such as are dissimilar: and if it is impossible to conjoin with eternals, things formed in a part of time: (for as things generated differ from such as are self-subsistent, and things which have a partial, from such as have a perpetual existence, but the middle of these and those, are partly similar to them, and partly dissimilar.) Hence between things which are sometimes generated, and such as are eternal, the medium must either be that which is always in generation, or that which is sometimes, or that which is not truly, or does not possess true being. But it is impossible that the medium should be that which sometimes truly is. And that which is not true being, is the same with that which is sometimes in generation. Hence the medium cannot be that which *sometimes is*. It remains therefore that that which is always in generation, or in becoming to be, must be the middle of both: for on account of its passing, or flowing existence, it is conjoined with the worse nature, but on account of its perpetuity it imitates an eternal nature.

## COLLAR Y.

From hence it is evident that eternity is two-fold: for some things are of themselves eternal, but others according to time. And the former of these, is an *abiding* eternity; but the other a *flowing* eternity, or such as exists in becoming to be. And the former of these has its being united, and at once total; but the latter diffused, and unfolded according to temporal extension. And the former of these is essentially total; but the latter is composed from parts, each of which are separated according to prior, and posterior.

## PROPOSITION LVI.

Every thing which is produced from secondary causes, is also produced from those prior and more principal causes, from which secondary causes are produced.

FOR if that which is secondary possesses its whole essence from that which is prior to itself, its power of producing emanates also from thence. For productive powers reside in producing causes according to essence, and replenish the essence of these. But if they are allotted a productive power from a superior cause, they possess from this

the cause of being, measured from thence according to an hypostatic, or fabricative power. But if this be the case, the productions of this secondary cause, are caused on account of that which is prior to its nature: for that which perfects a cause perfects also the thing caused. And that the thing caused is more perfected from thence, is manifest. For if that which is first gives to the second, the cause of producing, it will primarily possess this cause; and on this account that which is second generates, receiving from thence a secondary generative power. But if the one becomes productive through participation, but the other by communication; on this hypothesis likewise that will be the primary and more principal cause, which bestows a power of generating on another proximate to its nature.

### P R O P O S I T I O N   L V I I .

**E**very cause both energizes prior to the thing caused, and is productive of more effects posterior to the things caused.

**F**OR so far as it is cause, it is more perfect and powerful than that which is posterior to its nature. And if this be the case it is the cause of more effects. For it is the property of a greater power to produce more, of that which is equal, equal, and of that which is less, less effects. And that which in things similar can effect greater things, can also accomplish such as are less. But that which can accomplish less effects, cannot necessarily effect greater. If, therefore, the cause is more powerful than the thing caused, it is also productive of more effects. But whatever the thing caused can accomplish, the cause is much more capable of effecting. For every thing which is produced from secondary causes, is much more produced from prior and more principal causes. Whatever, therefore, the thing caused is naturally adapted to produce, co-exists with the cause. But if the cause produces prior to the thing caused, it is evident that it energizes before the thing caused, according to its productive energy. Every cause therefore energizes prior to the thing caused; and in conjunction with, and posterior to its nature constitutes other effects.

### C O R O L L A R Y .

From hence it is manifest, that whatever is caused by soul, is also caused by intellect, but whatever is caused by intellect, is not also caused by soul. For intellect energizes prior to soul; and whatever the soul confers on secondary natures, intellect also confers in a more ample manner. And when soul no longer energizes, intellect illuminates with its gifts, natures to which soul does not communicate its essence. For that which is inanimate, so far as it participates of form, participates of intellect, and the formation

of

of intellect. Besides, this likewise follows that whatever is caused by intellect is also caused by the good, but not the contrary. For the privations of forms emanate from the good: since all things flow from this. But intellect since it is form, is not the fabricator of privation.

## PROPOSITION LVIII.

**E**very thing produced from many causes, is more compounded, than that which is produced from a few.

**F**OR if every cause confers something on that which proceeds from it, many causes will confer many gifts, but fewer causes will bestow fewer gifts. Of participants, therefore, some will consist from many, but others from a few of the things which each participates; the former indeed on account of their progression from many causes, but the latter on account of their progression from a few. But the former proceeding from many causes are more composite: and things proceeding from a few, are more simple than those which proceed from many causes. Hence every thing produced from many causes, is more compounded; but that which proceeds from a few is more simple. For the more compounded participates of that which the more simple participates, but the contrary to this, is not true.

## PROPOSITION LIX.

**E**very thing essentially simple, is either more excellent, or worse than composite natures.

**F**OR if the *highest* of beings are produced from things fewer and more simple, but such as are in the *middle*, from a many, these will be composite. And with respect to the *extremes*, some are more simple, according to that which is more excellent, but others according to that which is worse. But that the highest beings are produced from fewer causes, is evident from their being superior, and originating prior to inferiors, and extending themselves over beings, beyond the progressions of subordinate natures, on account of their diminution of power. For on this account the last of beings \*, is most simple, as well as the first, because it proceeds from the first alone. But one kind of simplicity subsists according to a nature more excellent, but another kind, according to that which is more base than every compound; and there will be the same proportion in all things.

\* By the last of beings, he means matter, which on account of its formless nature may be considered as nothing more than the shadow of being; or something if possible still more privative and simple.

## PROPOSITION LX.

**E**very thing which is the cause of a multitude of effects, is more excellent than that which is allotted a power productive of a few; when the few are parts of the many.

**F**OR if this is the cause of a few, but that of a many, and the few are parts of the many, that which forms the one, will also form the rest, if fabricative of a many. It is, therefore, more powerful and comprehensive of a greater multitude. For as production is to production, so is one producing cause to another, according to a mutual relation. But that which is capable of accomplishing more, possesses a greater, and more universal power. And this is nearer to the cause of all: but that which is nearer is a greater good: since the cause of all, is the good itself. Hence that which is the cause of many effects, is essentially more excellent than that which produces but a few.

## PROPOSITION LXI.

**E**very power when impartible is greater, but when divided becomes less.

**F**OR if it is divided, it passes into multitude. And if this is the case, it becomes more distant from unity; and on this account is diminished in power; since it departs from unity by which it is contained, and acquires imperfection. Since the good of every thing subsists through the benefit of union.

## PROPOSITION LXII.

**E**very multitude which is near to unity, is less in quantity than things farther distant, but is greater in power.

**F**OR that which is near is more similar to unity. But unity is constitutive of all things without multiplication. The cause, therefore, of many effects, is more similar to unity. Since the cause of all is the most uniform and imparible of all things; if the cause of all is *one*. As, therefore, that which is less multiplied is more allied to the *one*; so that which is productive of a multitude of effects, is more allied to the cause of all. But a nature of this kind is more powerful.

C O R O L

## C O R O L L A R Y.

From hence it is evident, that there are more corporeal natures than souls; more souls than intellects; and more intellects than divine unities. And in all other natures there is the same proportion.

## P R O P O S I T I O N   L X I I I .

Every imparticipable produces twofold orders of things participated; one in things which sometimes participate; but the other in such as participate always, and in a connate manner.

**F**OR that which is always participated, is more similar to an imparticipable than that which is sometimes participated. Hence that which is always participable, will subsist prior to that which is sometimes participated. Because it is participated indeed, differing from that which is posterior to itself, but because it is always more allied, it is also more similar to an imparticipable. Nor are there alone things, which are sometimes participated: for prior to these are the natures which are always participated; through which they are conjoined with imparticipables according to a certain well ordered progression. Nor are there alone natures which are always participated: for these, since they possess an unextinguishable power (on account of their perpetuity), bear other natures in their essence, viz. the natures which are sometimes participated. And as far as to these diminution and subjection extends.

## C O R O L L A R Y.

From hence it is manifest, that of the unions, which illustrate beings from the one, some are always participated, but others sometimes: and that intellectual participations are in the same manner twofold; and likewise the animations of souls, and of other forms. For beauty, and similitude, and station, and identity, are imparticipable, but they are participated, through things which always participate, and by things which sometimes participate in a secondary manner, according to the same order.

## P R O P O S I T I O N   L X I V .

Every principal unity produces a twofold number; one indeed of self-perfect substances; but the other of illuminations, possessing their subsistence in others.

**F**OR if its progression takes place by subjection, and through things proper to fabricative causes; and if perfect natures proceed orderly from the perfect, and things imperfect

imperfect through these as mediums: hence some will be self-perfect substances, but others imperfect; and these last will become the forms of participants. For since they are imperfect, they will be indigent of subjects to their existence. But the perfect natures will make themselves their own participants: for since they are perfect, they will replenish and establish themselves. But they will require nothing of inferior natures, to their proper subsistence. Self-perfect substances, therefore, on account of their distinction into multitude, are diminished from their principal unity; but on account of their self-perfect essence, they are in a certain respect assimilated to it. But imperfect substances because they reside in others, are remote from that which is self-subsistent; and because they are imperfect, they are distant from that which perfects all things. But progressions are accomplished by things similar, as far as to things perfectly dissimilar. Hence every principal unity produces a twofold number.

#### C O R O L L A R Y.

From hence it is evident, that with respect to unities, some are self-perfect proceeding from *the one*: but that others are illuminations of unities and intellect. And again, that some of these are self-perfect essences, but others nothing more than resemblances of souls which are animated. And hence, neither is every unity a god, but this is peculiar to a self-perfect unity alone. Nor is every intellectual property an intellect, but that which is essential alone. Nor is every illustration of soul, a soul: but there are likewise images of souls.

#### P R O P O S I T I O N   L X V .

**E**very thing which subsists in any manner whatever, either subsists according to cause, in a primary manner (or possessing the form of a principle *ἀρχοειδῶς*) or according to hyparxis \*, or according to participation, after the manner of an image (*εἰκονικῶς*.)

**F**OR either the thing produced is beheld in its producing cause, as in a pre-existent cause: (because every cause previously assumes in itself, the thing caused, being that primarily, which its effect is secondarily) or the producer, is beheld in the thing produced. For since that which is produced participates of its producing cause, it exhibits in itself, in a secondary manner, that which its producer is primarily. Or every

\* By *image*, *hyparxis*, in these Elements is meant that *characteristic* or *summit* of any nature, through which it subsists: and in the gods is the same with the *unity* and *deity* of their natures. And it is likewise necessary to inform the reader, that by *image*, *hypostasis*, or subsistence, is meant any individual nature whether essential, or super-essential, considered as something distinct and different from *accident*.

thing

thing is to be considered in its own order, and is not to be contemplated either in its cause, or in the thing caused. For the one so far as it exists, subsists after a more excellent mode; but the other, so far as it is, in a subordinate manner. But it is requisite that this last also, should be such as it is. And every thing subsists in its own order, according to hyparxis.

## PROPOSITION LXVI.

All beings are to one another either wholes, or parts, and are either *the same, or different.*

**F**OR either one comprehends, but the remainder are comprehended; or they neither comprehend, nor are comprehended. And they either suffer that which is the same, as participating one; or they are distinguished from one another. But if they comprehend they are wholes: and if they are comprehended they are parts. But if many things participate one, they are the same according to one. But if they are many only; so far as many they are different from each other.

## PROPOSITION LXVII.

Every totality is either prior to parts, or composed from parts, or contained in a part.

**F**OR we either contemplate the form of every thing in its cause, and affirm that the whole which subsists in its cause is prior to parts; or we contemplate the form of a thing in the parts which participate of that form. And this in a twofold respect. For the form is either collectively in all the parts; and this is a whole composed from parts, any one of which when absent diminishes the whole itself. Or it is in each of the parts; so that every part according to participation becomes a whole, i. e. a partial whole. But the whole composed from parts subsists on account of essence. But that which is prior to parts according to cause: and that which subsists in a part, according to participation. For this is a whole according to ultimate subjection, so far as it imitates the whole consisting from parts; since it is not any part indifferently, but that which is capable of being assimilated to a whole, whose parts also are wholes.

## PROPOSITION LXVII.

**E**very whole contained in a part, is a part of that whole which is composed from parts.

**F**OR if it is a part, it is a part of some whole: and is either a part of that whole which abides in itself, according to which it is called a whole in a part. But on this hypothesis the whole would be a part of itself, and a part would be equal to the whole, and each would be the same. Or it will be a part of some other whole; and if of some other, it is either only a part of that other, in such a manner as again to differ in no respect from the whole. Or it will be a part together with some other whole. For the parts of every whole, are more than one; and this will be a whole composed from many parts. And thus the whole contained in a part, is a part of that whole which is composed from parts.

## PROPOSITION LXIX.

**E**very whole composed from parts, participates of that totality which is prior to parts.

**F**OR if it is composed from parts, it becomes passive to a whole. For the parts since, they are made one, suffer a whole, on account of their union: and this is a whole subsisting in parts which are not wholes. But that which is imparticipable has an existence prior to every thing participated. An imparticipable totality, therefore, exists prior to a participated totality. And hence there is a certain species of totality, prior to that whole, which is composed from parts. And this is not a passive whole, but is an essential totality; from which the totality resulting from parts proceeds. Since likewise that whole which is composed from parts, subsists in many places, and in various ways, in many other things composed from parts. But it is requisite that there should be an essential monad or unity of all totalities. For each of these wholes is not sincere, because indigent of the parts from which it is composed, and which are themselves different from wholes. Nor if this whole was generated in any thing particular, could it be the cause that all others are wholes. The cause, therefore, by which all things are wholes, is prior to parts. For if this also was composed from parts, it would be a *certain whole*, and not that which is *simply whole*. And this again would subsist from another whole: and this must either be the case in infinitum, or there must be a first whole; a whole not composed from parts, but that which is a perfect totality.

P R O P O-

## PROPOSITION LXX.

That which is more universal subsists in principal causes, and prior to particulars illuminates participants; and leaves that which participates as second in order from principal causes.

FOR a more universal cause begins its energy in secondary natures, prior to that which is posterior to a more universal cause: and it is present not only when that which is posterior is present, but even when the energy of that which is posterior is no more; and it energises in a more causal manner, and this not only in different subjects, but also in each of the things which sometimes participate. Thus for example it is requisite, that being should be first, afterwards animal, and then man. And the species man no longer exists, when deserted by the rational power: but animal, breathing and sentient, will still subsist. And again, though life is taken away, being remains: for when man ceases to live, being is present. And the same reasoning may be adopted in all things. But a cause which is more efficacious, and which is on this account more causal, energises first in a more causal nature: for it suffers the same from a cause more powerful, and prior to itself; and it co-energises with that which is secondary when in energy. For every thing which generates that which is secondary, congenerates also that which is more causal: and when that which is secondary deserts the more causal nature, that which generates the secondary nature is present. For the communication of a more powerful cause, when it is more efficacious, leaves that which participates it, the last of all. For through the communication of that which is second, it strengthens its own illumination.

## PROPOSITION LXXI.

All things which abide in principal causes, and which possess a more universal and superior order in effects, according to the illuminations proceeding from them, become in a certain respect the subjects of the communications of particulars. And the illustrations emanating from superior, receive the progressions of secondary natures. And thus some participations antecede others, and repre-

sentations, or resemblances (*εμφάσεις*) supernally coalesce one after another in the same subject: things more universal energizing first, but particulars posterior to the energy of universals, bestowing their communications on their participants.

**F**OR if things which partake more of cause, energize prior to things secondary, on account of their exuberance of power, and are present with, and illuminate things endued with a more imperfect aptitude: but if things more subordinate, and which are second in order, receive their communications from these; it is evident that the illustrations of superior natures pre-occupy that which participates of both, and establish the communications of subordinate natures. But these illustrations use the resemblances emanating from superior natures, as supports and foundations, and operate in a participant prepared by superior natures.

#### P R O P O S I T I O N   LXXII.

All things which in their participants have the relation of a subject, proceed from more perfect and universal causes.

**F**OR the causes of many effects are more powerful and universal, and nearer to *the one*, than the causes of a few effects. But things producing the subjects of others, are the causes of many effects, because they likewise produce aptitudes, before forms are present. And hence these are more universal and perfect in the order of causes.

#### C O R O L L A R Y:

From hence it is evident why matter which derives its subsistence from *the one*, is essentially destitute of form. And why body is effectually destitute of soul, although it participates of being. For matter which is the subject of all things proceeds from the cause of all \*: but body which is the subject of animation, subsists from that which is more universal than soul, because it participates of being in a certain respect.

\* By matter proceeding from the cause of all, nothing more is meant than that it depends entirely on the first cause for its shadowy and unreal subsistence: for as the emanations of causes are extended in proportion to their eminence; hence the processions of the one extend beyond those of every other cause, and even leave faint traces of their illuminations in the dark receptacle of matter.

P R O P O-

## P R O P O S I T I O N . LXXIII.

**E**very whole is at the same time a certain being, and participates of being: but every being is not a whole.

**F**OR either *being* and *whole* is the same; or the one is prior, and the other posterior.

But a part also, so far as a part, is a certain being (for that which is a whole consists from partial beings) yet is not an essential whole. And hence *being* and *whole* is not the same: for on this hypothesis, a part would be a *non-entity*. But if the part is a *non-entity*, neither can the whole be *being*. For every whole is a whole of parts, either considered as existing prior to the parts \*, or as inherent in the parts. But the part being a *non-entity*, it is impossible that the whole can exist. But if the whole is prior to being, every being will be immediately a whole; and so a part, will not be a part, which is impossible. For if a whole is a whole, and is a whole of parts; the part also existing as a part, will be a part of the whole. It remains therefore that every *whole* is *being*, but not every *being* a *whole*.

## C O R O L L A R Y.

For hence it is evident that the first being is above totality, since being is present to a multitude of things: for it affords essence to parts, so far as parts. But totality is present to fewer natures. For the cause of a multitude of effects is more excellent: but that of a few is subordinate, as is demonstrated.

## P R O P O S I T I O N . LXXIV.

**E**very form is a certain whole..

**F**OR it is composed from a multitude of things, each of which replenishes the form.

But every whole is not a form. For that which is particular, and an indivisible, is indeed a whole, so far as it is an indivisible; but is not a form: For every whole consists from parts. But form or species, is that which may be divided into many particular forms. Hence *whole*, and *form* differ from each other: and the former is inherent in more natures than the latter. That which is *whole*, therefore, is above the *forms* of beings.

## C O R O L L A R Y.

From hence it is evident that *whole* possesses a middle order between *being*, and *forms*; and hence it follows that *being* is prior to *forms*, and that *forms*, are *beings*;

\* This follows from Prop. 69. For every whole prior to parts, may be said to be a *whole* of parts, because every whole formed from parts, participates of that totality which is prior to parts.

and.

and yet every being is not a form. From hence likewise in effects, privations are after a certain manner beings, yet they are not forms \*. But on account of the unifying power of being, they likewise receive a certain debile representation of being.

## PROPOSITION LXXV.

**E**very thing which is called a cause properly, is exempt from its effect.

**F**OR if it subsisted in its effect, it must either replenish its effect, or be indigent of it in a certain respect, in order to its being; and thus be more imperfect than its effect. But that which abides in its effect, is more an assistant-cause than cause itself: because it is either a part of that which is made, or an instrument of that which makes. For the part subsisting in that which is made, is more imperfect than the whole: and the instrument which supplies the measures of fabrication to the efficient, is not able to separate itself. Every thing, therefore, which is properly a cause, if it is more perfect, than that which proceeds from it, affords likewise a measure to generation, and is exempt from instruments, and elements, and from every thing which is simply called an assisting cause.

## PROPOSITION LXXVI.

**E**very thing which is produced from an immoveable cause, possesses an immutable hyparxis: but every thing which emanates from a moveable cause, possesses a mutable hyparxis.

**F**OR if every thing which fabricates is entirely immovable, it produces that which is second from itself, not by motion, but by being. But if this be the case, that which emanates from it, concurs with its essence. And if this be the case, as long as it exists, it will produce. But it always is, and therefore will always produce that which is posterior to itself. Hence too that which is posterior always emanates from thence, and always is; conjoining its own *progressive ever*, with the *ever according to energy*, of an immoveable cause. If, therefore, the cause is moved, that also which is produced by it will be essentially mutable. For that which derives its essence through motion, when

\* Thus matter possesses a certain obscure image of being, but does not preserve the most debile impression of form. For as the gradations of being are more extended than those of form, and as matter is the last of things; hence matter may be said to retain the footprint of being in its dark receptacle, whilst the processions of form are reflected like echoes from its rebounding seat.

the

the motion is changed, changes its being. For if that which is produced from motion abides immutable, it will be more excellent than its producing cause: but this is impossible. It will not, therefore, be immutable; and will consequently be changed, and moved according to essence, imitating the motion by which it is produced.

## PROPOSITION LXXVII.

Every being in *capacity*, emanates from that which is *energy*; and that which is in capacity proceeds into energy. But that which is in a certain respect in capacity, so far as it is in capacity, emanates from that which is in a certain respect in energy. And that which is all things in capacity proceeds from that which is all things in energy.

FOR that which is in capacity is not adapted to produce itself into energy, because it is imperfect. For if that which is imperfect should become the cause of perfection to itself, and this in energy, the cause will be more imperfect than that which it produces. Hence that which is in capacity, so far as in capacity, will not be to itself, the cause of subsisting in energy. For on this hypothesis, so far as it is imperfect, it will be to itself the cause of the perfect: since every thing in capacity, so far as in capacity, is imperfect. But every thing which is in energy, so far as in energy, is perfect. If then that which is in capacity becomes in energy, it will inherit perfection from something else. And this again, will be either in capacity; (but then again, the imperfect will be generative of the perfect), or it will be in energy. And either something else, or this which is in capacity, will rise into energy. But if something else in energy operates, acting according to its propriety, it will produce into energy, that which is in capacity, in another. Nor will this again be in energy; unless it rises into energy from capacity. It remains, therefore, that from that which is in energy, that which is in capacity must be changed into energy.

## PROPOSITION LXXVIII.

All power or capacity is either perfect, or imperfect.

FOR that which produces energy is a perfect power: for it makes other things perfect through its energies. But that which is perfective of others, is greater, because it is more self-perfect. But the power which is indigent of something pre-existing

ing in energy, according to which it is something in capacity, is imperfect : for it is indigent of the perfect abiding in another, that it may become perfect through its participation. And hence such a power is essentially imperfect. Hence too the power which subsists in energy is perfect, because prolific of energy. But the power which subsists in capacity is imperfect ; deriving its perfection from power in energy.

### PROPOSITION LXXIX.

**E**very thing which is generated, is generated from a twofold power.

**F**OR it is requisite that it should be adapted to generation, and that it should possess an imperfect power. It is likewise requisite that the agent being in energy, such as that which is generated is in capacity, should previously assume a perfect power. For every energy proceeds from inherent power. For if the agent does not possess power, how can it energise, and operate in another? But if that which is generated, does not possess power according to aptitude, how can it be fabricated? For that which produces, produces every thing, in that which possesses a passive power; but not in every thing; nor in that which is not naturally passive to the energies of the producing cause.

### PROPOSITION LXXX.

**E**very body is naturally adapted to passivity : but every thing incorporeal is naturally adapted to fabricate. And the former is essentially inefficacious, but the latter is impassive. Yet that which is incorporeal becomes passive through its communion with body : just in the same manner, as bodies are enabled to fabricate, through the participation of incorporeals.

**F**OR body, as body is divisible alone, and through this becomes passive; being every way partible, and this every way infinitum. But that which is incorporeal, because it is simple, is impassive. For neither can that which is impartible be divided, nor can that which is incomposite be altered. Hence nothing will be fabricative, or this must be affirmed of an incorporeal : since body so far as body, does not operate, because it is alone exposed to division and passivity ; while on the contrary every agent possesses an active power. Hence it will not fabricate so far as body, but according to a power of operating, which it contains. But body is essentially inefficacious, and impotent : and hence when it fabricates, it fabricates by a participation of power. But incorporeals  
likewise

likewise partake of passions, when they abide in bodies; because in this case they are divided in conjunction with bodies, and enjoy their partible nature, though at the same time, they are impartible according to their proper essence.

## PROPOSITION LXXXI.

Every thing which is participated in a separable manner, is present by a certain inseparable power, which it inserts in its participant.

**F**OR if it is separable from its participant, and does not abide in it, as that which possesses a subsistence in itself, a certain medium is requisite, which may connect the one with the other, and which is more similar to that which is participated, and to that which participates. For if this medium is separable, how can it be participated by the participant? Since neither the participant contains the medium nor any thing proceeding from this medium. A power, therefore, and illumination emanating from this medium, into its participant, conjoins both. One thing, therefore, is that through which participation subsists, but the second is that which is participated, and the third is the participant.

## PROPOSITION LXXXII.

Every thing incorporeal, because converted to itself, when it is participated by others, is participated in a separable manner.

**F**OR if in an inseparable manner, its energy will not be separable from its participant, any more than its essence. But if this be the case, it will not be converted to itself. For if it is converted it will be separable and different from its participant. If, therefore, it is capable of being converted to itself, it is participated in a separable manner, when it is participated by others.

## PROPOSITION LXXXIII.

Every thing endued with a self-gnostic power, is entirely converted to itself.

**F**OR that which is converted to itself, in energy, manifestly knows itself: for that which knows is one and the same with that which is known; and the knowledge

of itself reverts to itself, as that which is known. And as this knowledge belongs to that which knows, it is a certain energy: but it is an energy of itself to itself, because it possesses a power of knowing itself. But that this also subsists in essence if in energy has been demonstrated. For every thing which is converted to itself in energy, contains also an essence verging to itself.

## PROPOSITION LXXXIV.

**E**very thing which always *is*, possesses an infinite power.

**F**OR if its essence is never-failing, its power also, according to which it is what it *is*, and is able to be, must be infinite. For if the power according to which it subsists was finite, it would some time or other fail. But if it should fail, the being also of that which possesses this power must fail; nor would it be any longer an eternal being. It is requisite, therefore, that the power belonging to, and containing that which always *is*, should be essentially infinite.

## PROPOSITION LXXXV.

**E**very thing which is always *in generation*. (*αεὶ γενόμενον*), possesses an infinite power of *being generated* (*τὸ γίγενθαι*).

**F**OR if it is always in the act of becoming to be, it contains a never-failing power of generation. For if its power was finite, it would cease in a infinite time. But if its power of being generated ceases, that also which is in generation will cease; viz. that which is in generation according to this power will cease; nor will it any longer be always in generation \*. But it is always in generation, according to the hypothesis: and consequently it possesses an infinite power of being generated.

\* The proposition ends here in the Greek, though very erroneously: and its conclusion forms the beginning of the next proposition. It is strange that Portus should not have detected, this egregious mistake, which the alteration of a single letter, would have enabled him to rectify. Thus if after *αλλὰ μονὸν διεπέσεις γίγενεται*, *αντίον ἀφεὶς ίχει τῷ το γίγενθαι δυναμον*, which is the conclusion of the proposition; *μονὸν διεπέσεις γίγενεται*, &c. is retained, instead of *τῷ το γίγενθαι δυναμον*, and is made the beginning of the next proposition, the whole will be correct and plain. Such mistakes, are dreadful instances of the danger attending the understanding from the study of words alone.

## PROPOSITION LXXXVI.

*Every true being* is infinite, not according to multitude, nor according to magnitude, but according to power alone.

FOR every infinite is either in multitude, or in magnitude, or in power. But true being is indeed infinite, as possessing an inextinguishable life, a never-failing essence, and an undiminished energy. Nor is it infinite on account of magnitude; since that which is true being, is without magnitude, and self-subsistent. For every self-subsistent being, is impartible, and simple. Nor is it infinite, on account of multitude, for it is most uniform, on account of its vicinity, and alliance to the one. But the one is infinite according to power: and hence through this, true being will be impartible and infinite. And indeed by how much the more it is one and imparible, by so much the more will it be infinite. For power when distributed into parts, becomes debile, and finite. And powers entirely partible, are entirely finite. For such as are last, and most distant from the one, are after a certain manner finite: but first powers are on account of their imparibility infinite. For partition dissipates and dissolves the power of every thing. But imparibility, from its binding and collective nature, contains in itself, that which is never-failing and undiminished. But infinite according to magnitude and multitude, is entirely a privation, and desertion of imparibility. For that which is finite proximately approaches to that which is imparible; while that which is infinite is most distant from an imparible nature, because it on all sides departs from the one. Hence that which is infinite according to power, does not belong to the infinite, either of multitude, or magnitude: since infinite power is co-existent with imparibility. But infinite, either in multitude or magnitude is most distant from an imparible nature. If, therefore, true being was infinite, either in magnitude, or multitude, it would not be endowed with infinite power. But it is endowed with infinite power, and is, therefore, not infinite, either according to magnitude, or multitude.

## PROPOSITION LXXXVII.

*Every thing eternal, is being; but not every being is eternal.*

FOR in generated natures the participation of being, is after a certain manner inherent, so far as they are not that, which is in no respect being. But if that which is in generation, is not that which is in no respect being (*οὐδαποτε ἔνεστι*) it is being, *in a certain respect* (*ἐν τῷ μηδενὶ ἔνεστι*). But that which is eternal, or eternity itself, is *in no respect* inherent

in generated natures ; and is particularly separated from things which do not participate of eternity, according to the whole of time. But every thing eternal always is ; for it participates of eternity itself, which confers perpetual being, on the natures by which it is participated. *Being*, therefore, is participated by more natures than *eternity* : and hence *being* is above *eternity*. For things which participate eternity, participate also of being : but not all that participate of being, participate also of eternity.

### PROPOSITION LXXXVIII.

**E**very true being, is either prior to eternity, or abides in eternity, or participates of eternity.

**B**UT that it is above, or prior to eternity, is demonstrated in the preceding proposition. And it likewise abides in eternity : for eternity possesses perpetuity together with being. And this is also true of that which participates of eternity : for every thing eternal, is called eternal from its participation of perpetuity and being. For this according to participation possesses both perpetuity and being. But *eternity* possesses perpetuity the first of all ; but *being*, through participation \*. And being itself, is the first being.

### PROPOSITION LXXXIX.

**E**very primary being ( $\pi\varrho\tau\omega\varsigma \circ\nu$ ) consists of bound, and infinite.

**F**OR if it is endued with infinite power, it is evident that it is infinite, and through this subsists from infinite. But if it is impartible and uniform, through this it participates of bound. For that which participates of the one, is bounded. But that which is impartible, is at the same time endued with infinite power. Every true being, therefore, consists from bound and infinite.

\* To a reader not skilled in the Platonic philosophy, it will doubtless appear strange, that *being* should be prior to *eternity*, and yet each participate of one another. This apparent paradox may be easily solved, by considering that the mode of participation is different in each. For *being* participates of eternity, as establishing, illuminating, replenishing, and deifying eternity : but *eternity* participates of being, as depending upon, established and deified by *being*. So that when a superior, participates of an inferior nature, the participation consists in the energy of the former on the latter : but when an inferior participates of a superior nature, the participation consists in its receiving the communications of the superior nature.

## PROPOSITION XC.

*First bound, and first infinity, have a self-subsistence prior to all things which consist from bound and infinite.*

FOR if beings which subsist from themselves, subsist prior to certain beings, because common to all, and primary causes, and this not to some in particular, but simply to all; it is requisite that there should be a first bound, and a first infinity prior to that which consists from both. For in that which is mixed, *bound* participates of *infinity*, and *infinity* of *bound*. But the first of each, is no other than that which it is. It is requisite, therefore, that that which is *primarily infinite* should not possess the form of *bound*, and that that which is *primarily bound*, should not possess the form of *infinity*. And hence these subsist primarily prior to that which is mixed.

## PROPOSITION XCI.

Every power is either bounded, or infinite. But every terminated power subsists from infinite power: and infinite power from *first infinity*.

FOR powers which have a partial existence, or subsist sometimes, are bounded; because they have fallen from the infinity of perpetual being. But the powers of eternal beings, are infinite, because they never desert their own hyparxis.

## PROPOSITION XCII.

Every multitude of infinite powers, depends on one first infinity, which is not as a participated power, and which does not subsist in things endued with power, but is essential; not existing as the power of any participant, but as the cause of all beings.

FOR though the first being possesses power, yet it is not power itself: for it likewise possesses bound. But the first power is infinity: for infinite powers, are infinite, through the participation of infinity. Infinity itself, therefore, will be before all powers; through which being also possesses infinite power, and all things participate of infinity. For *that which is first*, is not infinity: for that is the measure of all things, because it is the good, and the one. Nor is *being* infinity: for this is infinite, but not infinity (or infinite itself.) Hence between that which is first, and being itself, infinity subsists, as the cause of all things endued with infinite power, and of all the infinity in beings.

P. R. O. P. Q.

## PROPOSITION XCIII.

**E**very infinite subsisting in beings, is neither infinite with respect to superior natures ; nor is it infinite to itself.

**F**OR that by which every thing is infinite, by this also it is without circumscription. But every thing in natures superior to beings, is bounded in itself, and in all things prior to itself. It remains, therefore, that the infinite belonging to inferior natures, belongs to them alone, above which it is expanded in such a manner, that it is incomprehensible by them all. For however they may extend themselves towards this infinite, yet it possesses something entirely exempt from their nature. And though all things enter into this infinite, yet it possesses something occult, and incomprehensible by secondary natures. And again, though it expands its powers, yet it contains something on account of its union, invincible, convolved, and surpassing their involutions. Likewise containing and bounding itself, it will not be infinite to itself ; and much less will it be infinite with respect to superior natures, because it possesses a portion of the infinity which they contain. For the powers of more universal natures are more infinite \*, because they are more universal, and are placed nearer to the first infinity.

## PROPOSITION XCIV.

**E**very eternity is a certain infinity, but every infinity is not an eternity.

**F**OR many infinites possess *the infinite*, not on account of their perpetuity ; as is evident in the infinity according to multitude, and according to magnitude, and in the infinity of matter ; and whatever else may be infinite, either because it cannot be passed over, or on account of the indefinite nature of its essence. But that eternity is an infinity, is evident : for that which never fails is infinite : and this because it possesses a never-failing subsistence. Infinity, therefore, is prior to eternity. For that which constitutes a greater multitude, and is more universal, is a more causal nature. *First infinity*, therefore, is above eternity, and *infinity itself*, is prior to eternity.

\* The reader must not be surprised to find that among infinites, some are more infinite than others. For as among beings some are truer than others, and possess more of real being in proportion as they approach nearer to being itself ; at the same time that they are all in a certain respect beings : so infinites possess more of infinity, as they approach nearer to the infinite itself. Thus for instance eternity possesses infinity more truly than time, though time also is infinite ; because the infinity of eternity, is a stable indivisible life, but the infinity of time consists in an unceasing progression, or as it were an unwearied pursuit of infinity, which it can alone obtain in an extended and perishable manner. And this difference among infinites extends even to matter itself, which is the most degraded and abject of all infinites, because it is infinite only in the most dormant capacity.

PROPO-

## PROPOSITION XCV.

Every power which possesses more of the nature of the one than of multitude, is more infinite than the power which is multiplied.

FOR if the first infinity is the nearest to the one, hence of the powers which are more allied to the one, that which is less distant from the one, is more infinite. For when multiplied it loses its uniform nature; in which when it abides, it will possess a superiority among other natures, being contained by imparibility. For among partible natures, collected powers are multiplied, but such as are divided, are debilitated.

## PROPOSITION XCVI.

The power of every finite body, when infinite is incorporeal.

FOR if this power is corporeal, since in this case it would be an infinite body, infinite would reside in that which is finite. But if this power is a finite body, it will not be an infinite power, on account of body, but on account of something else. For if through body it is finite, but through power infinite; it will not be power, on account of body. Hence the infinite power, which resides in a finite body is incorporeal.

## PROPOSITION XCVII.

Every principal cause in every series, communicates its property to all that series: and the series is that by remission, or subjection, which this cause is after a primary manner.

FOR if it is the leader of the whole series, and all kindred natures, are co-ordinated to this cause; it is manifest, that it confers on all one idea, through which they are allotted an order under the same series. For either all things participate of similitude with this principal cause, without a cause; or the sameness which is in all, proceeds from this principal cause. But that this should be the case, without a cause, is impossible: for that which is without a cause is fortuitous. But among things in which there is order, and a connection with each other, and which always abide in the same state,

state, chance can never take place. From this principal cause, therefore, the whole series receives the property of its subsistence. And if from this cause, it is evident that it receives this property with remission, and a descent accommodated to secondary natures. For either this property subsists in a similar manner, in that which is principal, and in things secondary, and the former presides, but the latter are allotted a subsistence posterior to the principal; or it subsists in a dissimilar manner. And if this is the case, it is manifest that identity in multitude proceeds from one, and not the contrary; and that the property which primarily pre-exists in one, is secondarily in multitude, and is exempt from the series.

### P R O P O S I T I O N XCVIII.

**E**very separate cause, is at once, every where, and no where.

**F**OR by the communication of its power, it is every where. For this is a cause, which replenishes things naturally adapted to participate of its nature, and is the leader of all secondary natures, and is present to all the prolific progressions of illustrations. But, on account of an essence unmixed with things in place, and through its excellent purity, it is no where. For if it is separate from effects, it is placed above all things. In like manner it resides in none of the natures subordinate to itself. For if it was alone every where, it would not indeed be hindered from being a cause, and it would be in all participants: but it would not be in a separate manner prior to all. But if it was no where, without being every where, it would not indeed be restrained from being prior to all things, and it would not be any one of subordinate natures, but it would not be in all things; as causes are naturally in things caused, through their abundant communications. On account of its being a cause, therefore, it is in all things which are able to participate its nature: and from its being separate in itself, it is prior to all the natures which it replenishes; and is at once every where and no where. And indeed it is not according to a part every where, and according to a part no where: for thus it would suffer a divulsion and separation from itself: since one part of itself would be every where in all things, and another part would be no where, and prior to all things. But it is total, every where and no where, after the same manner. For things which are able to participate of this cause, abide in the *whole*, and find the *whole* present with their nature; while this whole is exempt from its participants. For its participant does not establish this whole in itself, but participates of it as much as it is able to receive. Nor in communicating does it contract itself, through the participations of a multitude of things: for it is separate. Nor do the participants participate in a defective manner: for that which communicates is every where.

P R O P O-

## PROPOSITION XCIX.

Every imparticipable, so far as it is an impar-

ticipable, does not sub-

sist from another cause, but is the principle and cause of all parti-

cipated natures. And in consequence of this every principle in

every series is without generation.

**F**OR if it is impar-

ticipable, it is allotted a principality in its own proper series, and

does not proceed from others: for it would not be the first, if it received that pro-

perty on account of which it is impar-

ticipable, from any other. But if it is worse than

others and proceeds from them, it is not allotted a progression, so far as it is impar-

ticipable, but so far as it is a participant. For in this case it participates of the natures

from which it proceeds, and the things which it participates, have not a primary sub-

sistence. But that which is impar-

ticipable has a primary subsistence. And hence so

far as it is impar-

ticipable it does not flow from a cause\*. For if it proceeded from a

cause, it would be a participant, and not an impar-

ticipable. But so far as it is an im-

participable, it is the cause of participants, and not that which participates of others.

## PROPOSITION C.

Every series of wholes is extended to an impar-

ticipable cause and

principle. But all impar-

ticipables depend on one first principle

of all.

**F**OR if every series suffers a certain sameness, there is something in every ruling na-

ture, which is the cause of identity. For as all beings proceed from *the one*, so like-

wise every series emanates from *one*. But all impar-

ticipable unities are reduced to the

*one* itself: because all of them are analogous to the *one*. So far, therefore, as they suf-

fer a certain sameness, through their analogy to the *one*, so far they are reduced to the

*one* itself. And so far as they all proceed from *the one*, none of them is a principle, but

they flow from *the one*, as from a principle. But so far as each of them is impar-

\* Thus for instance the first being, or *being itself*, does not flow from any being as its cause, because it is with-

out any participation of being; and though it proceeds from a superior cause, i. e. the *one itself*, yet it does not

proceed from this cause, on account of its being an impar-

ticipable, but on account of its being subordinate to *the one*, and consequently a participant of the *one*. And this is likewise true in every other impar-

ble, so far each of them is a principle. Because, therefore, they are principles of certain things, they depend on the principle of all: for that is the principle of all, of which all participate. But the first is alone participated by all things; while some things only participate of the rest. And hence *the one* is that which is simply the first. But others are firsts with relation to a certain order, but are not simply firsts.

## PROPOSITION CI.

The leader of all things participating of intellect, is an imparticipable intellect: of all things participating life, an imparticipable life: and of all things participating being, an imparticipable being. But of these, being is prior to life, and life is prior to intellect.

FOR since in every order of beings, imparticipables are prior to participants, it is requisite that there should be an intellect prior to intellects, life prior to things vital, and being prior to beings. But since that which is the cause of more effects precedes that which is the cause of a fewer; hence among these being will be the first: for it is present to all things, to which life and intellect is present. For every thing vital and intellectual participates of being; but the contrary is not a necessary consequence: since all beings are not endued with life, and intellect. But the second in order is life: for all things to which intellect is present, participate also of life; but the contrary is not true. For many things are endued with life, but are destitute of cognition. But the third is intellect. For every thing which is endued in any respect with cognition, both lives, and possesses being. If, therefore, being is the cause of more effects; but life of fewer; and intellect of still fewer: hence *being* is the first in order, *life* the second, and *intellect* the third.

## PROPOSITION CII.

All beings, in whatever manner they may possess being, consist from bound and infinite, through the first being. But all vital natures, are self-motive, through the first life. And all gnostic natures, participate of cognition, through the first intellect.

FOR if that which is in every series imparticipable, communicates its peculiar property, to all things under the same series; it is evident that being first communicates.

cates to all things *bound* together with *infinite*; since it is primarily mixed from these. And life imparts the motion resident in its nature: for life is the first progression and motion, from the stable subsistence of being. And lastly, intellect imparts cognition: for the summit of all cognition, is in intellect; and intellect is the first gnostic nature.

## PROPOSITION CIII.

All things are in all, but subsist peculiarly in each.

**F**OR in being there is both life and intellect; and in life, being and intellection; and in intellect, being and life. But in intellect all things subsist intellectually, in life vitally, and in being, according to true beings. For since every thing subsists, either, according to cause, or according to essence, or according to participation: and since in the first the rest subsist according to cause; and in the second, the first subsists through participation, and the third through cause: and in the third, natures prior to its own, subsist through participation; hence in *being*, *life* and *intellect* preside. But since every thing receives its characteristic, according to hyparxis, and not according to cause (for cause pertains to other things, or to effects); nor yet according to participation (for it receives externally that which it participates): hence in *being* there is *true*\* life and *true* intelligence, essential life and essential intellect. And in *life*, there is being according to participation; but intelligence according to cause. But both of these are vitally inherent in *life*: for its hyparxis is according to *life*. And in *intellect* there is both life and essence, through participation: but both these subsist intellectually. For the being and life of intellect is knowledge.

## PROPOSITION CIV.

Every thing which is primarily eternal, has both its essence, and energy eternal.

**F**OR if it is primarily allotted the perpetuity of eternity, it does not partly participate of this, and partly not; but it entirely participates of perpetuity. For either participating according to energy, it does not participate according to essence: but this is impossible, since in this case, energy would be more excellent than essence. Or parti-

\* οὐρανός, in the original, which means indeed literally, *truly*. But the philosophical reader will please to observe that as it is an adverb derived from the participle ὡραῖος, or *being*; its full meaning is *truly according to being*. So that by *true life*, we must understand *life according to the truth of being*; and so of the rest.

icipating according to essence, it does not participate according to energy. And thus that which is primarily eternal, will be the same with that, which primarily participates of time \*. And time will primarily measure the essence of some things, but eternity which is more excellent than all time, will be the measure of nothing: since that which is primarily eternal, will not be contained by eternity according to energy †. Every thing, therefore, primarily eternal, has both its essence and energy eternal.

## PROPOSITION CV.

Every thing immortal is eternal, but every thing eternal is not immortal.

FOR if that is immortal which always participates of life, and that which always participates of life, participates also of being, and that which is always vital is perpetual; hence every thing immortal is eternal. For that is *immortal* which does not receive death, and perpetually *lives*: but that is *eternal* which cannot receive non-being, and which always *is*. But if there are many beings more excellent and worse than life, but which are not susceptible of immortality, though they are perpetual beings; it follows that every thing eternal is not immortal. But that many perpetual beings; are not immortal is evident. For there are certain beings, destitute indeed of life; yet perpetual and incorruptible ‡: since as being is to life, so is that which is eternal to that which is immortal. For that life which cannot be taken away, is *immortality itself*. And being which cannot be destroyed, is *eternity itself*. But being is more comprehensive than life: and hence that which is eternal is more comprehensive than that which is immortal §.

\* For if that which is primarily eternal, is eternal according to essence, but not according to energy, it will be the same with the world, which is the first participant of time. For the world is essentially eternal, because though its parts are subject to change, yet considered as a whole, it is perpetually the same. But then it is not eternal in energy, but in capacity alone. For it subsists in a perpetual capacity of existence; and its flowing essence is perpetually composed from the past, present, and future circulations of time.

† In the original it is *καὶ τὸν ζεῖν*, but it should doubtless be read, according to our translation *καὶ τὸν ζητεῖν*.

‡ Thus for instance the qualities which subsist about bodies are incorporeal, and consequently eternal and incorruptible; but yet they are not immortal, because they are destitute of life.

§ In the Greek *ζωάντις* is erroneously printed, instead of *ζωάντες*, which the reader will evidently perceive must be the true reading. But though this is sufficiently obvious to those who understand the proposition; yet Portus, who seldom attends to the meaning, has, from not rectifying this mistake, given the most ridiculous translation of this concluding sentence, that can possibly be imagined. The original is: *τὸ δὲ ὃ τὸ τὸ ζεῖν μεγάλως προσθέτειν, καὶ τὸ ζωάντις ἀπε τὸ ζεῖν.* The translation of Portus: " Illud vero, quod est, vitam, et mortem magis complectitur, vel, ipsum vero ens, plura complectitur, et latius patet, quam vita et mors. Ergo ipsum eternum est illud, quod non recipit ipsum non esse, et quod semper est; i. e. " But that which is, comprehends in a greater degree, life and death; or being itself comprehends more, and is of greater extent than life and death. Therefore, eternity itself, is that which does not receive non-being, and which always is."

P. R. O. P. O.

## PROPOSITION CVI.

The medium of every thing entirely eternal, both according to essence, and according to energy, and of that which has its essence in time, is that which is in a certain respect eternal, but which in a certain respect is measured by time.

FOR that which has its essence comprehended by time is entirely temporal: for this in a most primary manner, is allotted a temporal energy. But that which according to all things is temporal, is perfectly dissimilar to that which is according to all things eternal. But all progressions subsist through similars. There is, therefore, some medium between these. Hence either that is the medium which is eternal in essence, but temporal in energy; or the contrary. But this is impossible: for energy on this latter hypothesis would be more excellent than essence. It remains, therefore, that the former hypothesis must be the medium.

## PROPOSITION CVII.

Every thing which is in a certain respect eternal, but in a certain respect temporal, is at the same time *being* and *generation*.

FOR every thing eternal, is *being*, and that which is measured by time is *generation*. And hence, if the same thing participates both of time and eternity, yet not according to the same, or after the same manner; this same thing will be, both *being* and *generation*, yet will not be both according to one of these alone\*.

## C O R O L L A R Y.

From hence it is evident that *generation*, since it has a temporal energy, depends on that which partly participates of *being*, and partly of *generation*; and which at once participates of eternity and time. But this is related to that which is eternal according to all things. But that which is eternal according to all things is related to eternity itself: and eternity itself is related to *being*, which is prior to eternity.

\* That is neither according to time alone, nor eternity alone: but from the participation of both.

## PROPOSITION CVIII.

Every thing which is particular in each order, is capable of participating in a twofold manner, that unity which is placed in a proximate superior disposition; either by its own proper totality, or through that which it contains, of a partial nature, and which is allied to something particular, according to an analogy to the whole series.

FOR if all things are converted through similitude, the particular nature which subsists in an inferior order, is dissimilar to that which in a superior order is monadic, and total; and is as that which is particular to that which is universal, and as different orders are to each other. But this particular nature is similar to a *whole* of the same series, on account of a communion of peculiarity; and to that superior proximately co-ordinated property, through an analogous subsistence. It is, therefore, evident that through these mediums a conversion from one to the other is effected, as through similars, to that which is similar \*. For *particular*, is similar to that which is *particular*, but that which is of the same series is peculiar. And the universal, or whole which is placed above the series, is dissimilar according to each of these.

## PROPOSITION CIX.

Every particular intellect participates of that unity, which is above intellect, and is the first; both through that which is *universal*, and through a *particular* unity, co-ordinated to its nature. And every particular soul, participates of universal intellect; both through *universal* soul, and a *particular* intellect. And every particular nature of body, participates of universal soul, both through *universal* nature, and a *particular* soul.

FOR every thing particular (by the preceding proposition) participates of that unity placed in an order above it; either by its own proper universality; or by that particular nature which it contains, and which is co-ordinated to something particular.

\* Instead of *καὶ τὸ μονικὸν διάθεμα*, read *καὶ τὸ μονικὸν τὸ εἶμα*. And read with astonishment the version of Portus; who not perceiving the error of the text, has made nonsense of the passage, as usual.

PROPO-

## PROPOSITION CX.

Of all things placed in order, according to every series, such as are first, and are conjoined with their unity, may participate of those which are proximately established in a superior order, through analogy. But such as are more imperfect, and remote from their proper principle, are not naturally adapted to participate of their superiors.

FOR because some things are allied to their superiors being allotted in their proper order, a more excellent and divine nature; but others are more distant, because they are allotted a secondary and ministerial, but not a primary and principal progression in every series: hence some things are naturally conjoined with those of a superior order; but others are not conjoined with a superior order. For all things are not of equal dignity, though they belong to the same distribution. For there is not one reason, of all things: but all things proceed from one, and return to one, from their own proper unity. And hence they are not allotted the same power. But some are able to receive continually the participations of their superiors. But others of a dissimilar nature, are deprived of such a power, through their far distant progressions from their principles.

## PROPOSITION CXI.

Of every intellectual series, some are divine intellects receiving the participations of the gods; but others are intellects alone. And of every animastic series (i. e. a series composed of souls) some are intellectual souls, depending on their proper intellects; but others are souls alone. And of every corporeal nature, some have supernally-presiding souls, but others are natures alone, destitute of the presence of souls.

FOR in every series the whole genus is not naturally adapted to depend on that which is prior to itself; but this belongs to that more perfect nature which the genus contains, and which is sufficient to coalesce with superior natures. Hence every intellect is not connected with a god; but this belongs to the highest and most uniform intellects,

lects, (i. e. to intellects which participate most of unity). For these are allied to the divine unities. Nor do all souls participate of participable intellect; but this belongs to such as are most intellectual. Nor do all corporeal natures enjoy the presence of soul, and of that soul which is participated; but such only as are more perfect and rational. And the mode of demonstration is the same in all.

## P R O P O S I T I O N C X I I .

**F**irst natures in every order, possess the form of things prior to themselves.

**F**OR the highest genera in every order, are conjoined with their superiors by similitude, and through a continuation of the progression of universals. Hence such as superior natures are primarily, such is the form which things first in every order are allotted, and which is allied to the nature of superiors. And through the peculiarity of their subsistence, they appear such as natures prior to themselves.

## P R O P O S I T I O N C X I I I .

**E**very divine number is unical (*εναίος*), i. e. possesses the form of unity.

**F**OR if a divine number has the one itself, as its preceding cause, in the same manner as an intellectual number has intellect, and an animastic number (*ψυχικός*, or number possessing the form of soul) soul as its preceding cause, and if multitude is everywhere analogous to its cause; it is evident that a divine number also, is uniform. Since *the one itself*, is *the deity*: and this because *the good itself* is the same with *the one*. For the *good itself*, and *the deity* are the same: since that above which there is nothing, and which all things desire, is *the deity*. And that from which all things proceed, and to which all things tend, is *the good*. If, therefore, there is a multitude of gods, it is an uniform multitude. But that there is a multitude is evident: since every principal cause is the leader of a proper multitude \*; and to this multitude it is similar and allied.

## P R O P O-

\* To such as understand these Elements, this argument for the existence of a multitude of gods, is perfectly demonstrative and clear. Indeed as every production of nature possesses the power of generating its similar, it is much more necessary that the first cause of all should generate a multitude the most similar to himself, that can possibly be conceived. For every being produces that which is similar prior to the dissimilar; as indeed a contrary mode of proceeding would be absurd and impossible. The immediate or first productions therefore of the first god, must be a multitude of gods: or otherwise his first progeny would not be perfectly similar to himself. Nor does this doctrine, in any respect derogate from the dignity of the supreme god, as the ignorant suppose;

## PROPOSITION CXIV.

**E**very god is a self-perfect unity: and every self-perfect unity is a god.

**F**OR if there is a two-fold number of unities, as we have previously demonstrated, and some of these are self-perfect, but others illuminations emanating from the self-perfect unities; and if a divine number is allied, and of a similar nature to the good; hence the gods are self-perfect unities. And on the contrary every self-perfect unity is a god. For as unity is most excellently allied to the one itself, and that which is self-perfect to the good, and through both the one, and the good, participates of a divine property; so likewise that which is self-perfect is a god.

## COROLLARY.

But if a god was a unity, yet not a self-perfect unity; or a self-perfect hypostasis, yet not a unity, he would be placed in another order, on account of a mutation of his property.

## PROPOSITION CXV.

**E**very god is super-essential, supervital, and super-intellectual.

**F**OR if every god is a self-perfect unity, but each of these (viz. essence, life, and intellect) is not a unity, but united; it is evident that every god, is above essence, pose; but on the contrary tends to exalt his majesty, and evince the ineffable beneficence and perfection of his nature. For though it establishes a multitude of gods, yet it teaches that they are dependant on the first, who is perfectly incomprehensible, and without participation. So that it leads us to consider the subordinate deities, as so many lesser luminaries shining before the presence of the sun of good, and encircling with awful grandeur his ineffable radiance, and occult retreats. And that this doctrine fully displays his superlative goodness, is sufficiently manifest; since by a contrary assertion we must ascribe imperfection to the fountain of excellence, and leave deity impotent and barren. Indeed this opinion is so natural and reasonable, that, excepting the Jews, it was embraced by every nation of the ancient world. Nor ought we to wonder that the Jews were an exception to the universal consent of mankind, in this important particular. For with respect to the origin of this despicable, though chosen people, it was scarcely known to the Heathens; "the greater part of whom," as Dr. Burnet justly observes (De Origin. Rer. cap. 7.) "supposed them to be natives of Egypt, sprung from the same root, or considered them as a vile, and inconsiderable people." And as to their learning we may remark with the same author, "that they never excelled in philosophical, or mathematical knowledge; and never gave the world a famous example of the strength of human wit; from whence arose that bitter reproach of Apollonius, That the Jews were the most trifling of all the barbarians, and that they were the only people who had never found out any thing useful for life." The example of the Jews, therefore, ought to be so far from deciding against Polytheism, that when their character is impartially estimated, it will strengthen the evidence of its reasonableness and truth.

life, and intellect. For if these differ from each other, but all are in all, every one of these being *all things* will not be *one* alone. Besides, if the first deity is super-essential, but every god, so far as a god is of the first series \*; hence every god will be super-essential. But that the first deity is super-essential is evident: for *essence* is not the same with *unity*; nor is *to be*, and *to be united* one and the same. But if *essence*, is not the same with *unity*, that which is first will either be both of these, and so will not be *one* alone, but something besides *one*, and will participate of *the one*, without being *the one itself*; or it will be either of these. But if indeed it is essence, it will be indigent of the one. But it is impossible that *the good*, and *the first* should be indigent. It will, therefore, be *the one* alone; and will consequently be super-essential. But if every thing subsisting in a primary manner, confers the property of its primary subsistence on the whole series; hence every divine number is super-essential. For every principal cause produces similars prior to dissimilars. If, therefore, the first god is super-essential, all the gods, will be super-essential: for by this means they will be perfectly similar. But if they were essences they would be produced from the first essence, as the unities of essences.

## PROPOSITION CXVI.

**Every deity, except *the one*, is participable.**

**F**OR that *the one* is imparticipable, is evident; since if he participated any thing, and thus became dependant on some other nature, he would no longer be the cause of all things; both of such as are prior to beings, and of beings themselves. But that other

\* Though the first cause or *the one itself*, confers on every thing a proper symbol of his ineffable nature; yet this occult unity, or impression is not divine in things subject to generation and decay, but in true essences alone, in the number of which rational souls must be ranked. Such of these, however, as are of a partial nature, and on this account are not the immediate progeny of *the first one*, do not contain a unity which can be called a god; because they are connected with motion, and are in a certain respect composite essences. But where there is a most true essence, as in separate intellects, and celestial souls, the unity of each is a god. And indeed on account of these unities, which are as it were expressive characters of the first unity, the essences of the gods contain all things, and extend their providential care to every part of the universe, with unbounded beneficence, and immaculate power. But these divine unities are perpetually united to the first one, like rays to light, and lines to a centre. They likewise subsist in the most perfect union with each other. For since union in other natures is effected through the power of unity, these divine unities must be much more closely united, through their subsisting much nearer to the first and most perfect one. Every divine unity, therefore, though it is neither essence, nor obnoxious to essential multitude, yet abides in essence; or is rather the summit, and as it were blossom of essence. And as every thing is established in its proper species through form, and as we derive our being through soul; so every god is a deity, from the secret unity which he contains. Hence these divine unities subsist in the intelligible world, and in the essences of the gods, like so many splendid lamps in diaphanous spheres, mingling their rays with an ineffable union, energy, and consent. And situated in most admirable order, in the vestibule of the good, they occultly signify divine silence, and solitary beauty; and perpicuously announce to posterior natures the awful sanctuary of their incomprehensible cause.

unitica.

unities are participants we shall now demonstrate as follows: For if there is another im-participable unity after the first, in what does it differ from *the one*? For it either subsists in the same manner as that: but how in this case, is the one second, and the other first? Or it does not subsist in the same manner. And so *that* will be the one itself, but *this* will be both *one*, and *non-one*. But this *non-one*, if it is no hypostasis (or subsistence) will be one alone. But if it is some other hypostasis besides the one, *the one* will be participated by *non-one*: and that will be a self-perfect one, by which it is conjoined. Hence this again, will be the deity. But that which subsists as *non-one*, will subsist in the participation of the one. Every unity, therefore, which subsists after *the one* is participable, and every god is participable.

## PROPOSITION CXVII.

**E**very god is the measure of beings.

**F**OR if every god possesses the form of one (*inā.ō.*), he defines and measures all the multitudes of beings. For since all multitudes, are naturally indefinite, they are bounded by *the one*. But that which is one, measuring and bounding whatever it supervenes, is willing to lead into bound, by its terminating power, whatever is indefinite. For that which is one becomes uniform through participation: but that which is indefinite recedes from the one, through its interminable and infinite nature. And by how much the less it is uniform, by so much the more is it indefinite and immense. And hence every multitude of beings, is measured by the divine unities.

## PROPOSITION CXVIII.

**E**very thing which is in the gods, according to their idioms (or properties), pre-exists in their natures. And the property of the gods, is uniform and super-essential. And hence all things are contained in the gods, uniformly, and super-essentially.

**F**OR if every thing subsists in a three-fold manner, either through cause, or through hyparxis, or through participation, but the first of all numbers is the divine number; hence nothing will be inherent in the gods according to participation. But all things will reside in them, either through hyparxis, or through cause. But likewise, whatever the gods, as the authors of all things previously receive, they previously re-

ceive in a manner convenient, and apposite to their union. For every thing which presides over seconday natures according to cause, naturally contains the cause of inferiors. All things, therefore, are in the gods uniformly, and super-essentially.

## PROPOSITION CXIX.

**E**very god subsists according to a super-essential goodness, and is good neither through habit, nor through essence.

**F**OR both habit and essence are allotted an order seconday and remote from the gods: but these have a super-essential subsistence. For if the first one, is also the good, and so far as the one is the good itself, and so far as the good the one itself; hence every series of gods, is both uniform, and beneficent (*ἀγαθούς*), on account of one property alone, and not through more than one. But every god, so far as a unity is also a goodness, and so far as a goodness is also a unity; and on account of progression from the first is also beneficent and uniform. For the first cause of all, is both the one itself, and the good; and consequently all the gods are unities and goodnesses. As therefore *the one* of the gods is super-essential, so likewise *the good*, which they contain, is super-essential, and is nothing else than *one*. For every god is not first of all something different from good, and afterwards *good*; but is good alone. Nor is first of all something besides *one*, and afterwards *one*; but is *one* alone.

## CONCERNING PROVIDENCE.

## PROPOSITION CXX.

**E**very god contains in his hyparxis a providence of the universe; and primary providence resides in the gods.

**F**OR all things posterior to the gods, provide through the communion of the gods: but providence is connate with the gods. For if to communicate good to things provided for, is the peculiar employment of a providential property, but all the gods are goodnesses; hence they will either communicate themselves to nothing, and so there will be no good in seconday natures (for whence can that which subsists by participation emanate, but from natures which are primarily endued with properties); or if they communicate, they will communicate good, and through this communication provide for the universe. Providence, therefore, primarily subsists in the gods. For where can

can an energy prior to intellect abide, but in super-essential natures? And hence providence as the name indicates, is an energy prior to intellect. The gods, therefore, on account of their being, and because they are goodnesses, provide for all things \*, and fill all things with that goodness which is prior to intellect.

## PROPOSITION CXXI.

**E**very thing divine has for its hyparxis goodness itself, and possesses an uniform power; and a knowledge occult, and incomprehensible by all secondary natures.

**F**OR if it provides for the universe, it contains a power comprehensive of the things for which it provides; and by this invincible and indiscernible power, it fills all things with itself, and subjects every thing to its own nature. For every principal and ruling cause, rules through its abundance of power, and contains according to nature. There is, therefore, a primary power in the gods, which does not govern some things, and not others, but it equally assumes in itself in a primary manner, the powers of all beings: and this is neither an essential power, nor much more unessential, but it is conate to the hyparxis of the gods, and is super-essential. But likewise the boundaries of all cognitions, pre-exist uniformly in the gods. For all other cognitions subsist, on ac-

\* We may farther infer the necessity of providence in the gods, from considering that as they are the producing causes of all things, so all things abide, and are radically established in their natures. For where can any thing subsist, which is not contained in their unknown and all-pervading comprehensions? But if this be the case, since all things are in reality the offspring of the gods, they must continually be the objects of their providential exertions. For as goodness is the characteristic of those divine natures, it is impossible that they should abandon their progeny, or cease to impart their beneficent, unenvying, and all-powerful communications. Nor must we think that these providential exertions are laborious to the gods; since, as Proclus well observes, (Theol. Plat. p. 41.) that which is according to nature, is not laborious to any thing. "For neither (says he) is it laborious for fire to give warmth, nor for snow to refrigerate, nor for bodies themselves to energize according to their peculiar powers. Nor prior to bodies, is it laborious to natures themselves to nourish, or generate, or increase: for these are the operations of natures. Nor again, prior to these, is it laborious to souls to exert their peculiar energies: for many of their energies are attended with delight, many are the result of their essence; and many motions are produced by their presence alone." Hence if the communication of good naturally belongs to the gods, providence also is natural to their divinites, which they exert in a tranquil, unpolluted, and incorporeal manner.

Should it be enquired in what manner providence operates, the following beautiful passage from Proclus on the Parmenides, as cited by Ficinus, in his commentary on that dialogue, will give us abundant satisfaction: "Let us conceive (says he) a ship agitated by the winds and waves, and let us suppose, that the imagination of some one is so powerful, that while he imagines the sea, the sea immediately flows; that while he imagines the ship, the ship is constructed; and that the winds and waves, arise agreeable to his imagination, and as the consequences of its vehement energy. Now it is evident, that such a one would not be compelled, in surveying these particulars, to employ a confused and distracted vision; but both his knowledge and operation, would equally subsist in a uniform manner. And such is the simplicity of divine intelligence, with respect to the intuition and fabrication of inferior concerns."

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count of divine cognition, which is abstracted from the universality of things. And this cognition is neither intellectual, nor much less does it belong to the cognitions posterior to intellect: but according to its divine property, it is constituted above intellect. If then this knowledge is divine, it is an occult and uniform cognition. But if the power is divine, it is uncircumscribed by all things, and, in a similar manner, comprehensive of all things. But if the goodness is divine, it gives bound to the hyparxis of the gods. For if all things are contained in the gods, and among these knowledge, power, and goodness; and if their hyparxis is characterized with that which is best, the subsistence also of the gods will take place according to the best: and this is no other than goodness.

### P R O P O S I T I O N CXXII.

**E**very thing divine both provides for secondary natures, and is separated from the things for which it provides; providence neither remitting its unmixed and uniform excellence, nor a separate union obscuring providence.

**F**OR the gods abiding in their uniform nature, and hyparxis, fill all things with their power. And every thing which is able to participate, enjoys the goods, which it is capable of receiving according to the measure of its proper subsistence; the gods in the mean time, through being itself, or rather through a nature prior to being, pouring their illuminations on every thing which exists. For since they are no other than goodness, they abundantly confer good upon all things, through being itself; not making a distribution according to a reasoning energy, but because these receive according to their dignity, and those confer according to their hyparxis. Hence, in their providential operations, they receive no impediment from the natures for which they provide: for they benefit all things through their very being itself. But every thing which operates essentially, operates without habitude or respect: for respect, is an addition to being itself: and is on this account contrary to nature. Nor again, because they are separate, do they take away their providential care; for thus they would remove (which it is unlawful to say) their peculiar hyparxis whose property is goodness. For the communication of good extends to every thing capable of its participation: and that which is greatest, is not that which is endued with a form of good, but that which is beneficent. This beneficent nature, therefore, either no being will possess, or the gods will possess it prior to beings. For to goods subsisting through communication, it is impossible that a greater good should be present, but a less good only, to such goods as are first.

PROPO-

## PROPOSITION CXXIII.

Every thing divine, on account of its super-essential union, is ineffable and unknown to all secondary natures ; but it is comprehensible, and knowable by its participants. And hence *that which is first*, is alone entirely unknown, because it is imparticipable.

FOR all knowledge subsisting through reason, belongs to *beings*, and in *beings* possesses the apprehension of its truth : for it is conversant with *conceptions*, and subsists in intellections. But the gods are above all beings. Hence that which is divine, is neither to be apprehended by opinion, nor by a rational energy, nor by intellection. For every being, is either sensible, and on that account the object of opinion, or true being, and on that account intelligible. Or it subsists between these, and is at the same time *being*, and *generable*, and is on this account the subject of a rational energy. If, therefore, the gods are super-essential\*, and prior to beings, there can neither be any opinion of their natures, nor science, nor cogitation, nor intellection. But they are known by dependent natures in a manner correspondent to their properties : and this by a necessary consequence. For the diversities of participants are divided together with the properties of the things participated. Nor does every thing participate every thing : for neither is there a composition of things perfectly dissimilar, nor does any thing participate fortuitously of another : but a kindred nature is conjoined with every thing kindred, and derives its progression from that to which it is allied.

## PROPOSITION CXXIV.

Every god knows partible natures, in an imitable manner, things subsisting in time without time, things not necessary, necessarily, things mutable, immutably ; and universally, all things, in a manner more excellent than the order of the things known.

FOR if every thing which is present with the gods is present, according to their *characteristic* ; it is evident that the knowledge of the gods will not subsist according to

\* The reader must remember that the gods are alone super-essential, through their *unities*, which are the characteristics of their natures : for as irrationality is the essential signature of a brute, and rationality of a man ; so a divine unity, is the invariable characteristic of a god.

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the nature of things inferior, but according to the singular excellence which the gods contain. Their knowledge, therefore, of multiplied, and passive natures, will be uniform and without passion. Likewise if that which is the object of cognition, is partible, divine knowledge will be impartible. If the subjects are mutable, the gnostic energy of the gods will be immutable: if contingent, divine knowledge will be necessary; and if indefinite, definite. For that which is divine does not receive knowledge into itself from subordinate natures, that so cognition may correspond to the object of knowledge; but inferiors receive their indefinite subsistence, about the terminated nature of the gods, are changed about their immutability, receive with passivity that which is impassive, and temporally, that which subsists without time. For it is possible that subordinate, may be surpassed by more excellent natures: but it is not lawful for the gods to receive in themselves any thing from natures inferior to their own.

## PROPOSITION CXXV.

Every god proceeds through all secondary natures, in the order from which he begins to indicate himself. Always indeed multiplying and dividing his communications, yet preserving the characteristic of his own proper subsistence.

FOR since progressions are produced through remission, things first, every where multiply into the decrements of secondary natures. But proceeding according to a similitude to their producing causes, they receive the same ordination; so that the whole, is in a certain respect the same and different, and that which proceeds, with that which abides. For on account of its remission, it appears different, but on account of its coherence with the whole, it does not depart from identity. But such as *that* is among first natures, such is the subsistence of *this* among things secondary, and such is its preservation of the indissoluble communion of the series. Every god, therefore, appears in a manner adapted to the orders in which he exhibits his presence: but he proceeds from thence even to the last of things, through the generative power of primary natures. But he always multiplies the progression from one into multitude: but preserves identity, in the progression, on account of the similitude of the progressions to the governing and first operative cause of every series.

PRO

## PROPOSITION CXXVI.

Of every deity, he is the more universal, who is nearer to the one: but he is more particular, who is more distant.

FOR he who is the cause of more effects, is nearer to the cause of all, but he who produces fewer effects, is more distant. And he who is the author of many is more universal, but he who produces fewer effects, is more particular. And each of these is a unity. But the one is greater, and the other less according to power. And the more particular goods are generated from such as are more universal, without the latter receiving any division, (for they are unities) or alteration (for they are immovable) or being multiplied according to habitude (for they are unmixed). But they generate from themselves through an abundance of power, secondary progressions, diminished from such as are first.

## PROPOSITION CXXVII.

Every thing divine is primarily, and especially simple, and through this is most sufficient.

FOR that it is simple, is evident from its unity: for the whole is eminently uniform. But a nature of this kind, is most eminently simple. But that it is likewise most sufficient, may be learnt by any one who considers, that a composite, is indigent, though not of things external to its nature, yet of those from which it is composed. But that which is most simple, and uniform, and one, is the same with *the good*, in which good establishing itself, it becomes most sufficient. But every thing divine, is of this kind. And hence it is neither indigent of externals, because it is goodness itself, nor of things requisite to composition, because it is uniform.

## PROPOSITION CXXVIII.

Every god, who is participated by natures nearer to his own, is immediately participated: but when he is participated by far distant natures, this is effected through *mediums* more or less numerous.

FOR the former, since they are by their alliance uniform, are on this account enabled to participate the divine unities. But such as through their diminution, and

extension into multitude become far distant, are indigent of other things more united, that they may participate such as are no longer united, but are essential unities. For multitude united, subsists between essential unity, and divided multitude. And thus united multitude is able to coalesce with unity, through *union*; but is at the same time allied to divided multitude, through *the manifest appearance of multitude*.

## PROPOSITION CXXIX.

Every divine body is divine, through a divine soul. But every soul, is divine, through a divine intellect. And every intellect is divine through the participation of a divine unity. And unity indeed, is a god from itself (*αὐτόθεν θεος*) but intellect, is most divine: and soul is divine, but body deiform, or endued with a divine form.

FOR if every number of gods is above intellect, but participations are effected through kindred and similar natures, an imitable essence will first of all participate the super-essential unities. But in the second place things conjoined with generation. And in the third place, generation itself. And each particular will participate through its proximate superior; the peculiarity of the gods proceeding even to the extremities of things in participants, through mediums allied to their natures. For unity confers on the first intellect, its own illustrious power among divine concerns, and causes this intellect to be like itself, according to an uniform multitude. But through intellect it is present also to soul, adapting and inflaming its conjunction with intellect, when this intellect is participable. And by the resounding echo \* as it were of soul, it imparts its idiom or peculiarity to body, if it is a body participating in any respect of soul. And thus body becomes not only animated, and intellectual, but also divine. For it receives life and motion from soul, but indissoluble permanency from intellect, and divine union from participated unity. For each of these communicates its subsistence to subsequent natures.

\* By this *εχήσης*, or *resounding echo of soul*, we must understand that vital quality, by which the soul is united to the body; and which is nothing more than the extreme image and shadow of the soul. The necessity of such a connecting quality, will easily appear, from considering that an *incorporeal* nature, like that of soul, cannot be connected with *body*, without a vital medium. In consequence of this we may consider with Plotinus, (Ennead. 3v. lib. 4.) the animated body as resembling illuminated and heated air; and the pains and pleasures of the body, will be convervant with this shadow of the soul.

## PROPOSITION CXXX.

In every divine order, things first, are more exempt from the natures proximately placed under them, than these last are from things subsequent: and secondary natures are more dependant on their proximate superiors, than following natures are dependant on these.

FOR by how much the more uniform, and universal any thing is, by so much the more is it allotted an excellency greater than subsequent natures: and by how much the more it is diminished according to power, by so much the more is its alliance increased with things posterior to its nature. And sublimer natures are indeed more united with their more principal causes: but inferiors are less united. For it is the property of a greater power to be more exempt from its inferiors, and to be more united with more excellent natures. And on the contrary to recede more, and to be passive together with these, implies a diminution of power. And this indeed happens to secondary natures in every order, but not to such as are first.

## PROPOSITION CXXXI.

Every god begins his own proper energy from himself.

FOR he first exhibits in himself the peculiarity of his presence in secondary natures, because he likewise communicates himself to others, according to his own exuberant plenitude. For neither is defect, nor plenitude alone, peculiar to the gods: since every thing deficient is imperfect; and it is impossible that the imperfect, can cause any thing to be perfect. But that which is full, is alone sufficient, and is not yet prepared for communication. It is requisite, therefore, that the nature which fills, and extends its beneficence to others, should be above measure full. Hence, if that which is divine, fills all things through itself, with the goods which it contains in itself, every thing divine is beyond measure full. And if this be the case, it will primarily possess in itself, the property which it confers on others. For thus it will extend to others the communications of over-flowing goodness.

## PROPOSITION CXXXII.

All the orders of the gods, are bound in union, by a medium.

**F**OR all the progressions of beings, are effected through similars; and much more is it necessary that the orders of the gods, should possess an indissoluble continuity, because they subsist uniformly, and are terminated according to one principal cause of their subsistence. Their revolutions, therefore, take place in an united manner, and through that similitude alone which is found among beings, of things secondary to such as are first: and this because the subsistence of the gods, much more consists in union, than the subsistence of beings. All the divine genera, therefore, are bound together by proper mediums; so that first natures do not immediately proceed into progressions entirely different, but through genera common to each, and of which they are the immediate causes. For these genera combine the extremes into one union, being subjected to some, through an alliance of nature, but proximately separated from others: and they preserve the well-ordered progeny of divine causes.

## PROPOSITION CXXXIII.

Every god is a beneficent unity, or a goodness unific (*ὑπονοής*); and each possesses this hyparxis, so far as a god. But the first god is simply *good*, and simply *one*. And every god posterior to the first, is a certain *goodness*, and a certain *unity*.

**F**OR a divine property or idiom distinguishes the unities and goodnesses of the gods, so that every god confers goodness on all things, according to a certain characteristic of goodness; such as that of *perfecting*, or *containing*, or *defending*. For each of these is a certain good, but not every good. But that which is first primarily establishes a uniform cause. And this is no other than the good, constitutive as it were of all goodness. For all the hyparxes of the gods are not together equal to *the one*; so great is the super-eminence of *the first*, with respect to the multitude of the gods.

## PROPOSITION CXXXIV.

Every divine intellect, understands as intellect, but provides as a god.

**F**OR to possess a knowledge of beings, and a perfection in intellectual conceptions, is the property of intellect. But it is the province of a god to exercise a providential care,

care, and to fill all things with good. But this communication, and replenishing, subsists through a union of the things replenished, with natures prior to their own. And intellect imitating this, becomes in its intellections the same with intelligibles. So far, therefore, as a divine intellect provides, it is a god; because providence is an energy prior to intellect. Hence, as a god, it communicates itself to all things; but as intellect, it is not present to all things. For a divine unity, extends beyond the progressions of an intellectual property. And this will be evident by considering, that natures void of intelligence, desire to provide, and to participate something of good: and this because all things do not desire intellect, even among such as are capable of its participation; but all things desire good, and hasten to acquire its possession.

## PROPOSITION CXXXV.

Every divine unity is immediately participated by some being; and every thing which is deified, is extended to one divine unity; and the number of the participating genera of beings is the same as that of the participated unities.

FOR neither two, or more unities are participated by one being\*. For how is it possible, that when the properties, which the unities contain, are changed, that which is connate to each, can remain without alteration; since conjunction subsists through similitude? Nor is one unity participated in a divisible manner by many beings: for many beings are unjoined with unity; both considered as beings, with respect to that which is prior to beings, and as multitude to unity. But it is requisite that the participant, should be partly similar to that which it participates, and partly different and dissimilar. Since, therefore, that which participates, is something belonging to beings, but unity is super-essential; and the two are on this account dissimilars; it is requisite that that which participates should be one, that by this means it may become similar to the participated one; though the latter is one, because it is a unity, but the former is one, because it is passive to the communications of one, and is united through its participation.

\* This must be understood of immediate participation; and on this hypothesis, it is certainly true, that neither two or more unities are participated by one being. For since there is an order among the unities, and some are more universal than others, two cannot be immediately participated by one being, without an alteration taking place in the subsistence of each. This is evident from the cxxxii. Proposition, in which it is proved that all the orders of the gods are bound in union by a medium: and consequently since all the unities are connected by proper mediums, it is impossible that any two can be immediately participated.

P R O P O

## PROPOSITION CXXXVI.

**E**very god having a more universal subsistence, and being placed nearer to *the first*, is participated by a more universal genus of beings. But every god who is more particular and remote, is participated by a more particular genus of beings. And as being is to being, so is unity to divine unity.

**F**OR if the number of unities, is the same with that of beings, and on the other hand one unity is participated by one being; it is evident that the order of beings proceeds according to the order of unities, assimilated to an order prior to that of beings. And more universal beings coalesce with more universal unities, but more particular beings, with more particular unities. For if this be not the case, dissimilars will again be joined with dissimilars, and distribution will not subsist according to dignity of nature: but both these cases are impossible; since the one itself, and a proper measure, through the divine unities illuminates and supervenes all other natures. Much more, therefore, will there be an order of participation in the divine unities; similars depending on similars according to the power which they contain.

## PROPOSITION CXXXVII.

**E**very unity, together with *the one* constitutes being participating of its nature.

**F**OR *the one*, as it is hypostatic, (*ὑποστάτης*) or constitutive of all things, so likewise it is the cause of participated unities, and of beings depending on unities. But the unity belonging to every being, produces the property, which shines forth to view in that particular being. And *the one*, indeed, is the cause of *simple being*; but *unity* is the cause of alliance, because it is connate to *the one*. Hence unity, is that which of itself defines the being, which is its participant, and *essentially* exhibits in it a *super-essential* characteristic. For universally, from that which is primary, that which is secondary obtains its subsistence. If, therefore, there is any *super-essential* property of deity, it must belong to being, which participates it essentially.

PROPO-

## PROPOSITION CXXXVIII.

Of all things which participate of a divine property, and which are deified, the first and highest is *being itself*.

FOR if being is above intellect and life, as we have demonstrated, and is the most abundant cause after the one; hence *being* will be the highest after *the one*. For it is more uniform than intellect and life, and is on this account more venerable. But there is no other prior to this, except *the one*: for what besides *the one*, can be prior to uniform multitude? But *being itself* is uniform multitude, because it consists from *bound* and *infinite*. And universally, super-essential being \* is prior to essence. For in the illuminations which are imparted to secondary natures, *the one* alone extends beyond *being*. But *being* subsists immediately after *the one*. For that which is *being* in capacity, and is not as yet *being* in energy, is nevertheless according to its nature one. And the *being*, which subsists after *the one*, is *being* in energy. Among the principles of being, therefore, *non-being* † subsists immediately above *being*, as something more excellent, and no other than *the one itself*.

## PROPOSITION CXXXIX.

All the participants of the divine unities originate from *being*, and end in a corporeal nature.

FOR *being* is the first of participants, but body the last: for we say that there are divine bodies. For the highest of all genera are attributed to the gods, whether they are bodies, souls, or intellects; as in every order, things analogous to the gods, contain, and preserve secondary natures, and every number is a whole, containing all things in itself according to that whole which is contained in a part, and possessing before all things a divine characteristic. The divine genus, therefore, subsists both *corporeally*, and *animistically* (or according to the nature of soul *ψυχής*) and *intellectually*: and it is evident that all these are divine through participation. For that which is primarily divine subsists in the unities. The participants, therefore, of the divine unities, originate from *being*, but end in a corporeal nature.

\* For as *being itself*, is no other than the highest order of the gods and the most uniform multitude, and as the characteristic of every god is a divine unity; hence the characteristic of *being itself*, will be the unity proceeding from bound. But as all the divine unities are super-essential; hence *being itself* according to its characteristic will be super-essential.

† For as matter is deservedly called *non-being*, because it is *worse* than all things; in like manner this appellation is proper to the first cause, as he is *better* than all things.

R R. O. P. Q.

## PROPOSITION CXL.

All the powers of divine natures, having a supernal origin, and proceeding through proper mediums, extend to the extremity of things, and to places situated about the earth.

FOR nothing intercepts these powers, and restrains their universal presence; because they are in no respect indigent of places and intervals, on account of their invincible excellence in all things, and a presence every where pure and unmixed. Nor is that which is adapted to the participation of these powers, prohibited from participation: but as soon as any thing is prepared for their communications, they are immediately present, neither then approaching, nor being prior to this absent, but always possessing themselves in the same uniform manner. If, therefore, any terrene nature is adapted to the participation of these divine powers, they are present to this; and fill all things with themselves. And indeed they are more present to superior natures, but they are present to such as are middle according to the order of these, and to last natures, in an ultimate respect. They supernally, therefore, extend themselves to the extremities of things: and on this account last natures contain the images of such as are first; and *all things sympathize with all*\*. For seconday pre-exist in first natures; and first natures manifestly appear in such as are second. For every thing subsists in a three-fold manner; either through *cause*, or through *hyparxis*, or through *participation*.

\* He who understands this divine sentence, *that all things sympathize with all*, will see that the *magic* cultivated by the ancient philosophers, is founded in a theory no less sublime than rational and true. Such a one will contemplate the universe as one great animal, all whose parts are in union and concert with each other; so that nothing is foreign and detached; nothing strictly speaking void of sympathy and life. For though various parts of the world, when considered as separated from the whole, are destitute of *peculiar* life; yet they possess some degree of animation however inconsiderable, when considered with relation to the universe. Life indeed may be compared to a perpetual and universal sound; and the soul of the world resembles a lyre, or some other musical instrument, from which we may suppose this sound to be emitted. But from the unbounded diffusion as it were of the mundane soul, every thing participates of this harmonical sound, in greater or less perfection, according to the dignity of its nature. So that while life every where resounds, the most abject of beings may be said to retain a faint echo, of the melody produced from the mundane lyre. It was doubtless from profoundly considering this sympathy between the mundane soul, and the parts of the world, that the ancient philosophers were enabled to procure the presence of divinity, and perform effects, beyond the comprehension of the *vulgar*. And that this was the opinion of Plotinus, the following passage evinces: "It appears to me that the ancient wise men, who wished to procure the presence of the gods, by fabricating statues and performing sacred rites, directed their intellectual eye to the nature of the universe, and perceived that the nature of soul was every where easy to be attracted, when a proper subject was at hand, easily passive to its influence. But every thing adapted to imitation, is readily passive; and is like a mirror able to seize a certain form, and reflect it to the view." Ennead iv. lib. 3.

PROPO.

## PROPOSITION CXLI.

Every providence of the gods, is partly exempt from the natures for which it provides, and is partly co-ordinated with them.

FOR one kind of providence is entirely extended above the things which are illuminated, according to its subsistence, and the characteristic of its order. But another kind provides for subjects of the same co-ordination with itself; these subjects themselves imitating the providential energy of the gods, who are separated from the concerns for which they provide; and desiring to fill secondary natures with the goods, they are capable of receiving.

## PROPOSITION CXLII.

The gods are present to all things after the same manner, but all things are not after the same manner present to the gods. For every thing participates of their presence according to its order and capacity. And this is accomplished by some things uniformly, and by others variously; by some things eternally, and by others according to time; and by some things incorporeally, and by others in a corporeal manner.

FOR it is necessary that the different participation of these, should either proceed from the participant, or from the thing participated. But every thing divine always possesses the same order: and with respect to all things, is without restraint, and without mixture. It remains, therefore, that mutation must subsist through the participant; and that in these that which is not perpetually the same must abide; and that these are differently present to the gods. Hence the gods are present to all things, in the same uniform manner, though all things are not equally present to them. But particulars are present according to their ability; and they enjoy the divinities, agreeable to the manner in which they are present to their illuminations. For the participation of these is according to the measure of their presence.

## PROPOSITION CXLIII.

All inferior natures fail before the presence of the gods, though a participant among these may be adapted to participation. Indeed every thing foreign departs from divine light, but all things are at once illuminated by the gods.

FOR divine natures always possess a more comprehensive capacity, and are more powerful than their progressions. But the inaptitude of the participants, is the cause of the privation of divine light: for it obscures divine light by its debility\*. But this obscured light, appears to receive another domination, not according to its own power, but according to the impotency of the participant, which seems to fail and die away, before the illumination of a divine form.

## PROPOSITION CXLIV.

All beings, and all the distributions of beings, extend as far in their progressions as the orders of the gods.

FOR the gods produce beings together with themselves, nor is any thing able to subsist, and to receive measure, and order beyond the influence of the gods. For all things are perfected, dispos'd, and measured through the power of the gods. Hence the gods have a subsistence prior to the last genera of beings; who also dispose these, and impart to them life, formation, and perfection; who convert them to *the good*, and who are in like manner prior to middle, and primary natures. And all things are bound, and stably rooted in the gods, and through this derive the continuance, and preservation of their being. But when any thing apostatizes, or recedes from the gods, and becomes on this account solitary and destitute, it entirely departs into non-entity, and perishes: because perfectly deprived of those natures, by which it was contained.

\* For as this divine light operates according to the debilitated nature of the subject into which it is received (and there is no other way in which it can operate), it necessarily appears to receive a dominion foreign from its own. Hence it appears both obscure and impotent, though in reality neither: for these are the imperfections of the subject which it actuates and illuminates.

PROPO-

## PROPOSITION CXLV.

The characteristic of every divine order, pervades through all secondary natures, and imparts itself to all the subordinate genera of beings.

**F**OR if the distributions of beings, extend as far as the orders of the gods, there must be in every genus of beings, a supernally-illuminated property of the divine powers. For every thing receives from its proximate cause, that characteristic, or property, by which it is allotted its peculiar subsistence. I say, for example, if any deity possesses a cathartic, or purgative power, there will also be a purgation in souls and in animals, in plants and in stones. And in the same manner with respect to a defensive, converting, perfective, and vivifying power. And a stone indeed participates of a purgative virtue, but in a corporeal manner only. But a plant participates it more clearly according to life. An animal possesses this form, according to the motion of appetite : but a rational soul, in a rational manner ; and intellect, intellectually. But the gods possess this super-essentially, and uniformly. And the whole series is endued with this power, from one divine cause : and there is the same mode of reasoning in the rest. For all things depend on the gods. And different natures are illuminated by different gods ; the divine series, descending even to the extremity of things. And some things are connected with the gods immediately, but others through more or fewer mediums ; while all things in the mean time are full of gods. And whatever any being naturally possesses it possesses from the gods.

## PROPOSITION CXLVI.

The extremities of all the divine progressions, are assimilated to their principles ; preserving a circle without beginning and end, through a conversion to their principles.

**F**OR if every progression returns to the principle from which it proceeds, much more must total orders, proceeding from their summit, be converted to it again. But the conversion of the extreme to its principle, forms one whole, finite, and verging to itself ; and exhibiting through its inclination uniformity in multitude.

## PROPOSITION CXLVII.

The summits of all the divine orders, are assimilated to the extremes of their superiors.

**F**OR if it is requisite that there should be a coherence, and continuity in a divine progression, and that every order should be connected by proper mediums; it is necessary that the summits of secondary orders, should be conjoined with the extremes of such as are first. But conjunction subsists through similitude: and hence there will be a similitude of the principles of an inferior order, to the extremes of one superior.

## PROPOSITION CXLVIII.

Every divine order is united to itself in a triple respect; by the summit which it contains; and by its middle, and end.

**F**OR its *summit* possessing a most united power, transmits this power into a total union, and unites every thing supernally flowing into itself. But its *middle* extending to each extreme, connects every thing about itself: transfusing the gifts of primary natures, but extending the powers of such as are last; and inserting in all things a communion and connection with each other. For by this means one co-ordination is produced from replenishing and replenished natures, mutually verging to the middle, as to a certain centre. But the *end* returning again to the beginning, and reducing to this the progressive powers, affords similitude and a mutual inclination to the whole order. And thus the whole order is one, through the unifying power of its primary parts; through the coherence subsisting in its middle; and through the conversion of the extreme, to the principle of the progressions.

## PROPOSITION CXLIX.

Every multitude of divine unities, is bounded according to number.

**F**OR if it is proximate to the one, it is not infinite; since that which is infinite is not connate to the one, but foreign from its nature. For if multitude essentially recedes from *the one*, it is evident that infinite multitude is perfectly destitute of the one: and hence it is likewise impotent and inefficacious. The multitude of the gods, therefore, is not infinite: and consequently, it is uniform and bounded,

bounded, and more bounded than any other multitude, because it is more allied to the one. If, therefore, multitude was the principle of things, it would be requisite that every thing nearer to, should be a greater multitude than that which is more distant from the principle: for that which is nearer is more similar. But since that which is first is *the one itself*, the multitude conjoined with it must be less multitude than that which is more remote from *the one*. But infinite is not a less multitude, but multitude in the most eminent degree.

### P R O P O S I T I O N C L.

Every thing progressive in the divine orders, is not naturally adapted to receive all the powers of its producing cause. Nor do secondary natures entirely receive all the powers of natures prior to themselves: but these possess some powers abstracted from inferiors; and incomprehensible by things posterior to themselves.

**F**OR if there is a difference in the characteristics of the gods, those of the inferior must pre-exist in the superior gods: but the characteristics of the superior, as being more universal, do not reside in the inferior divinities. But the more excellent characteristics impart some powers to their productions, but eminently pre-occupy others in themselves. For it has been demonstrated that those are more universal, which are nearer to the one, but more particular, which are more distant. But if the more universal possess powers comprehensive of the more particular characteristics; hence those which possess a secondary, and more particular order, will not contain the power of such as are more universal. Hence in the superior, there is something incomprehensible, and uncircumscribed by the inferior properties. For every thing divine is truly infinite; nor does it exhibit itself to itself; nor to things of a much prior superiority to itself: but to all such as are posterior to its nature. But infinity resides in these last, according to capacity. And infinite is incomprehensible by those to whom it is infinite. Hence inferiors do not participate of all the powers, which more excellent natures pre-occupy in themselves. For the latter are incomprehensible by the former. Hence things secondary, from their more particular subsistence, will neither possess the whole of superior natures, nor will they contain the properties which they possess, in the same manner, as their superiors; on account of that infinity through which superior excel subordinate natures.

P R O P O-

## PROPOSITION CLI.

Every thing paternal\* in the gods has a primary subsistence, and pre-exists in the order of *the good*, according to all the divine distributions.

FOR that which is paternal, produces the hyparxes of secondary natures, and universal powers, and essences, according to one ineffable excellence. And on this account it is denominated *paternal*, indicating the uniform and beneficent power of *the one*, and the hypostatical, or procreative cause of secondary natures. And in every order of the gods, that which is paternal, obtains the principality, producing and adorning all things from itself; because it is established analogous to *the good*. And with respect to these divine fathers, some are more universal, but others more particular; just as the orders of the gods differ in the proportion of cause, according to more universal, and more particular. As many therefore as are the universal progressions of the gods, so many also, are the differences of fathers. For if in every order there is something analogous to the good, it is requisite that *the paternal* should reside in all, and that each order should proceed from a paternal union.

## PROPOSITION CLII.

Every thing generative in the gods proceeds according to the infinity of a divine power, multiplying itself, penetrating through all things; and eminently demonstrating a never failing energy, in the progressions of secondary natures.

FOR what else but the infinite power of the gods, through which all divine natures are filled with prolific good, can multiply progressions, and produce them into offspring from their occult comprehension in causes? For that which is universally full,

\* According to the Chaldaic theology, which is the same with the Platonic, the intelligible order, subsisting immediately after *the one*, consists of three triads; the summit of each being called *father*, the middle, *power*, and the third *intellect*. This is profoundly and beautifully explained by Proclus in Theol. Plat. lib. 3. to which I refer the reader desirous of copious information, on this sublime and interesting subject. I only add that Plato in the Timaeus, ascribes these appellations to the demiurgus of the universe: and this sufficiently proves that words of this kind are not the invention of the latter Platonists, according to the ignorant assertion of Hyde, in the preface to his book, "On the Religion of the Persians;" a work of an alluring title, but of insignificant execution.

produces

produces other things from itself, through its overflowing power. Hence a dominion of power, is the characteristic of generative deity : and this absolute dominion multiplies the powers of generated natures, causes them to be prolific, and excites them to the generation and production of others. For if every thing imparts its primary characteristic to others, every thing prolific must insert in natures posterior to itself, a prolific progression, and form a figurative representation of that infinity, which is the first progeny of the universe ; from which every generative power proceeds, and which eminently scatters as from a fountain, the perennial progressions of divine natures.

## PROPOSITION CLIII.

**E**very thing perfect in the gods, is the cause of divine perfection.

**F**OR as with respect to hypostases, or subsistences, some belong to beings, and others are super-essential ; so likewise of perfections, some belong to the gods themselves according to hyparxis, but others to secondary beings posterior to the gods. And the former indeed are self-perfect, and first-artificers, because in these good is contained in a primary manner : but the latter possess perfection through participation. On this account, therefore, the perfection of the gods is different from the perfection of things deified. But that which is primarily perfect in the gods, is not only the cause of perfection to things deified, but to the gods themselves. For if every thing perfect is converted to its domestic principle, the cause of every divine conversion, is the perfective genus of the gods.

## PROPOSITION CLIV.

**E**very thing in the gods endued with a protecting power, preserves every thing in its proper order ; uniformly separating secondary natures, and establishing them in such as are first.

**F**OR if the preservation of every order, preserves measure in an immutable manner, and contains all the protected natures, in their proper perfection, divine protection will insert in all things an eminence above their inferiors, and will permanently establish in itself every thing, without mixture. It will likewise be the cause of immaculate purity, to protected natures, and will establish them in their superiors. For every thing adhering to primary natures is perfect ; but at the same time it abides in itself, and is extended above inferior natures.

PRO-

## PROPOSITION CLV.

Every thing vivific in the divine genera, is a generative cause ; but every prolific order is not also vivific.

FOR a generative power is more universal than that which is vivific, and is nearer to the principle of all. For generation manifests a cause producing beings into multitude : but vivifying (*ζωοντια*) represents deity the supplier of universal life. If, therefore, the former multiplies the hypostases of beings, but the latter the progressions of life ; it will be as *being* is to *life*, so is the *generative* order to the *vivific* series. And hence the generative order will be more universal, and the cause of more effects, and on this account nearer to the principle of all.

## PROPOSITION CLVI.

Every cause of purity, is contained in the protecting order. But the protecting is not the same with the purifying genus.

FOR purity inserts an unmixed nature in every thing inferior to the gods, and an unpolluted power, in the providence of secondary natures. But protection likewise produces this, comprehending all things in itself, and firmly establishing them in their superiors. Hence the protecting is more universal than the purgative genus. For it is simply the property of protection, to preserve the order of every thing, both with respect to itself, and to things prior and posterior to its nature. But it is the property of purity to separate things more excellent from such as are more base \* : and the former of these are primarily contained in the gods. For it is requisite that there should be one antecedent cause of that which is contained in all things. And universally the uniform measures of every thing good, are first received from the gods ; and there is no good in secondary natures, which does not pre-exist in the gods according to cause. For what other origin, or cause, can this possess ? In the gods, therefore, purity is likewise a primary good, together with protection, and every thing of this kind.

\* In consequence of this the cause of protection must be superior to the cause of purgation, or purity. For since protection preserves things in their proper order, but purity separates things excellent from such as are base, and the latter is preparatory to the former; hence protection must be superior in the order of causes to purgation.

## PROPOSITION CLVII.

Every paternal cause supplies every thing with being, and constitutes the hyparxes of beings. But every demiurgic, or fabricative cause of forms, precedes composite natures, together with their order, and division according to number: and is of the same order with a paternal cause, in the more particular genera of things.

**F**OR each of these belongs to the order of bound, because both *hyparxis*, and *number*, and *form*, are all of them endued with the form of bound: and hence through this they are co-ordinate to one another. But that which is *a demiurgic cause*, deduces fabrication into multitude. And that which is *uniform*, supplies the progressions of beings. And the former indeed is the artificer of forms, but the latter produces essence. In whatever respect, therefore, *form* and *being*\* differ from each each other, in the same respect that which is *demiurgic* differs from that which is *paternal*†. But form itself, is a certain one. A paternal cause, therefore, is both more universal and causal, and is superior to the demiurgic genus; in the same manner as being itself is more universal than form.

## PROPOSITION CLVIII.

Every reductorial cause ( $\tauὸ\ \alphaὐαγωγὸν$ ) in the gods differs both from a cathartic or purifying cause, and from converte genera.

**F**OR that a reductorial cause, ought to be primarily resident in the gods, is evident; as in these all the causes of universal good pre-exist. But it subsists prior to a cathartic cause; because that liberates from baser, but a reductorial cause connects with more excellent natures. It has, however, an order more particular than the converte genus; because every thing converte, is either converted to itself, or to a more excel-

\* That *being* and *form* differ from each other is evident from the 73d and 74th proposition of these Elements, in which it is demonstrated that *being* is superior to *form*; because *being* is above that which is total, and that which is total is above *form*.

† He who understands this will see the propriety of the appellation *fabricator*, and *father*, given by Plato in the Timaeus to the artificer of the world; and why *fabricator* is placed before *father*; concerning which consult Proclus on Plato's Theology, lib. v. cap. 16.

## ELEMENTS OF THEOLOGY.

lent nature. But the operation of that which is reductorial, is characterized according to a conversion to that which is more excellent; because it leads that which is converted to something superior, and more divine.

### PROPOSITION CLIX.

**E**very order of the gods consists from the first principles, *bound* and *infinity*. But one order consists more from the cause of *bound*, and another from that of *infinity*.

**F**OR every order indeed proceeds from both, because the communications of primary, penetrate through all secondary causes. But in some orders *bound* predominates in the mixture, and in others *infinity*. And hence that in which *bound* prevails, becomes a genus possessing the form of *bound*; but that in which *infinity* has the dominion, becomes a genus endued with the form of *infinity*.

### CONCERNING INTELLECT.

#### PROPOSITION CLX.

**E**very divine intellect is uniform, and perfect; and is a primary intellect subsisting from itself, and producing other intellects.

**F**OR if it be a god, it is full of divine unities, and is uniform. But if this be the case it is also perfect, being full of divine goodness. And again, if this be the case, it is a primary intellect, as being united to the gods: for deified intellect is more excellent than every intellect. But since it is a primary intellect, it also confers subsistence on other intellects: for from first entities, all secondary beings obtain their hyparxis.

#### PROPOSITION CLXI.

**E**very true being depending on the gods, is a divine intelligible, and is imparticipable.

**F**OR since true being as we have demonstrated is that which first participates a divine unity, it also fills intellect, from itself. For intellect is being, as that which is

is replenished with being: and consequently true being is a divine intelligible. It is divine indeed, as that which is deified; but as that which is filled with intellect, which it also participates, it is intelligible. And intellect indeed is being, through the first being. But the first being is separated from intellect, because intellect is posterior to being. And imparticiples are prior to things participated. Hence being united with intellect, pre-exists by itself, and is imparticipable. For it is intelligible, not as co-ordinated with intellect, but as eminently perfecting intellect; because it communicates being to intellect, and fills it with essence substantial and real.

## PROPOSITION CLXII.

**E**very multitude of unities illustrating true being, is occult and intelligible. *Occult* indeed, as conjoined with the one; but *intelligible*, as participated by being.

**F**OR all the gods are denominated from their dependants, because the different hypostases of the gods may be known from these. For every thing divine is of itself ineffable and unknown, because connate to *the ineffable one*. But by the permutation of participants, it happens that the properties of the gods become known to subordinate natures. Indeed the unities which illustrate true being are intelligible; because true being is a divine intelligible, and is likewise imparticipable, as subsisting prior to intellect. For this would not depend on the first gods, unless they possessed a primary hypostasis, and a power perfective of other gods: since as participants are to each other, so likewise are the hyparxes of participated natures.

## PROPOSITION CLXIII.

**E**very multitude of unities participated by imparcieipable intellect, is intellectual.

**F**OR as intellect is to true being, so are these unities, to intelligible unities. So far, therefore, as they illuminate divine and imparcieipable intellect, they are intellectual: but they are not so intellectual, as subsisting in intellect, but as subsisting through cause prior to, and generating intellect.

## PROPOSITION CLXIV.

**E**very multitude of unities participated by imparticipable soul, is super-mundane.

**F**OR since imparticipable soul, is primarily super-mundane, the gods also participated by this soul, are super-mundane; possessing the same proportion to the intellectual, and intelligible gods, which soul has to intellect, and intellect to true being. As therefore every soul is extended to intellect, and intellect is converted to that which is intelligible; so likewise the super-mundane depend on the intellectual gods, in the same manner as these last, on such as are intelligible.

## PROPOSITION CLXV.

**E**very multitude of unities participated by any sensible body, is mundane.

**F**OR it supernally illuminates the parts of the world, through the mediums of intellect and soul. For neither is intellect present without soul to any mundane body; nor are deity, and soul immediately conjoined: for participations subsist through similars. And intellect according to the intelligible which it contains, and the summit of its nature, participates of unity. Unities, therefore, are mundane, so far as they fill the whole world, and deify apparent bodies. For each of these is divine, not through soul; (for soul is not the first god) nor through intellect; (for this is not the same with *the one*), but is animated and self-motive, through soul. But it always contains itself in the same manner, and is carried in the best order through intellect; being at the same time divine through a divine unity. And if it possesses a providential power, it is such through unity as the cause.

## PROPOSITION CLXVI.

**E**very intellect is either imparticipable, or participable. And if participable, it is either participated by super-mundane, or mundane souls.

**F**OR an imparticipable intellect possessing a primary hyparxis presides over every multitude of intellects. But of participated intellects, some are super-mundane, and illustrate

lustrate imparticipable soul; but others are mundane. For multitude emanating from an imparticipable, is not immediately mundane; since progressions subsist through similars. But that which is separated from the world, is more similar to an imparticipable, than that which is divided about it. And there is not only a super-mundane, but likewise a mundane multitude. Since there is likewise a mundane multitude of gods, and the world is at the same time animated and endued with intellect. And the participation of super-mundane gods by mundane souls, subsists through mundane intellects as the connecting mediums.

## PROPOSITION CLXVII.

**E**very intellect understands itself. But the first intellect understands itself alone \*. And in this, intellect and that which is intelligible is one in number. But all succeeding intellects, understand both themselves and prior intellects. † And the intelligible to this first intellect, is partly that which it is itself, and partly that from which it proceeds.

**F**OR every intellect either understands itself, or that which is above, or that which is posterior to itself. But if it understands that which is posterior to itself; since it is intellect it will be converted to a worse nature, and will not even know that to which it is converted, because the object of its intellection will not reside in its nature, but will be external. And thus it will only possess in itself a type, or figure, of this external object. For it knows that which it possesses, and that to which it is passive, but not that which it does not possess, and by which it is not affected. But if it understands that which is above itself, since this is accomplished by the knowledge of itself, it will both understand itself, and the nature superior to its own. But if it knows that alone, it will at the same time that it is intellect, be ignorant of itself. But by knowing that which is superior to itself, it knows also that it is a cause, and of what it is the cause: for if it is ignorant of these, it will also be ignorant of that superior

\* This must be understood, not as if the first intellect understood nothing but itself; but that it understands no other intellect besides itself. For the divine unities, and the first one, are the objects of its continual speculation: and in this exalted employment its life and felicity invariably and eternally consists.

† This last sentence of the proposition, is in the original: *καὶ τοῦτο εἰλ. τρώτο, τὸ μὲν, δίκι, τὸ δε, ἀφ' οὐ λέγεται,* And this Portus renders with his usual unskillfulness: *et est mente-perceptum. Hoc autem partim quidem, εἴτε illud, quod est, partim vero, εἴτε illud, a quo est.* That is, "and it is intelligible. But this is partly that which it is, but partly is that from which it is." The wrong pointing after *εἴτε*, which instead of a period should be a comma, and *τρώτο* being erroneously printed for *τρώη* were the causes of Portus' mistake: and this he would have rectified, had he understood the preposition.

nature.

nature. And hence by knowing that which is prior to itself, it will also know itself. If therefore any intellect is intelligible, this by knowing itself will understand an intelligible, and will be itself its own intelligible. But each of the intellects posterior to this, will at the same time understand that which is intelligible in itself, and that which is prior to itself. There is, therefore, in intellect, that which is intelligible, and in that which is intelligible intellect. But the one, is the same with that which is intelligible \*, and the other is the same with the intelligible in itself, but is not the same with the intelligible prior to itself. For the one is simply intelligible, and the other is an intelligible in an intelligent nature.

### PROPOSITION CLXVIII.

**E**very intellect knows in energy that which it understands. And it is not the property of one part of its nature to know, and of another to understand that which it knows.

**F**OR if it is intellect in energy, and knows itself as not different from the object of its intellection †; it will both know and perceive itself. But beholding that which is intelligent, and knowing that which beholds, it will know that it is intellect in energy. And knowing this, it will know that it understands, and will not alone know the object of its intellection. It will, therefore, at the same time both know that which is intelligible, and that it understands this: and by intellection it will be understood by itself.

### PROPOSITION CLXIX.

**E**very intellect possesses in eternity, its essence, power, and energy.

**F**OR if it understands itself, and intellect is the same with that which is intelligible; intellection also is the same with intellect, and intelligible. For since intelligence

\* Thus for instance intellect in *being itself*, which comprehends the highest order of intelligibles, is *not* *intelligible*, or an *intelligible intellect*; because it is the object of intelligence to all subordinate natures, and because its vision is transcendently simple and occult. But every intellect is indeed the same with the intelligible in its own nature, but is subordinate to *intelligible itself*.

† Intellect in energy, or in the act of understanding is the same with the object of its intellection. For the object of its perception, must be resident in its essence, or it would perceive externally like sense; and thus would not behold the thing itself, but only its image. But if that which is intelligible is seated in the essence of intellect, it will in no respect differ from intellect: for it will be essential to its nature, and will consequently be intellectual, as well as intelligible.

is a medium between that which knows, and that which is known ; and since these two are the same, intelligence also will be one and the same with each of these. But since the essence of intellect is eternal (for the whole subsists at once) intelligence also will be eternal : for it is the same with the essence of intellect. But if intellect is eternal, it will by no means be measured by time, neither according to essence, nor according to energy. And since these subsist in the same manner, the power also of intellect is eternal.

## PROPOSITION CLXX.

**E**very intellect, at once understands all things. But an imparticipable intellect understands all things simply. And each of the intellects posterior to this understands all things according to one.

**F**OR if every intellect establishes its essence in eternity, and together with its essence, its energy, it will understand all things at once. And all things indeed exist according to parts, and a successive energy, which do not subsist in eternity. For every thing successive subsists in time ; since it possesses prior and posterior, which are successive, and do not subsist all at once. If, therefore, all intellects understand similarly, they will not differ from each other : for if they understand all things similarly, they are all things similarly ; since they are no other than the things which they understand. But if they are all things similarly, one intellect will not be imparticipable, and another not : for their essences are the same with the objects of their intellections ; since the intellection of each is the same with its essence, and every intellect is both intelligence and essence. It remains, therefore, either that every intellect does not equally know all things but one or more, and not all things together ; or that it knows all things according to one \*. But to assert that intellect does not understand all things, is to make it ignorant of some particular being. For if it is affected with transition, and does not understand *at once*, but according to prior and posterior, at the same time possessing an immovable nature, it will be inferior to soul, understanding all things according to motion, or a mutable energy ; because intellect on this hypothesis, will only understand one thing by its permanent energy. It will, therefore, understand all things according to one. For it either understands all things ; or one thing ; or all things according to one. And the intelligence indeed of all things perpetually subsists in all intellects : but they terminate

\* By an intellection *according to one* (μόνῳ) we must understand a various intelligence subsisting indivisibly, and without mutation. Just as when by one and the same energy of vision, we survey the various parts of the same countenance though distant from each other. And an intellection of this kind belongs to all intellects subordinate to the first : for the intelligence of this is perfectly simple, and comprehends all things in one.

all

all things, according to one intelligence of all. Hence there is something pre-dominant in intellect, and the objects of intelligence ; since all things are at once understood as one, through the dominion of one, which characterizes all things with itself.

## PROPOSITION CLXXI.

**E**very intellect is an imitable, or indivisible essence.

**F**OR if it is without magnitude, incorporeal, and immovable, it is imitable.

For every thing in any respect partible, is either partible on account of magnitude, or multitude, or on account of energies subsisting in time. But intellect is eternal according to all things, and is beyond a corporeal nature ; and the multitude which it contains is united. It is, therefore, imitable. But that intellect is incorporeal, is manifest, from its conversion to itself : for no body possesses a self-convertive power. But that intellect is also eternal, the identity of its energy with its essence evinces : for this we have already demonstrated. And that its multitude is united, is evident from the coherence of intellectual multitude, with the divine unities : for these are the first multitude, and after these intellects subsist. Hence though every intellect is a multitude, yet it is an united multitude. For prior to that which is divided, that which is collected, and is nearer to the one, subsists.

## PROPOSITION CLXXII.

**E**very intellect is the proximate sustaining cause of natures eternal, and immutable according to essence.

**F**OR every thing produced from an immovable cause, is immutable according to essence. But immovable intellect being all things eternally, and abiding in eternity, essentially produces that which it produces. But if it is perpetual, and subsists after the same manner, it will always produce, and according to one uniform energy. Hence it is not the cause of things which are sometimes beings, and sometimes not, but it is the cause of eternal beings.

## PROPOSITION CLXXIII.

**E**very intellect is intellectually both the things which are prior and posterior to itself.

**F**OR it is the same with things posterior to itself according to cause, and with things prior to itself by participation : but still it is intellect, and is allotted an intellectual essence.

essence. Hence it defines all things according to its essence; both such as subsist according to cause in another, and such as subsist according to participation. For every thing according to its natural constitution, participates of more excellent natures: but not according to the subsistence of its superiors. For these indeed are participated by all things, though in a different respect, according to the various natures of the participants. And hence participations subsist according to the characteristic and power of the participants: and consequently in intellect things prior to its nature, subsist in an intellectual manner. But intellect is likewise intellectually things posterior to itself: for it does not consist from its effects, nor does it contain these, but the causes of these in itself. But intellect is the cause of all things by its essence, and its essence is intellectual; and consequently it contains the causes of all things intellectually. Hence every intellect possesses all things intellectually; both such as are prior and such as are posterior to itself. As, therefore, every intellect contains intelligibles intellectually, so likewise it contains sensibles according to an intellectual subsistence.

## PROPOSITION CLXXIV.

**E**very intellect constitutes through intelligence natures posterior to itself: and its fabrication is contained in intellection, and its intelligence in fabrication.

**F**OR if intelligible and intellect is the same; hence the being of every intellect consists in self-intellection. But it fabricates that which it fabricates by its essence, and produces that which is, according to being; and consequently its productions arise from intelligence. For in intellect being and intelligence are one: because intellect is the same with every being which it contains. If, therefore, intellect fabricates by its essence, and its essence is intellection, it will operate through intelligence, and intelligence will subsist in energy in intellection. But this is the same with its essence: and its essence consists in operating. For that which operates immovably, always possesses its essence in operating: and consequently intellection consists in fabrication.

## PROPOSITION CLXXV.

**E**very intellect is primarily participated by those natures, which are intellectual, both according to essence, and according to energy.

**F**OR it is necessary that it should either be participated by these, or by other natures, which possess indeed an intellectual essence, but are not always intelligent. But it

is impossible that it should be participated by these latter. For the energy of intellect is immovable. And hence the natures by which intellect is participated, always participate of an intellectual energy, which always causes the participants to be intellectual. For that which possesses its energy in any part of time, cannot be conjoined with an eternity of energy. But as in essences themselves, so also in the variations of energies, between every eternal energy, and that energy which receives its perfection in some period of time, that energy intervenes which possesses its perfection through the whole of time. For progressions subsist nowhere immediately, but are produced through kindred and similar natures, both according to hypostases, and the perfections of energies. Every intellect, therefore, is primarily participated by those natures which are able to understand through the whole of time, and which possess a perpetual intelligence; though their intellection may subsist according to time, and not according to the stability of eternity.

## C O R O L L A R Y.

From hence it is evident that the soul which sometimes understands, and at other times is void of intellection, cannot proximately participate of intellect.

## P R O P O S I T I O N CLXXVI.

All intellectual forms subsist in one another, and each is at the same time separate and distinct from the rest.

**F**OR if every intellect is imitable, and the multitude which it contains is united through an intellectual imitability: hence all that intellect contains will entirely subsist in one, and imitables will be united to each other, and all intellectual forms will penetrate through all. But if all intellectual forms subsist immaterially, and incorporeally, they are without confusion with respect to each other, and each separately preserves its own purity, and abides that which it is. But the characteristic participation of each distinct participant, declares the unconfused subsistence of intellectual forms. For if participated natures were not distinguished, and separate from each other, neither would their participants participate them distinctly, but there would be a much greater indistinct confusion in subordinate natures, from their subsisting in a more degraded order. For from whence could distinction arise, if the natures which constitute and perfect these, should be indistinct and confused? Again, the hypostasis of that which contains imitably, and an uniform essence, attest the union of forms. For things possessing their hyparxis, in that which is imitable and uniform, subsist imitably in the same. For how can that be divided, which is imitable and one? For natures of this kind.

kind subsist together, and penetrate totally through each other, without distance: since that which contains, is not distant; and one thing is not in this place, and another in that, as in things separated by interval from each other. But every thing at once subsists in that which is impenetrable and one: and consequently they all subsist in each other. All intellectual forms, therefore, subsist unitedly in each other, and each is at the same time distinctly separate from the rest.

## C O R O L L A R Y.

But if any one besides the above demonstrations requires examples, let him contemplate the theorems resident in one particular soul; all which subsist truly in the same soul, in an essence destitute of magnitude, and are united to each other. For the soul does not contain the things resident in its nature, according to magnitude, and locally, but impenetrably, and without distance, unitedly, and distinct. For the soul produces all things distinctly, and each at the same time separate and apart, without attracting any thing to itself from the rest, which unless they were always distinguished according to habit, would not be distinguished by the energy of the soul.

## P R O P O S I T I O N CLXXVII.

Every intellect since it is a plenitude of forms, comprehends either more universal or more particular forms. And superior intellects contain in a more universal manner, whatever posterior intellects contain in a more particular manner. But inferior intellects, contain according to a more partial mode, whatever prior intellects contain more universally.

**F**OR superior intellects employ greater powers, because they are more uniform than secondary intellects. But inferior intellects, from their being more multiplied, diminish the powers which they possess. For such as are more allied to the one, being contracted in quantity, are superior in power to such as are posterior; while such as are more distant from the one possess a contrary property. Superior intellects, therefore, establishing a greater power, but a less multitude, produce more effects through forms, less according to quantity. [But \* intellects posterior to these, produce fewer

\* That part of this proposition, within the crotchetts is in one place very defective in the Greek, which may be corrected as follows: Instead of reading of δὲ μεί τελέους, διὰ πλείστων, ἐλασσων. κατὰ τὴν δύναμιν οὐκ εἰλαμψεῖν ἔκπειν διὰ ἑλασσόνων πλέοντα παραχρότων, ὀδικηθεῖν εἰς αὐτὸς ἔτιδε: read after πλείστων:—ἐλασσων, κατὰ τὴν δύναμιν ἐλαυνεῖν εἰς οὐν τελέους δι, and the rest as above.

effects, through a greater multitude of forms, on account of their deficiency in power. If, therefore, superior intellects produce more effects, through a less number of forms, the forms which they contain are more universal.] And if inferior intellects, produce fewer effects through a greater multitude of forms, the forms which they contain, are more particular.

## COROLLARY.

From hence it happens that the natures which are generated from the superior orders according to one form, are produced in a divided manner from secondary orders, according to a greater multitude of ideas. And on the contrary those natures which are produced from things subordinate, through many, and distinct forms, are produced by superior natures, through fewer, and more universal forms. And that which is universal and common, supernally accedes to all participants. But that which is divided, and peculiar proceeds from secondary natures. Hence secondary intellects by the more particular separation of characteristics, articulately distinguish, and attenuate the formations of primary intellects.

## PROPOSITION CLXXVIII.

**E**very intellectual form, is the framer of eternal natures.

**F**OR if every intellectual form is eternal, and immovable, it is essentially the cause of immutable and eternal hypostases; but not of such as subsist in generation, and are corruptible. And hence every thing fabricated according to an intellectual form, is an *intellectual eternal*. For if it produces all forms posterior to such as are intellectual, through being; and if the being of intellectual forms is eternally the same, their productions also will subsist after the same manner, and will be eternal. Hence neither the genera which according to some particular time, are fabricated by a formal cause, nor things corruptible, so far as corruptible, possess a pre-existent intellectual form. For they would be void of corruption and generation, if they possessed their hypostasis, according to a pre-existent intellectual form.

## PROPOSITION CLXXIX.

**E**very intellectual number is bounded.

**F**OR if there is another multitude posterior to this diminished according to essence, and so more remote from the one, while intellectual number is nearer to the one; and

and if that which is nearer to the one, is less according to quantity, and that which is far distant is more according to quantity; intellectual number also will be less than every multitude posterior to its nature. It is not, therefore, infinite: and so the multitude of intellects is bounded. For that which is less than another, is not infinite: because infinite, so far as infinite, is not less than any thing.

## PROPOSITION CLXXX.

Every intellect is a whole, as composed from parts, and is united with others, and at the same time distinguished from them. But imparticipable intellect is simply universal; and contains in itself, as it were all parts universally. But each particular intellect possesses the whole as in a part; and thus contains all things particularly.

FOR if it is all things according to one thing; and if that which is all things according to one, is something particular alone: hence, the whole subsists in each of these particularly, on account of something particular, determinately predominating in them all.

## PROPOSITION CLXXXI.

Every intellect which is participated is either divine, as depending on the gods; or is intellectual only.

FOR if there is a divine and imparticipable intellect, that which is primarily allied to this, does not differ from it in both these respects; that it is not divine, and that it is not imparticipable. For things dissimilar in both these respects, cannot be conjoined with each other. It is evident, therefore, that the medium between these, is partly similar to the first intellect, and partly dissimilar. Either, therefore, it is imparticipable, and not divine; or it is participated, and divine. But every thing imparticipable is divine, as being allotted an order in multitude, analogous to the one. And hence there will be some one intellect, divine, and at the same time participated. But it is requisite that there should be an intellect, not participating the divine unities, but intelligent only. For in every series, first natures, and which are conjoined with their unity, are able to participate their proximate superiors. But such as are far distant from their primary unity, cannot depend on the natures placed in an order proximately superior to their

their own. There is, therefore, both a divine intellect, and an intellect intellectual alone. And the latter subsists according to an intellectual characteristic which it possesses from its own unity, and from imparticipable intellect: but the former according to a union, which it receives from participated unity.

## PROPOSITION CLXXXII.

**E**very divine intellect, which is participated, is participated by divine souls.

**F**OR if participation renders the participant similar, and causes it to be allied to that which is participated; it is evident that that which participates a divine intellect, must be a divine soul \*. It is likewise evident that it must depend on a divine intellect, and that it must participate the deity which it contains, through intellect as a medium. For † intellect connects with deity (*θεόν*) its participant soul, and conjoins one divine nature with another.

## PROPOSITION CLXXXIII.

**E**very intellect, which is participated indeed, but is intellectual alone, is participated by souls neither divine, nor subsisting in a mutation from intellect, into a privation of intellect.

**F**OR neither are divine souls of this kind; nor such as participate of intellect. For souls participate of the gods through a divine intellect, as we have already demonstrated. Nor are such as participate of an intellectual intellect susceptible of mutation. For every intellect is participated by natures, which are always intellectual, both according to essence, and according to energy; as is evident from the preceding propositions.

\* The reader must observe that this is to be understood of immediate participation.

† Instead of *ι γαρ διοτε ευηγέρτει τῷ μ. κ. τ. λ.* as in the original, the sense requires that we should read *ι γαρ εὐηγέρτει τῷ διοτε. κ. τ. λ.*

## CONCERNING

## CONCERNING SOUL.

## PROPOSITION CLXXXIV.

Every soul is either divine, or capable of being changed from intellect into a privation of intellect; or it always remains as a medium between these, and is at the same time inferior to divine souls.

FOR if a divine intellect is participated by divine souls, but an intellectual intellect, by those souls alone, which are neither divine, nor susceptible of a mutation from intellect into a privation of intellect (for there are souls of this kind, which sometimes understand, and are sometimes destitute of intelligence); it is evident that there are three genera of souls. And the first indeed are divine. But the second are not divine, yet they always participate of intellect. And the third are those, which are sometimes changed into an intellectual condition, and sometimes into a privation of intellect.

## PROPOSITION CLXXXV.

All divine souls, are gods animaftically, ( $\psi\chi\nu\kappa\omega\varsigma$ , or according to the nature of soul). But all souls participating an intellectual intellect, are the perpetual attendants of the gods. And all souls susceptible of mutation, are some time or other attendants of the gods.

FOR if some souls posses a divine light, supernally illustrating their nature, but others are endued with perpetual intelligence, and others again, are sometimes only allotted this perfection: hence the first of these will among the multitude of souls, be analogous to the gods; but the second, will perpetually attend the gods, on account of their perpetually energizing intellect, and will depend on divine souls, to which they will have the same proportion, as that which is intellectual to that which is divine. And those which are sometimes endued with intelligence, will also sometimes attend the gods; but they will neither always participate intellect after the same manner, nor will they always be conversant with divine souls. For that which is only sometimes allotted intellect, cannot by any means always attend the gods.

P R Q P Q

## PROPOSITION CLXXXVI.

**E**very soul is both an incorporeal essence, and separable from body.

**F**OR if it knows itself, and if every thing self-gnostic, is converted to itself, and every thing converted to itself is not a body (for every body is incapable of self-conversion), nor inseparable from body; for every thing inseparable from body, is not naturally adapted to be converted to itself, since through this, it would be separated from body; hence every soul, is neither a corporeal essence, nor inseparable from body. But that the soul, knows itself, is manifest. For if it knows things superior to itself, and is naturally adapted to know itself, it will much more know itself, through causes prior to its own nature \*.

## PROPOSITION CLXXXVII.

**E**very soul is immortal and incorruptible.

**F**OR every thing which is capable in any respect of dissolution and dispersion, is either corporeal and a composite, or is allotted an hypostasis in a subject. And that indeed which is dissolved, is corrupted, as subsisting from many things. But that which is naturally adapted to subsist in another, when separated from its subject, vanishes into non-entity. But soul is both incorporeal, and external to every subject, residing in itself, and being converted to itself. It is, therefore, immortal, and incorruptible.

## PROPOSITION CLXXXVIII.

**E**very soul, is both life, and vital.

**F**OR that to which soul accedes necessarily lives; and that which is deprived of soul, is immediately left destitute of life. For it either lives through soul, or through something else, and not through soul. But it is impossible, that it should live through something else alone. For every thing which is participated, either communicates it-

\* For inferiors are comprehended in superiors, and particulars in universals; so that he who knows universals, knows particulars also; though the reverse of this is not true. The soul, therefore, by possessing a natural capacity of knowing herself, and things superior to her own nature, will from the illuminations attending her knowledge of the latter, know herself in a much more eminent and perspicuous manner.

self,

self, or something of itself to its participant. But if it should do neither of these, neither will it be participated. But soul is participated by that to which it is present : and that is called animated, which participates of soul. If, therefore, that which is participated confers life on animated natures, it is either life, or vital alone, or at the same time both life and vital. But if it is vital alone, and not also life, it will be composed from life, and non-life \*: and thus it will neither know, nor be converted to itself. For *life is knowledge*†; and that which is gnostic, or endued with knowledge, so far as it is gnostic, lives. If, therefore, there is any thing in soul destitute of life, this something will not essentially possess a self-gnostic power. But if soul, is life alone, it will no longer participate an intellectual life. For that which participates of life, is vital, and not life alone ; since that which is life alone, is first and imparticipable life. But life posterior to this, is vital, and at the same time life. And soul is not imparticipable life. It is, therefore, both life, and vital.

## PROPOSITION CLXXXIX.

**Every soul is self-vital.**

**F**OR if it is converted to itself, and every thing self-convertive, is self-subsistent, soul also is self-subsistent, and sustains itself. But it is also both life, and vital, and its hyparxis is according to vitality. For to whatever natures it is present, it communicates life, through its essence. And if the participant is adapted to participation, it immediately becomes animated and vital ; soul neither reasoning nor chusing, nor vivifying by reasoning and judgement, but by its essence alone communicating life to the participant. Hence the being of soul, is the same with its life. If, therefore, it possesses being from itself, and this is the same with its life, it will essentially possess life, and will afford life to itself, and will possess life from itself. But if this be the case, soul will be self-vital.

\* Thus for instance the body of the world, or of any particular animal, is *vital* from its receiving the echo of soul ; but as it does not at the same time, essentially possess *life*, (or else it would be soul) it is composed from life, and non-life. For it is nothing more than body, or non-life, united with the last image of soul, or a debile life.

† This truly divine sentence, is derived from the most profound theory ; and can alone be understood by those who have deeply studied the six books of Proclus on Plato's Theology. I shall, therefore, only observe for the sake of the *intellectual reader*, that as *essence*, *life*, and *intellect* subsist in occult union, in the first being, which comprehends the highest order of the gods ; and as *intelligence* is the medium between that which is *intelligible*, and *intellect* ; it is evident that life itself is intelligence.

## PROPOSITION CXC.

**E**very soul is a medium between natures impartible, and such as are divisible about bodies.

**F**OR if it is self-vital, and self-subsistent, and has an hyparxis separate from bodies, it is separated from, and is more excellent than all partible natures, subsisting about bodies. For these are entirely inseparable from their subjects; because they are divided together with divisible weights, depart from themselves, and their own imparibility, and are co-extended with bodies. And though they subsist in vital natures, yet these are not the lives of partible essences, but of their participants: and though they abide in essence and forms, yet these are not their own forms, for they are forms of formed natures. Soul, therefore, is a self-subsistent, and self-vital essence; it is likewise a knowledge, gnostic of itself, and according to all these separable from bodies. But it likewise participates of life: and if this be admitted, it likewise participates of essence. But it participates also of knowledge from other causes. And hence it is evident, that it is worse than impartibles, because it is filled with life externally: and if with life, it is evident that it is also externally replenished with essence. For prior to every particular life, imparticipable life, and imparticipable essence subsists. But it is likewise manifest that soul, is not the first gnostic nature. For every soul so far as soul, possesses life indeed, but not knowledge also from its existing as soul. For certain souls, while they remain as souls, are at the same time ignorant of beings. Soul, therefore, is not the first gnostic nature, nor does it possess knowledge on account of its essence. And hence it possesses an essence the second from those, which are primarily, and essentially gnostic. Since, therefore, the essence of soul is divided from its knowledge, it does not rank among natures purely imparible. But it has been demonstrated, that neither does it subsist in the order of things divisible about bodies. It is, therefore, situated between both.

## PROPOSITION CXCI.

**E**very participable soul possesses an eternal essence, but its energy subsists in time.

**F**OR either it possesses both eternally, or both temporally; or one eternally, but the other temporally. But it cannot possess both eternally: for on this hypothesis, it would be an imparible essence; and the nature of soul would differ nothing from an intellectual

tellelctual hypostasis; viz. a self-motive from an immoveable nature. Nor can it possess both its energy and essence in time: for thus it would be generated alone; and would neither be self-vital, nor self-subsistent. For nothing measured by time is essentially self-subsistent. But soul is self-subsistent. For that which is converted to itself according to energy, is also essentially converted to itself, and proceeds from itself. It remains, therefore, that every soul is partly eternal, and partly a participant of time. It is either, therefore, eternal according to essence, but participating of time, according to energy; or the contrary. But this latter hypothesis is impossible. Every participable soul, therefore, is allotted an eternal essence, but possesses an energy according to time.

## PROPOSITION CXCII.

**E**very participable soul, ranks in the number of eternal beings, and among the first of generated natures.

**F**OR if it is eternal according to essence, it is true being according to its hyparxis, and is a perpetual being. For that which participates of eternity, participates likewise of perpetual being. But if it subsists in time according to energy, it is generated. For every thing participating of time, is always in generation (or in becoming to be) according to the prior and posterior of time, and is not at once, that which it is, but the whole of it is generated. But if every soul, is in a certain respect generated according to energy, it will be the first of generated natures. For that which is entirely generated, is more remote from eternal natures.

## PROPOSITION CXCIII.

**E**very soul subsists proximate to intellect.

**F**OR if it possesses an eternal, and immutable essence, it proceeds from an immoveable essence: since that which proceeds from a moveable essence, is entirely changed according to essence. The cause, therefore, of every soul is immoveable. But if it is proximately perfected by intellect, it is also converted to intellect, and participates the knowledge, which intellect confers on the natures able to participate cognition. For all knowledge proceeding from intellect, is inherent in all the natures, in which intellect resides. But that to which all things are naturally converted, is the source of their progression according to essence. Every soul, therefore, proceeds from intellect.

## PROPOSITION CXCIV.

**E**very soul possesses in a secondary manner, all the forms, which intellect primarily contains.

**F**OR if it proceeds from intellect, and intellect is the fabricator of soul; and if intellect subsisting immovably produces all things; it will also impart to soul, which it constitutes, the essential reasons of all things which it contains. For every thing which operates through essence, imparts secondarily to its production, that which it is itself primarily. Soul, therefore, contains in a secondary manner the representations of intellectual forms.

## PROPOSITION CXCV.

**E**very soul is all things, containing sensible natures, after the manner of an exemplar; ( $\pi\alpha\varphi\alpha\delta\epsilon\gamma\mu\alpha\lambda\omega\varsigma$ ) but intelligibles after the manner of images ( $\epsilon\iota\kappa\omega\iota\kappa\omega\varsigma$ ).

**F**OR subsisting as a medium between natures imitable, and such as are divided about bodies; it produces and constitutes the latter of these; but establishes in itself the prior causes from which it proceeds. Hence it previously receives after the manner of an exemplar the natures to which it is prior as their cause: but it possesses through participation, and as the blossoms of first natures, the causes of its subsistence. It previously receives in its essence, therefore, through cause all sensible natures, and contains immaterial reasons of things material, incorporeal of such as are corporeal, and indistinct of such as are distinguished by interval. But it contains intelligibles after the manner of an image, and receives partibly, their imitable forms, such as are uniform variously, and such as are immoveable according to a self motive condition. Soul, therefore, is all beings; containing such as are first through participation, but such as are posterior to its nature, after the manner of an exemplar.

## PROPOSITION CXCVI.

**E**very participable soul primarily uses an eternal body, which possesses an unbegotten and incorruptible hypostasis.

**F**OR if every soul is eternal according to essence, and through its essence first animates some particular body, it will always animate this body: for the essence of every

every soul is immutable. But if this be admitted, that which is animated must be always animated, and must always participate life. But that which always lives, is perpetual by far the first of all things\*. And that which is perpetual is eternal. Hence that body which is first animated, and which first depends on soul, is eternal. But every participable soul is primarily participated by some particular body; since it is not imparticipable, and essentially animates its participant. Every participable, or participated soul, therefore, uses a body primarily eternal, without generation, and incorruptible according to essence.

## PROPOSITION CXCVII.

Every soul is *an essence* vital and gnostic, and *a life* essential, and gnostic, and is both knowledge, essence, and life. It likewise contains all things together, the essential, the vital, and the gnostic; and all in all, and each separate and apart from the rest.

FOR if it has a middle subsistence between forms imparible, and such as are divisible about bodies; it is neither so imparible as all intellectual natures, nor so parible, as corporeal forms. Since, therefore, both essences, lives, and cognitions, are distributed in corporeal natures; all these subsist imparibly in souls, unitedly, and incorporeally, and are at the same time all things, on account of their immateriality, and imparibility. And since all things subsist in intellects according to union, they are distinguished and divided in souls. All things, therefore, subsist together, and apart in soul. But if all imparibles subsist together and in one, they mutually penetrate through each other: and if separate they are again divided without confusion; so that each subsists by itself, and all in all. For in essence there is both life and knowledge: since if essence was essentially deprived of life and knowledge, every soul would not know itself. And in life there is both essence and knowledge. For life without essence and knowledge belongs to material lives, which are neither able to know themselves, nor are sincere and pure essences. And knowledge which is both destitute of essence and life, is incapable of self subsistence. For all cognition belongs to that which is vital, and which is allotted essence essentially.

\* Because eternity subsists in life, and is no other than infinite life, as is copiously proved from Plotinus, in the note to Proposition LII.

## PROPOSITION CXCVIII.

Every thing which participates of time, and is always moved, is measured by periods.

**F**OR since it is measured by time it both participates a measure and bound of motion, and proceeds according to number. But because it is always moved, and this *always*, is not eternal \*, but temporal, it is necessary that it should use periods. For motion is a certain mutation from some things into others. But beings are terminated by multitudes and magnitudes. And these being terminated, there can neither be an infinite mutation, according to a right line, nor can that which is always moved proceed according to a finished progression. Hence that which is always moved will proceed from the same to the same ; and will thus form a period in its progression.

## PROPOSITION CXCIX.

Every mundane soul uses periods and restitutions of its proper life.

**F**OR if it is measured by time it operates transitively, and possesses a proper motion. But every thing which is moved and participates of time, when it is eternal, uses periods, revolves periodically, and proceeds from the same to the same. And hence every mundane soul, possessing motion and energizing according to time, will both possess periods of motion, and restitutions into its pristine state. For every period of eternal natures, returns to its pristine state.

## PROPOSITION CC.

Every period of soul is measured by time. But the period of particular souls, is measured by some particular time : and the period of the first soul, since it is measured by time, is measured by universal time.

**F**OR if all motions contain prior and posterior, they participate of a period, and on this account of time. And that which measures all the periods of souls is time.

\* For that which is properly eternal, is perfectly stable, and is never subject to mutation.

But

But if the periods of all souls were the same, and about the same ; the time of all would be the same. But if the restitutions of different souls are different, the periodic time of their restitutions also, is different. That the soul, therefore, which is first measured by time, is measured by universal time, is evident. For if time is the measure of every motion ; the first motion, will entirely participate of time, and will be measured by the whole of time. For if universal time, did not measure its first participant, neither would it measure any thing else, according to the whole of itself. But that all other souls are measured by the more particular measures of universal time, is evident from what we have now demonstrated. For if they are more particular than the soul which first participates of time, they cannot accommodate their periods to universal time. But the multitude of their restitutions, will be parts of that one period and restitution, by which the first participant of time, returns to its pristine state. For the participation of a lesser power is more particular, but of a greater, more universal. Other souls, therefore, are not naturally adapted to receive a universal temporal measure, through one life ; since they are allotted an order more remiss than that which is first measured by time, because they are allotted an inferior order..

## PROPOSITION CCI.

All divine souls possess triple energies ; one kind as souls ; another as receiving a divine intellect ; and a third kind, as depending on the gods. And they provide indeed for the universe, as gods ; but they know all things through an intellectual life ; and move bodies through a self-motive essence.

FOR since they naturally participate supermundane natures, and are not simply souls, but divine souls, bearing before themselves an order analogous to the gods, in an animastic latitude ; they will energize not only animastically, but also divinely ; because they are allotted a deified summit in their essence, and possess an intellectual hypostasis, through which they are spread under intellectual essences. They energize, therefore, not only divinely, but also intellectually ; possessing one energy according to *the one*, which they contain in the recesses of their natures, but another according to an intellectual operation. There is likewise present to these divine souls, an energy according to their proper hyparxis ; which is motive of natures moved by others, but vivific of such as possess an adventitious life. For this is the proper employment of every soul ; but such energies as intelligence and providence, they receive through participation.

PROPO-

## PROPOSITION CCII.

All souls attending upon, and always following the gods, are inferior to divine, but more eminent than particular souls.

FOR divine souls participate of intellect and deity. They are, therefore, at the same time intellectual and divine, and preside over other souls, in the same manner as the gods preside over the universality of things. But particular souls are deprived of a suspension from intellect, because they are not able to participate proximately of a divine essence. For they would not fall from an intellectual energy, if they essentially participated of intellect, as we have previously demonstrated\*. Hence the souls, which always follow the gods, are of a middle condition; participating indeed a perfect intellect, and through this surpassing particular souls, yet not depending on the divine unities. For the intellect which they participate is not a divine intellect.

## PROPOSITION CCIII.

Of every animastic multitude, (i. e. a multitude belonging to souls) divine souls since they are greater than others in power, are contracted according to number. But such as always follow the gods, retain a middle order among all souls, both in power, and quantity. And particular souls, are inferior to others in power, but proceed according to a greater number.

FOR divine souls are more allied to the one, on account of a divine essence; but those of a middle order, through the participation of intellect. And those of the last order, are essentially dissimilar to those of the middle and first kind. But among eternal natures such as are nearer to the one, are more united in number, and are more contracted in multitude, than such as are more distant. But such as are more remote, are more multiplied. Hence the powers of superior souls, are greater, and have the same proportion to secondary souls, as that which is divine to that which is intellectual, and as the intellectual to the animastic nature. And the quantities of inferior souls, are more in number. For that which is more distant from the one, is a greater, and that which is nearer a less multitude.

\* In Proposition cxxxiv. And from hence it appears, that by particular souls in this Proposition, are meant such as are capable of being changed from the possession of intellect, into its privation.

PRO-

## PROPOSITION CCIV.

**E**very divine soul presides over many souls, the perpetual attendants on the gods ; and over a still greater number of such as sometimes receive this order.

**F**OR if it is divine, it is requisite that it should be allotted an order, generative of all things, and first-operative among souls. For that which is divine, throughout all beings, presides over the universality of things. And it is requisite that it should neither alone preside over such souls, as perpetually follow the gods ; nor alone over such as are sometimes their attendants. For if any divine soul alone presides over such souls as sometimes attend the gods, how can these be united with a divine soul ; since they are entirely different from this, and neither proximately participate intellect, nor (by a much stronger reason,) the gods ? But if it alone presides over such as perpetually follow the gods, how can the series proceed to souls, the partial attendants on the gods ? For thus intellectual natures will be the last, and will be unable through their barrenness, both to perfect other natures, and reduce them to their original. It is necessary, therefore, that such souls as follow the gods, and energize through intellect, and are reduced to intellects more partial than divine intellects, should first depend from every divine soul. But the second to these are partial or particular souls, which are able through the former, as mediums, to participate intellect, and a divine life. For through those which always participate, those which sometimes participate a more excellent condition, are perfected. And again, it is necessary, that about every divine soul, there should be a greater number of souls which sometimes follow, than of those which always attend on the gods. For the power of unity, always proceeds into multitude, according to remission, and subjection ; failing indeed in power, but excelling in number. Since in a similar manner every soul perpetually following the gods, presides over a greater multitude of particular souls, imitating a divine soul ; and elevates many souls to the first-operative unity of the whole series. Every divine soul, therefore, presides over a multitude of souls, the perpetual attendants on the gods : but presides over a still greater multitude of such as are sometimes allotted this order.

## PROPOSITION CCV.

Every particular soul has the same proportion to the soul to which it is subjected according to essence, as the vehicle of the one to the vehicle of the other.

FOR if there is a natural distribution of vehicles in all souls, it is necessary that the vehicle of every particular soul, should have the same proportion to the vehicle of a universal soul, as the essence of the one, to the essence of the other. But the distribution of vehicles is according to nature: for first participants are naturally conjoined with the things participated \*. If, therefore, as a divine soul is to a divine body, so is a particular soul to a particular body, each being participated essentially; hence that is true, which was asserted in the beginning, that vehicles also have the same proportion, as their correspondent souls.

## PROPOSITION CCVI.

Every particular soul, possesses a power of descending infinitely into generation, and of ascending from generation to *being*.

FOR if it sometimes follows the gods, but sometimes falls from its pursuit of a divine nature, and alternately participates of intellect, and a privation of intellect; it is evident that it is conversant by parts in generation, and with the gods. But since it does not reside with the gods, through an infinite time, neither will it be conversant with bodies, through the whole succeeding time. For that which has no temporal beginning, cannot have any end: and that which has no end, is necessarily without a beginning †. It remains, therefore, that every soul must perform periods, both of ascensions from generation, and of descensions into generation; and that this will never fail, through an infinite time. Every particular soul, therefore, is capable of descending and ascending in infinitum: and this passion never ceases to take place about every particular soul.

\* The reader must observe that these vehicles or divine bodies, the first participants of their correspondent souls, are no other than those vehicles, so beautifully described by Synesius, and inserted in the preceding history of Theology.

† For a demonstration of the truth of this sentence, see the note to page 59, vol. I. of this work.

PRO-

## PROPOSITION CCVII.

The vehicle of every particular soul is fabricated by an immovable cause.

FOR if it eternally depends on the soul, by which it is used, and is by a natural sympathy immutable according to essence, it is allotted a subsistence from an immovable cause. For that which is produced from moveable causes, is wholly changed according to essence. But every soul possesses an eternal body, which is the first participant of its nature. Hence the cause of every particular soul\*, and consequently of its vehicle, is immovable, and on this account super-mundane.

## PROPOSITION CCVIII.

The vehicle of every particular soul, is immaterial, indivisible according to essence, and impassive.

FOR if it proceeds from an immovable fabrication, and is eternal, it possesses an immaterial and impassive hypostasis. For such things as are naturally passive according to essence, are all of them changed, and material: and from their subsisting differently at different times depend on mutable causes. And on this account they receive all various mutation, because they are moved with their primary causes. But that this vehicle is indivisible is manifest. For every thing which is divided, is corrupted so far as it is divided, because it relinquishes the whole, and departs from continuity and conjunction. If, therefore, the vehicle is essentially immutable, it will also be impassive, and indivisible.

## PROPOSITION CCIX.

The vehicle of every particular soul descends indeed with the addition of material vestments †; but is conciliated with the soul, by

\* Instead of οἵς καὶ ἡ μητρὸς φύσις, καὶ τοῖς αἰτίαις. n. r. λ.—I read, οἵς τοῖς αἰτίαις τοῦ μητρὸς φύσεως, καὶ τοῖς. n. r. λ.

† χρήματα is erroneously printed in the Greek, instead of χαράκην.

an ablation of every thing material, and by returning to a form proper to its nature, and analogous to the soul by which it is employed.

**F**OR the vehicle indeed descends, assuming irrational lives, but in its ascent, casts aside all the powers of generation, with which it was invested in its descent, and becoming [*pure returns to its proper form, and the pristine condition of its nature. It likewise\**] imitates the lives of the souls which employ it as an instrument, and is every where moved in conformity, with their motions. And by its circulations, it represents the intelllections of some souls, but the falling of others, through their inclination to the realms of generation; and the purgations of others through the revolutions which lead to an immaterial nature. But because it is essentially vivified by, and is connate with souls, it is all-variously changed along with their mutations; follows them every where; becomes passive, when they are exposed to passivity; returns with them when they are purified; and is elevated when they are elevated, and pursues its proper perfection. For every thing is perfected, when it pursues the perfection of its nature.

### PROPOSITION CCX.

Every connate vehicle of the soul, possesses both a form and magnitude perpetually the same. But it appears to be both greater and less, and endued with a dissimilar figure, through the additions and ablations of other bodies.

**F**OR if it derives its essence from an immoveable cause, it is evident that both its figure and magnitude is derived from this cause: and each is immutable and invariable. But it appears differently at different times, as likewise greater, and less. Hence through the intervention of other bodies added from the material elements, and again taken way, it exhibits a different appearance both in quantity and form.

### PROPOSITION CCXI.

Every particular soul, descending into generation descends totally. Nor does any part of it remain on high, and another part descend.

**F**OR if any thing belonging the soul remains in the intelligible world, it either perpetually understands without transfiguration, or transitively. But if without transfiguration,

\* That part of this proposition, within the crotchet, is neither in the Greek which is here defective, nor in the translation of Patricius, but is added from my own conjecture; whether or not with propriety, the reader may determine.

it

it will be intellect, and not a part of the soul; and this particular soul will be that which proximately participates of intellect. But this is impossible. And if transitively, that which is perpetually, and that which is only sometimes intelligent, will form one essence. But this likewise is impossible: for all these differ, as we have previously shewn. Add too, the absurdity which results from supposing that the summit of the soul is perpetually perfect, and yet does not rule over the other powers, and give them perfection. Every particular soul, therefore, totally descends.

T H E   E N D.

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## A P P E N D I X.

WHEN I first determined to give my labours to the public, in hopes of contributing to the restoration of the Platonic philosophy, I embraced the resolution of Dr. Johnson and Goldsmith, *to set the Reviewers at defiance*. For I was fully convinced that neither able criticism, nor candid attention could be expected, where composition is dictated by the spirit of malevolence, and influenced by the views of pecuniary reward. However, though contempt is the most philosophical mode of revenge, yet as a certain author well observes severe retaliation is sometimes requisite, in order to convince the subjects of our revenge, that we do not stoop to the meanness of abject submission. This mode of retaliation the defamation of the *Monthly Reviewers* in their *bundle of criticism* for August last obliges me to adopt: and they have afforded me in this review the most favourable opportunity I could desire, of exposing their *malevolence, ignorance, and pride*. I shall begin, therefore, with instancing their *malevolence*, as it is the first in our list of their bad qualities, and is the general characteristic of these assuming critics. In my preface to the translation of *Orpheus*, after representing the difficulty of well translating the compound epithets of the Greek, into English, and the necessity of possessing the philosophic genius for this purpose. I add: “*If some sparks of this celestial fire, shall appear to have animated the bosom of the translator, he will consider himself as well rewarded for his laborious undertaking.*” Upon which these *candid* reviewers observe, (p. 138.) “Mr. Taylor was aware of this difficulty, though he seems to claim the merit of subduing it.” In the second place they assert, (p. 138.) that after lamenting, that the Commentary of Proclus on Plato’s *Cratylus* is not likely to be published, “I comfort myself with the hope that my own labours will in some measure supply its place, by opening the pure sources of genuine wisdom. And that to this end I promise copious and truly philosophic notes.” Now the passage which furnished this malevolent assertion is the following: “What farther light we have been able to throw on these mysterious remains of antiquity, will appear in our following notes. If the valuable Commentary of Proclus on the *Cratylus* of Plato, was once published, I am persuaded we should find them full of the most recondite theology: but as this is not to be expected in the present

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sent age, the lovers of wisdom will I doubt not gratefully accept the preceding and subsequent elucidations. For on a subject so full of obscurity as the present, a glimmering light, is as conspicuous, and as agreeable to the eye of the mind, as a small spark in profound darkness, is to the corporeal sight." Dissertation, p. 106. The infamy of such misrepresentation is too glaring to require any illustration, too shameful to admit of any excuse, and in any other cause than that of verbal criticism, too contemptible either to rouse resentment, or deserve the most trifling attention. Let us now examine the specimens of *ignorance* which these *Reviewers* afford in great abundance; and which as I presume will appear much to the credit of my translation. In the first place I am charged with "universally translating the epithets φλεγός, φλεγτος, and μανικός, by the word *fanatic*, which I have employed in the sense of the Latin word, from which it is derived." To which I reply, that the former part of this charge is false. For in the hymn to Minerva φλεγός is translated *rage*; in the hymn to Diana, *fierce*; and in the hymn to Dionysius Baffareus, μανικός is translated *furious*. The latter part of this assertion is true. For as the word *fanatic* is immediately derived from the Latin word *fanaticus*, which according to their own confession means *numine affatus*, or *one inspired by a divine power*; and as the great Scaliger, whose authority is always decisive, constantly translates φλεγός, *fanaticus*, I made no scruple of adopting it in my translation. That, *fanatic* is never used in a good sense by any author of repute may perhaps be true: but I see no reason why it should not be employed according to the meaning of its original, especially as there is no other word in our language so expressive of the words to which it corresponds in the Greek. The example of Aristotle, and the greatest men of antiquity sufficiently justifies both the invention of new terms when the poverty of a language requires a supply, and the adoption of old ones in a different sense, when the difficulty of the subject demands verbal innovation. After this I am accused of *totally mistaking* the meaning of various passages, the greater part of which I shall expose to the view of the reader with a literal translation, and comment; that the ignorance of the *Reviewers* may appear without that veil which at present screens it from the eyes of the unlearned in Greek. In the hymn to Pluto then, I have translated the following line:

Μένος εφεκτον εργαν Φανερόν τι βραβεύεις.

Of unapparent works thou art alone  
The dispensator visible and known.

That is, literally, "Thou art alone the dispensator of apparent and unapparent works." Now there is nothing in my version can be objected to, but the omission of the word *apparent*, which the measure of the verse obliged me to neglect; and which the addition of *visible* and *known* in the second line renders superfluous, as the following observations:

observations will evince. According to the Orphic theology, *Pluto* belongs to the same order as the *sun*, and from his subsisting in occult union with this deity, he is celebrated as one and the same : a custom frequent with the Orphic theologists, as is well known to those who are skilled in their writings. Hence considered as the *sun*, he is the dispensator of *apparent*, and as *Pluto*, of *unapparent* works : and thus I presume, I have not *totally mistaken* the meaning of this line, in celebrating Pluto as a deity *visible and known*. But that the reader may be fully convinced of the truth of this assertion, concerning the occult union between Pluto and the sun, let him attend to the following Orphic verse, preserved by Justin Martyr, (in Cohortat. ad Gentes).

ΕἽ: ζῶς, οἵ ἀδει, οἱ ἥπα, οἱ δίνος. —————  
i. e. " Jupiter, Pluto, the Sun, and Bacchus are one."

Again, in the epithet *σύλαχος*, it seems I have *totally mistaken* the meaning of my author, by translating it *honored light*. This word means literally *exceedingly honoured*: and the preceding exposition sufficiently proves the propriety of calling *Pluto*, *lucid*. Every reader knows the necessity there is in poetical translations of adding something to the original : and this is always allowed, when the addition is not contrary to the sense of the text, but either expands it, if condensed, or enlightens it, if obscure. I am likewise charged with mistaking the meaning of *λόγος Γράμμος*, *τροφή*, or, *prophet of discourse to mortals*, which I have rendered,

—————' Prophet of discourse.'

Now as this is literal, the mistake must consist in not substituting another word for *prophet*, which might express what the author meant ; the *Reviewers* never dreaming that this word, when properly understood, is perfectly sufficient for the purpose. As they appear, therefore, to be *totally ignorant* of the original signification of a prophet, I shall subjoin its definition from Festus. "*Prophetas dicebant veteres antistites fanorum, oraculorumque interpres?*" i. e. "the ancients called prophets the priests of fanes, and the *interpreters of oracles*." *Prophet of discourse*, therefore, means *interpreter of discourse* : and as this epithet is applied to Mercury, it is doubtless highly proper ; if we consider that he first reduced the infinity of voice into bound, by dividing letters into species ; and thus truly became the interpreter of speech to mankind. In the hymn to Venus, I have translated,

Εἰτ' εἰ Κύπρη, αὐτοῦ τροφή στο —————  
"Or if in Cyprus with thy mother fair."

And it is literally "Or if in Cyprus O queen, with thy nurse." Fortunately for me, the *metaphrase* of Scaliger agrees with my version, "Sive in Cupro, matre tua." Perhaps the

the Reviewers forgot, or perhaps they are ignorant, that a mother and a nurse are frequently synonymous terms! I shall not trouble the reader with any more instances of my mistakes, as I can faithfully assure him, that the remaining passages adduced by the *Reviewers*, betray *if possible*, more malevolence and ignorance than the present. I shall, therefore, proceed to a defence of some epithets, and expressions which I have employed; and in which these *exquisite critics*, can neither discover *beauty*, nor even *propriety*.

In the first place then, they confess that they have too little *taste*, or too little *knowledge* to discover either *beauty*, or *propriety*, in my translation of the following line:

*Νύμφαι θυγατερες μεγαληπορος Ωκεανοιο* —————

‘Nymphs, who from *ocean's stream* derive your birth.’

i. e. literally, ‘Nymphs, daughters of the mighty ocean.’ Now as the exceptionable part of this line, is *ocean's stream*, as appears by its being printed in *italics*; I can only assure the reader that I can plead no less authority than that of both *Homer*, *Hesiod*, *Plato*, and *Milton* for its *propriety* and *beauty*. Thus *Homer*, (Iliad xviii. l. 606.) speaking of the fabrication of Achilles' shield by Vulcan, says :

*Ἐν δ' ἐτίθει ποταμοῦ μεγασθενος ὥκεανοιο.*

i. e. ‘But he placed in it the mighty strength of the *ocean's stream*?’

So likewise : (Iliad xx. l. 7.)

*Οὐτέ τις ἐν ποταμῶν ἀπένν νόσφ' ὥκεανοιο.*

i. e. ‘No *stream* was absent, except the *stream of the ocean*?’

Thus again, in the *Odyssley* : (lib. xi. l. 637.)

*Τὴν δὲ κατ' Ὀκεανὸν ποταμὸν φέρε κῦμα ἔσσιον.*

i. e. ‘But the waves of the current bore it (the vessel) through the *ocean stream*?’ And *Milton* had doubtless an eye to this last passage, when, speaking of the *Leviathan*, (*Paradise Lost*, book I.) he says :

————— or that sea beast  
Leviathan, whom god of all his works  
Created hugeft, that swim th' *ocean stream*.

For here, as the reader must observe, he uses the very same expression with *Homer*. But *Milton* was not only a great poet, but a man of great learning; and was doubtless much better acquainted with *Homer* than the *Reviewers*.

Thus too Hesiod: (in Theog. l. 241. &c.)

———— καὶ Δωρίδες ἡύχομοιο,  
Κύρης Ὄνταροῦ τελέκτος πολαρμοῖο.

i. e. ‘and from the fair haired Doris, the daughter of the perfect stream of the ocean.’ And the same epithet is used in l. 959. of the same work. And lastly, Plato in the Phædo, thus speaks of the ocean, as one of the four great rivers, of which Tartarus is the source: τὰ μὲν ἦν ἀλλα πολλά τε καὶ μέγαλα καὶ παντοδαπά ἔμφατά. οὐ. τυγχάνει δ' αὖτε ἄλλα :: τέλος τοῖς πολλοῖς τέττας ἀττα ἔμφατα, ὃν τὸ μὲν μέγιστον καὶ εἰντάται ἔισι. περιπλάνων, ὁ παλαιότερος Οὔπειρος ιστι. i. e. ‘There are many other both great and all-various rivers, but principally four; the greatest and last of which, flowing round the earth in a circle, is called the ocean.’

I only add that this expression is perfectly philosophical, as will be evident from considering the ever-flowing condition of the ocean, by means of which it admirably corresponds with the nature of a stream. Homer indeed was so sensible of this truth, that he generally (if not always) speaks of the ocean in this manner; and there is no doubt, but he derived his conviction from the first and most profound philosophy in the world. After this the expression, a *blameless tide* of abundance is objected to. But if the epithet *blameless* may be applied to *abundance*, which it is in the original; (*Ἄρδες αμυητη*) and if a *tide* of wealth, is an usual expression, I see no reason why abundance, when conferred with moderation, may not be said to be poured in a *blameless tide*. The objections to the translations of (*Ὥμηρος σημειώσεως*) ‘basis of mankind,’ and the first part of the hymn to Protogenus, are too contemptible to deserve any reply. This too would be the case with the epithet ‘*Bacchic King*,’ which is literally translated from the Greek; (*Βακχῖος ἄναττα*), but very fortunately these sagacious critics have employed a correspondent expression, in their Review of Wharton’s Milton: for in page 1. they speak of the *Miltonic muse*, which I presume must fall under the same imputation of *impropriety*, and want of *beauty* with *Bacchic king*. I shall only adduce one instance more, and then proceed to take notice of the pride of these uncandid and *ignorant* censors. In the hymn to Boreas, that deity is requested to dissolve the all-misty station of the air:

Δὺς τε πανηθεῖλος σασσος νέρος —————

Which I have accordingly translated,

‘The *misty station* of the air dissolve.’

And I must confess, that as I cannot find the least impropriety in speaking of the air as being in a *misty station*, I must conclude that this was exactly the *station* of the Reviewers, at the time when they composed the present criticism; the whole of which appears to have been the result of *misty visions*, *clouded conceptions*, and *uncertain conjectures*.

Let

Let us now proceed to a *review* of their *pride*. In the first place then, they very pompously inform us of their natural gravity as follows: ‘Grave though we be, our own risibility has been provoked,’ &c. As if it was of any consequence to the public, whether they are *grave* or *facetious*, *solemn* or *ludicrous*, *fanguine* or *bilious*: whether they possess the qualities of the *owl*, or the *ape*; and whether they laugh like the tickled *Hyæna*, or like Milton’s *death* ‘grin horribly a ghastly smile.’ In the next place, after having praised my paraphrase of Plotinus on the Beautiful, they add: ‘this praise ought to convince Mr. Taylor, that we are neither insensible to the real value of his author’s work, nor blind to the merits of the translation’. As if the praise of a Reviewer could be of any importance to a man, whose writings, are not calculated for the multitude: or as if the censure of ignorant judges, was not preferable to their most unbounded approbation! I only add that from men who are critics by profession on the writings of others, the most perfect composition may be justly expected: and yet the *Monthly Reviewers* have grossly failed in this respect, as the following instances will evince: Polybius makes use of the expression, *μετα τροπης ειναι τας των αυτων σκοτωσι*, i. e. ‘to verge to one and the same end’: and this our admirable critics *translate* (p. 122.) ‘to verge to one point, and conspire to one end’: which is obviously a most ridiculous tautology. For it is impossible that any thing can verge to one *point*, and at the same time conspire to an *end*, different from that *point*. Again, in their review of Bell’s Shakespeare, (p. 156.) they make use of the following simile: ‘Shakespeare, now stands (among the French) as a *Colossus*, while the most that can be done by Voltaire, and indeed the very best of our modern writers at home, is to *creep under his feet*.’ But here we may very justly enquire, what similitude there is between *modern wits* endeavouring to imitate Shakespeare, considered as a *dramatic writer*, and *men* crawling under his feet, considered as a *Colossus*? If Shakespeare indeed had been a quadruped, men by creeping under his feet might be considered as his *groveling* imitators: but I cannot conceive any similitude between a *creeping*, and an *upright* figure. I only add, that the Analytical Reviewers, are not more fortunate in their review of my translation of Proclus. For after asserting that the original is not remarkable for its elegance (though the contrary is the opinion of the best ancient and modern writers) and that I have too faithfully copied my author in this respect, they inform us, among other *interesting* particulars, ‘that the employment of an ancient philosopher did not consist in relieving the *distresses* of the *wretched*, and the *wants* of the *miserable*!’ After such a specimen of tautology, we cannot wonder that Proclus is considered as an inelegant writer: for though his language is always overflowing and majestic, it never degenerates into weak and needless repetition. While on the other hand, there is such a *perfect* sameness, in the above sentence, that, ‘to relieve the *distresses* of the *wretched*, and the *wants* of the *miserable*,’ is indeed no other, than ‘to verge to one point, and conspire to one end?’

And

## A P P E N D I X.

And thus much for the Reviewers, whom in any other cause than that of verbal criticism, I should consider as too mean for censure, and even too insignificant for contempt. For what attention can those writers deserve, who decide dogmatically on subjects they have never studied; who endeavour by malevolent aspersions to ruin the reputation of men they have never seen; and who abuse the credulity of the ignorant, by a monthly compilation of criticisms, which originate from vanity, and ultimately tend to illiberal gain?

T H E E N D.

T. A.